GENERAL MATHEMATICS 2

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October 5, 2022

Main Contents

Areas

Review

Graph of Some Functions

(1) Lines

The general linear equation in two variables x and y can be written in the form:

$$ax + by + c = 0 \quad \text{OR} \quad y = mx + b$$

where a, b and c are constants with a and b not both 0.

Example: $2x + y = 4$

$$a = 2, \; b = -1, \; c = -4$$

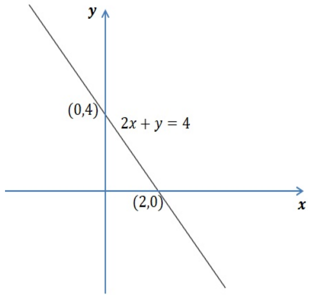
To plot the line, we rewrite the equation to become

$$y = -2x + 4$$

Then, we use the following table to make points on the plane:

x	0	2
y	4	0

The line $2x+y = 4$ passes through the points $(0, 4)$ and $(2, 0)$.



Review

- **Special cases of Lines**

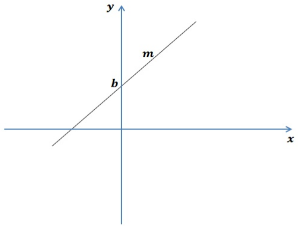
$y = mx + b$

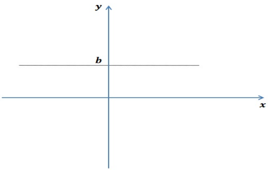
$y = b$

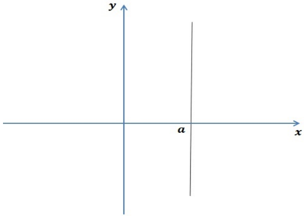
$x = a$

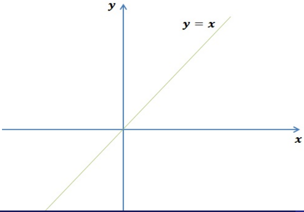
If $m = 0$, the line is horizontal.

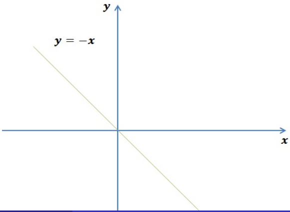
If m is undefined, the line is vertical.











Review

(2) Quadrature Functions $y = ax^2 + bx + c$

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Example: $y = 1 - x^2$

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(1) Intersection with x-axis: $y = 0$

$$1 - x^2 = 0 \Rightarrow x = \pm 1$$

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(2) Intersection with y-axis: $x = 0$

$$y = 1 - (0)^2 \Rightarrow y = 1$$

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The curve pass through the following points

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(3) First derivative test:

$$y' = -2x = 0$$

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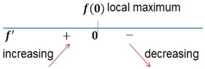

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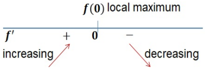

(4) Second derivative test:

$$y'' = -2$$

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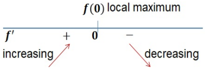
The curve pass through the following points

$$(1,0),(-1,0),(0,1)$$

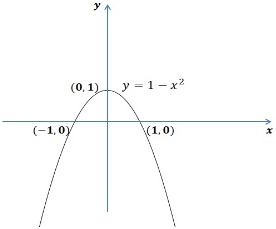
(3) First derivative test:

$$y' = -2x = 0 \Rightarrow x = 0$$



(4) Second derivative test:

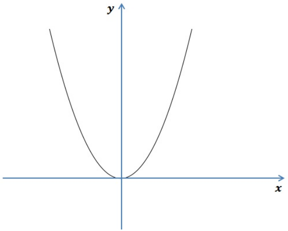
$$y'' = -2 \Rightarrow \quad \text{the curve concave downward}$$



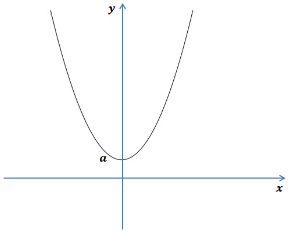
Review

- **Special cases of Quadrature Functions**

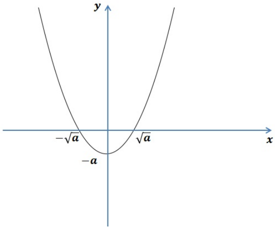
$y = x^2$



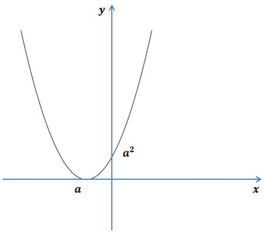
$y = x^2 + a$



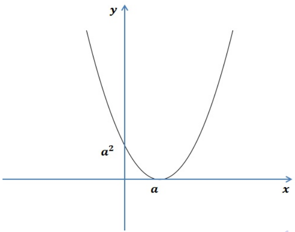
$y = x^2 - a$



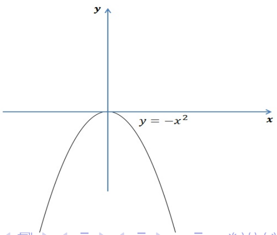
$y = (x + a)^2$



$y = (x - a)^2$

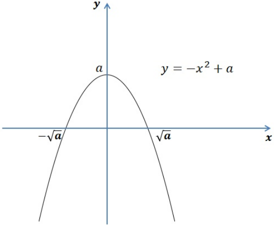


$y = -x^2$

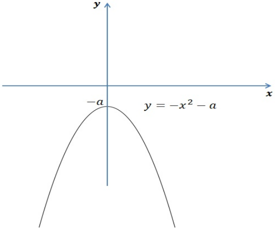


Review

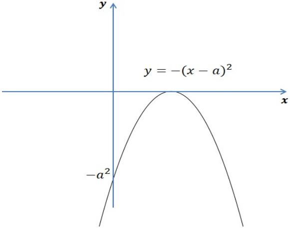
$y = -x^2 + a$



$y = -x^2 - a$



$y = -(x - a)^2$



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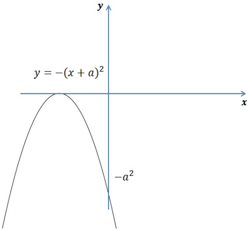
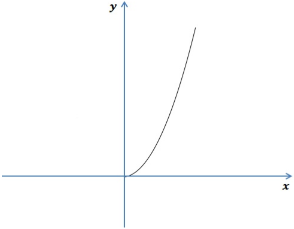
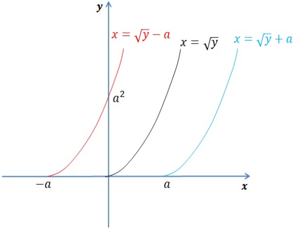


$x = \sqrt{y}$





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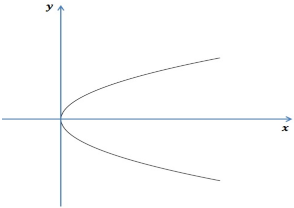

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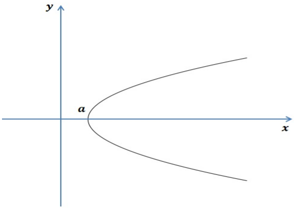

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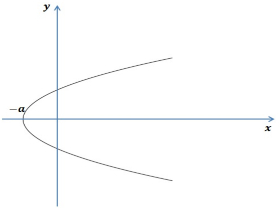

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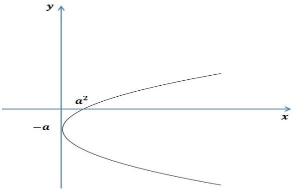
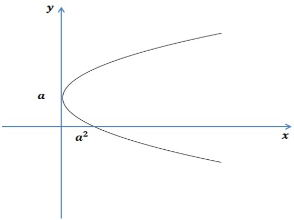
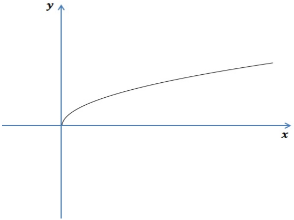


$x = (y - a)^2$

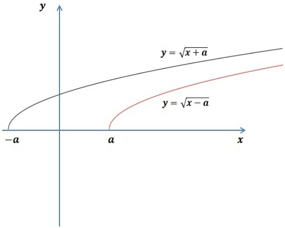


$y = \sqrt{x}$

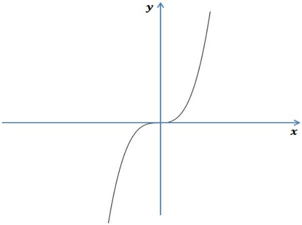


Review

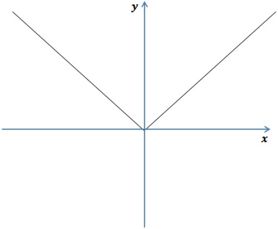
$y = \sqrt{x \pm a}$



$y = x^3$

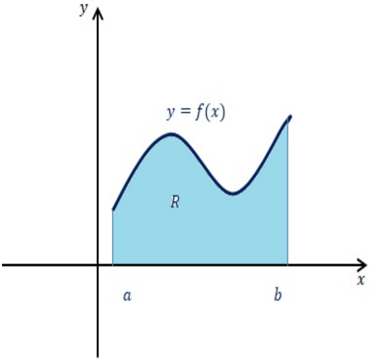


$y = | x |$



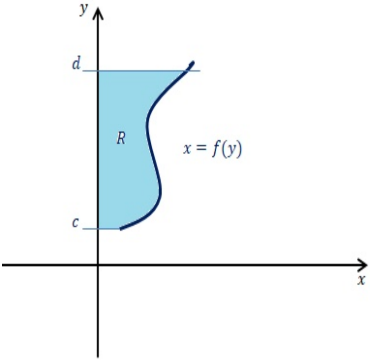
Areas

- If $y = f(x)$ is a continuous function on $[a, b]$ and $f(x) \geq 0$ for every $x \in [a, b]$, then the area of the region bounded by the graph of f and x-axis from $x = a$ to $x = b$ is given by the integral:



$$A = \int_a^b f(x)\ dx$$

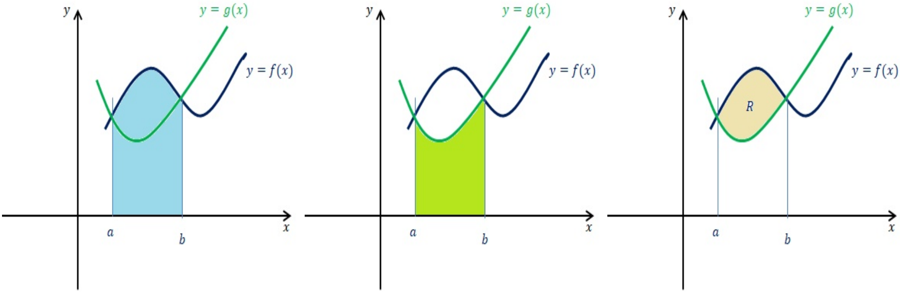
- If $x = f(y)$ is a continuous function on $[c, d]$ and $f(y) \geq 0$ $\forall y \in [c, d]$, then the area of the region bounded by the graph of f and y-axis from $y = c$ to $y = d$ is given by the integral:



$$A = \int_c^d f(y)\ dy$$

Areas

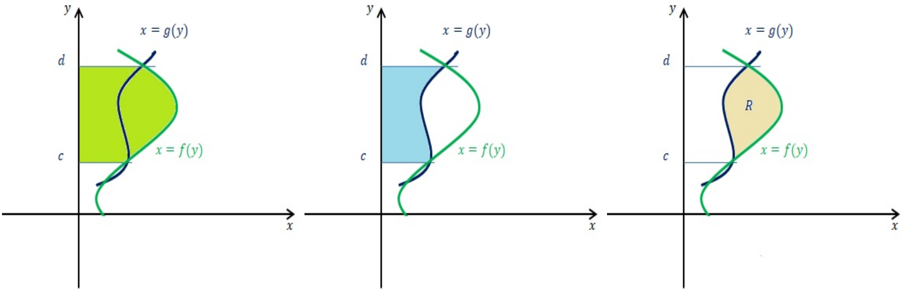
■ If the functions f and g are continuous and $f(x) \geq g(x)\ \forall x \in [a, b]$, then the area A of the region bounded by the graphs of f (the upper boundary of R) and g (the lower boundary of R) from $x = a$ to $x = b$ is subtracting the area of the region under g from the area of the region under f. This can be stated as follows:



$$A = \int_a^b \big(f(x) - g(x)\big)\, dx$$

Areas

■ If the functions f and g are continuous and $f(y) \geq g(y)\ \forall y \in [c, d]$, then the area A of the region bounded by the graphs of f (the right boundary of R) and g (the left boundary of R) from $y = c$ to $y = d$ is subtracting the area of the region bounded by $g(y)$ from the area of the region bounded by $f(y)$. This can be stated as follows:



$$A = \int_c^d \Big(f(y) - g(y)\Big)\, dy$$

Example

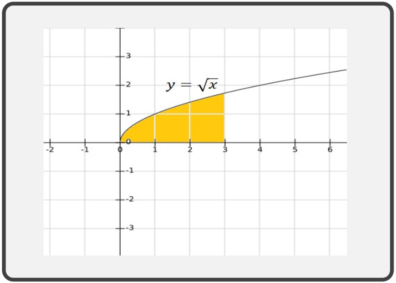
Sketch the region bounded by the graph of $y = \sqrt{x}$ *and* $x-$*axis from* $x = 0$ *to* $x = 3$*, then find its area.*

Example

Sketch the region bounded by the graph of $y = \sqrt{x}$ and $x-$axis from $x = 0$ to $x = 3$, then find its area.

Solution:
The area of the region is

$$A = \int_0^3 \sqrt{x}\, dx = \left[\frac{x^{3/2}}{\frac{3}{2}} \right]_0^3$$
$$= \frac{2}{3} \left[x^{3/2} \right]_0^3$$
$$= 2\sqrt{3}.$$



Example

Sketch the region bounded by the graph of $x = y + 1$ *and* $x-$*axis from* $y = -1$ *to* $y = 0$*, then find its area.*

Example

Sketch the region bounded by the graph of $x = y + 1$ and $x-$axis from $y = -1$ to $y = 0$, then find its area.

Solution:

x	0	1
y	-1	0

The line $x = y+1$ passes through the points $(0, -1)$ and $(1, 0)$.

Areas

Example

Sketch the region bounded by the graph of $x = y + 1$ and $x-$axis from $y = -1$ to $y = 0$, then find its area.

Solution:

x	0	1
y	-1	0

The line $x = y+1$ passes through the points $(0, -1)$ and $(1, 0)$.

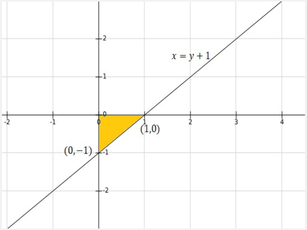
The area of the region is

$$A = \int_{-1}^{0} (y + 1)\, dy$$

$$= \left[\frac{y^2}{2} + y \right]_{-1}^{0}$$

$$= \left[0 - (\frac{(-1)^2}{2} - 1) \right]$$

$$= \frac{1}{2} \, .$$



Example

Sketch the region bounded by the graph of $x = y + 1$ *and* $y-$*axis over the interval* $[-1, 1]$*, then find its area.*

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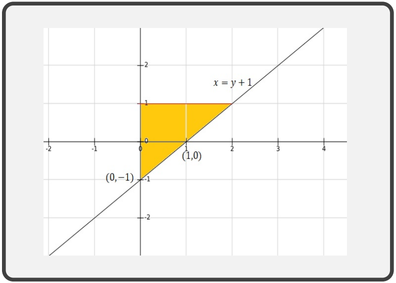


The line $x = y+1$ passes through the points $(0, -1)$ and $(1, 0)$.

The area of the region is

$$A = \int_{-1}^{1} (y+1)\, dy$$
$$= \left[\frac{y^2}{2} + y \right]_{-1}^{1}$$
$$= (\frac{(1)^2}{2} + 1) - (\frac{(-1)^2}{2} + (-1))$$
$$= 2 \,.$$

Example

Sketch the region bounded by the graph of $y = 2 - x^2$ and $x-$axis, then find its area.

Solution:

Areas

Example

Sketch the region bounded by the graph of $y = 2 - x^2$ and $x-$axis, then find its area.

Solution:

The area of the region is

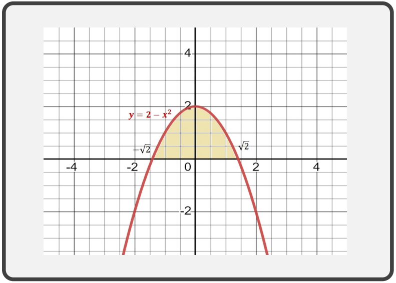
$$A = \int_{-\sqrt{2}}^{\sqrt{2}} (1 - x^2)\, dx$$

$$= \Big[\, x - \frac{x^3}{3} \,\Big]_{-\sqrt{2}}^{\sqrt{2}}$$

$$= (\sqrt{2} - \frac{(\sqrt{2})^3}{3}) - (\, -\sqrt{2} - \frac{(-\sqrt{2})^3}{3})$$

$$= \sqrt{2} + \sqrt{2} - \frac{(\sqrt{2})^3}{3} - \frac{(\sqrt{2})^3}{3}$$

$$= 2\sqrt{2} - \frac{2(\sqrt{2})^3}{3}\ .$$



Areas

Example

Sketch the region bounded by the graphs of $y = x^2$ *and* $y = x + 6$ *over the interval* $[-2, 3]$*, then find its area.*

Areas

Example

Sketch the region bounded by the graphs of $y = x^2$ *and* $y = x + 6$ *over the interval* $[-2, 3]$, *then find its area.*

Solution: The intersection points:

$$x^2 = x + 6 \Rightarrow x^2 - x - 6 = 0$$

$$\Rightarrow (x+2)(x-3) = 0 \Rightarrow x = -2 \text{ and } x = 3$$

$$x = -2 \Rightarrow y = 4 \Rightarrow (-2, 4)$$

$$x = 3 \Rightarrow y = 9 \Rightarrow (3, 9)$$

Areas

Example

Sketch the region bounded by the graphs of $y = x^2$ *and* $y = x + 6$ *over the interval* $[-2, 3]$*, then find its area.*

Solution: The intersection points:

$$x^2 = x + 6 \Rightarrow x^2 - x - 6 = 0$$

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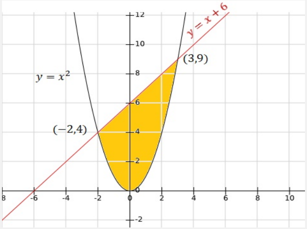


The area of the region is

$$A = \int_{-2}^{3} (x + 6 - x^2)\, dx$$

$$= \left[\frac{x^2}{2} + 6x - \frac{x^3}{3} \right]_{-2}^{3}$$

$$= \left(\frac{3^2}{2} + 6(3) - \frac{3^3}{3}\right) - \left(\frac{(-2)^2}{2} + 6(-2) - \frac{(-2)^3}{3}\right)$$

$$= \frac{27}{2} + \frac{22}{3} = \frac{125}{6}.$$

Example

Sketch the region bounded by the graphs of $y = x^2$ *and* $x = y^2$ *over* $[0, 1]$, *then find its area.*

Areas

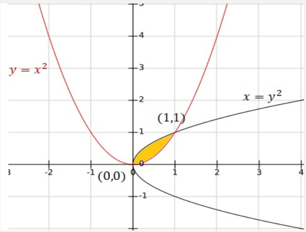
Example

Sketch the region bounded by the graphs of $y = x^2$ and $x = y^2$ over $[0, 1]$, then find its area.

Solution: We write the two functions in terms of x, so the upper graph: $x = y^2 \Rightarrow y = \sqrt{x}$.

The intersection points:
$x^2 = \sqrt{x} \Rightarrow x^4 = x$
$x^4 - x = 0 \Rightarrow x(x^3 - 1) = 0$
$x = 0 \Rightarrow (0, 0)$
$x = 1 \Rightarrow (1, 1)$

$$\begin{aligned} A &= \int_0^1 (\sqrt{x} - x^2)\, dx \\ &= \left[\ \frac{2}{3} x^{\frac{3}{2}} - \frac{x^3}{3}\ \right]_0^1 \\ &= \left[\ \frac{2}{3} - \frac{1}{3}\ \right] \\ &= \frac{1}{3}\ . \end{aligned}$$



Example

Sketch the region bounded by the graphs of $x = 2y$ *and* $x = \frac{y}{2} + 3$ *and* $x-$*axis, then find its area.*

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Sketch the region bounded by the graphs of $x = 2y$ and $x = \frac{y}{2} + 3$ and $x-$axis, then find its area.

Solution:
The intersection points:

$$\frac{y}{2} + 3 = 2y$$
$$\Rightarrow y + 6 = 4y$$
$$\Rightarrow y = 2.$$

Substitute $y = 2$ in both functions to have $x = 4$.
Thus, the two curves intersect at $(4, 2)$.

Example

Sketch the region bounded by the graphs of $x = 2y$ and $x = \frac{y}{2} + 3$ and $x-$axis, then find its area.

Solution:
The intersection points:

$$\begin{aligned} \frac{y}{2} + 3 &= 2y \\ \Rightarrow y + 6 &= 4y \\ \Rightarrow y &= 2. \end{aligned}$$

Substitute $y = 2$ in both functions to have $x = 4$.
Thus, the two curves intersect at $(4, 2)$.

$$\begin{aligned} A &= \int_0^2 (\frac{y}{2} + 3 - 2y)\, dy \\ &= \int_0^2 (-\frac{3}{2}y + 3)\, dy \\ &= \Big[-\frac{3}{4}y^2 + 3y \Big]_0^2 \\ &= -3 + 6 = 3. \end{aligned}$$