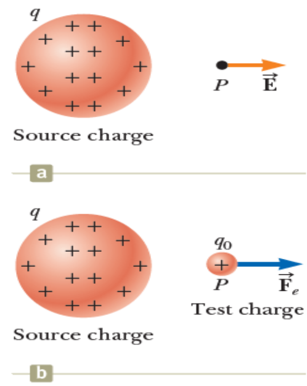
23.4 The Electric Field

Concept of electric field

- An electric field represents the region around a charged object where other charges (around the source charge) experience a force due to its influence.
- A charged particle, with charge q, produces an electric field in the region of space around it.
- A small test charge q_0 is placed near the source charge to detect and measure the presence and direction of the electric field.



Definition of the electric field:

the electric field E at a point in space is defined as the electric force $\mathbf{F}_e$ acting on a positive test charge q_0 placed at that point divided by the magnitude of the test charge:

$$\mathbf{E} \equiv \frac{\mathbf{F}_e}{q_0} \qquad (23.3)$$

As we discussed in the previous section, the electric force due to point charges can be written as:

$$\vec{F} = K_e \frac{|q_1||q_2|}{r^2} \hat{r}$$

So, one can re-write the electric force between q and q_0 as follows:

$$\vec{F} = K_e \frac{q\, q_0}{r^2} \hat{r}$$

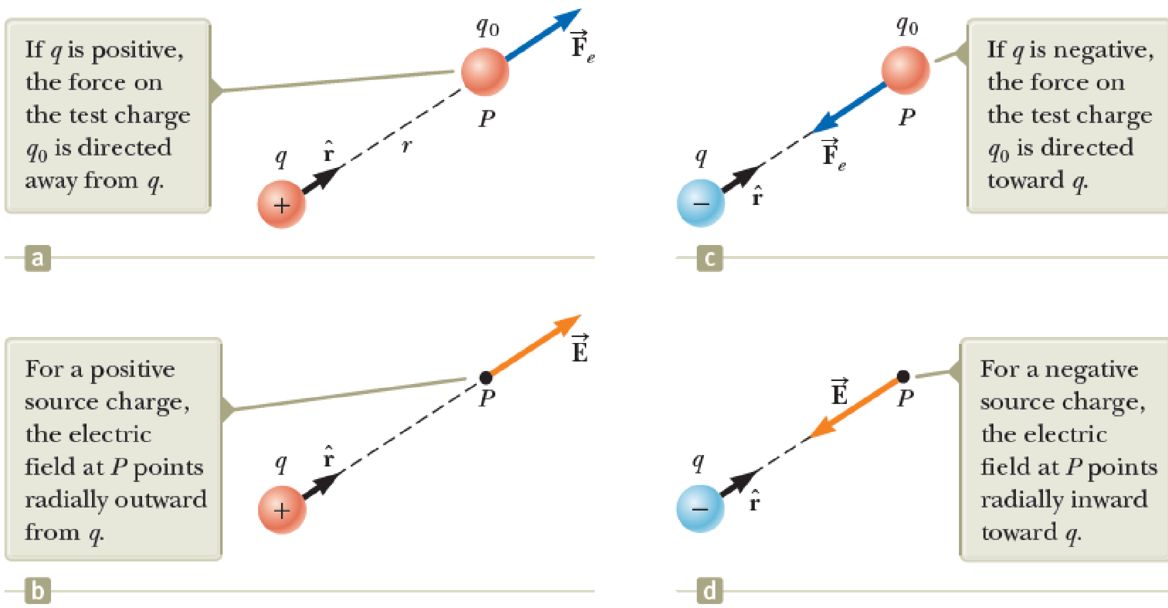
The electric field can also be derived:

$$\vec{E} = \frac{\vec{F}}{q_0} = K_e \frac{q\, q_0}{q_0 r^2} \hat{r} = K_e \frac{q}{r^2} \hat{r}$$

The vector $\vec{E}$ the SI units of (newtons per coulomb) (N/C).

<u>The direction of electric force and electric field:</u>

<u>Remember:</u> $\vec{F} = q\,\vec{E}$

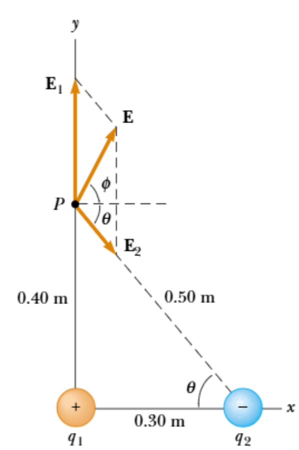


at any point P, the total electric field due to a group of charges equals the vector sum of the electric fields of the individual charges.

$$\mathbf{E} = k_e \sum_i \frac{q_i}{r_i^{\,2}} \hat{\mathbf{r}}_i$$

Example-1:

A charge $q_1 = 7$ µC is located at the origin and second charge $q_2 = -5$ µC is located on the x axis, 0.30 m from the origin. Find the electric field at the point P which has the coordinates (0, 0.40) m.



Solution:

$\vec{E} = \vec{E}_1 + \vec{E}_2$

1- $\vec{E}_1 = |\vec{E}_1|\,\hat{r} = K\frac{q_1}{r^2}\,\hat{r} = 9x10^9\,\frac{(7x10^{-6})}{(0.4)^2}\,(+\hat{j}) =$
   $3.9\ x\ 10^5\,\hat{j}\ N/C$

2- $\vec{E}_2 = |\vec{E}_2|\,\hat{r} = E_{2x}\hat{\imath} + E_{2y}\hat{\imath} = |\vec{E}_2|\cos\theta\ \hat{\imath} - |\vec{E}_2|\sin\theta\,\hat{\imath}$
   $r^2 = (0.3)^2 + (0.4)^2 = 0.25$ $\qquad\qquad r = 0.5\ m$
   $\cos\theta = \frac{0.3}{0.5}$ $\qquad\qquad \sin\theta = \frac{0.4}{0.5}$

   $|\vec{E}_2| = K\frac{q_2}{r^2} = 9x10^9\,\frac{(5x10^{-6})}{(0.5)^2} = 1.8\ x\ 10^5\ N/C$

   $\vec{E}_2 == |\vec{E}_2|\,(\cos\theta\ \hat{\imath} - \sin\theta\,\hat{\imath}) = 1.8x10^5\,(\frac{0.3}{0.5}\hat{\imath} - \frac{0.4}{0.5}\hat{j})$

3- $\vec{E} = \vec{E}_1 + \vec{E}_2 = 3.9\ x\ 10^5\,\hat{j} + 1.1\ x\ 10^5\,\hat{\imath} -$
   $1.44\ x\ 10^5\,\hat{j} = (1.1\,\hat{\imath} + 2.44\,\hat{j})x\ 10^5$

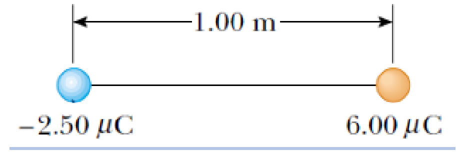
4- $|\vec{E}| = \sqrt{\vec{E}_1^2 + \vec{E}_2^2} = \sqrt{(1.1x10^5)^2 + (2.44x10^5)^2} =$
   $2.7\ x\ 10^5 N/C$

5- $\tan\emptyset = \frac{E_y}{E_x} = \frac{2.44\ x\ 10^5}{1.1\ x\ 10^5} = 2.22$

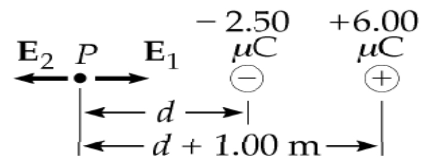
   $\emptyset = tan^{-1}(2.22) = 65.73^0$

Example-2:

In this figure, determine the point where the electric field is zero.



Solution:



$$\vec{E}_P = \vec{E}_1 + \vec{E}_2$$

$$\vec{E}_P = \mathbf{0} =====> \vec{E}_1 = \vec{E}_2$$

$$K\,\frac{q_1}{d^2} = K\,\frac{q_2}{(1+d)^2} =====> q_1(1+d)^2 = q_2 d^2$$

$$\sqrt{\frac{q_1}{q_2}}(1+d) = \pm\, d ====> d\left(\sqrt{\frac{q_1}{q_2}} \mp 1\right) = -\sqrt{\frac{q_1}{q_2}}$$

Please consider this step carefully, especially when given $q_1=q_2$.

$$d = 1.8\,m \quad from\; q_1$$