28.3 Kirchhoff's rules:

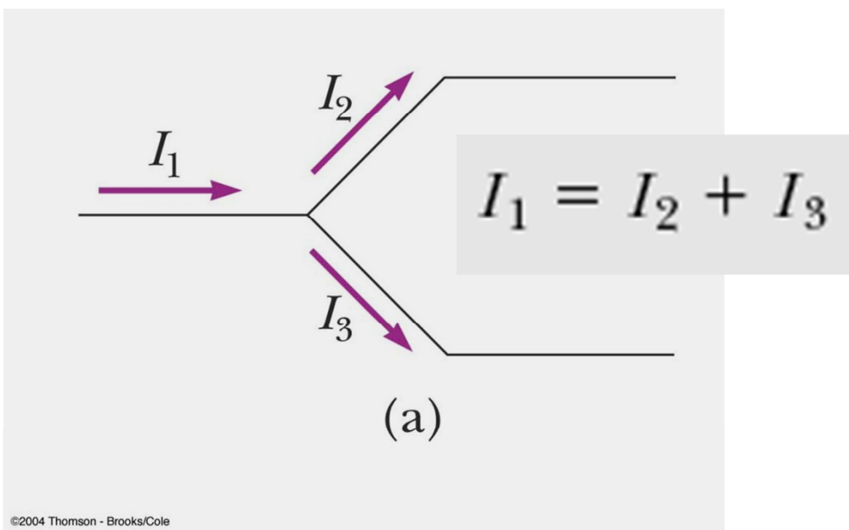
As we discussed that we can analyze simple circuits using the expression *V= IR and the rules for series and parallel combinations of resistors.* However, it is not often possible to reduce a circuit to a single loop.

-The procedure for analyzing more complex circuits is greatly simplified if we use two principles called Kirchhoff's rules:

Rule 1:

The sum of electric current entering any junction in a circuit must equal to the sum of electric current leaving that junction as given by:

$$\sum I_{in} = \sum I_{out}$$



The charge passes through some circuit elements must equal the sum of the decreases in energy as it passes through other elements. The potential energy decreases whenever the charge moves through a potential drop *IR* across a resistor or whenever it moves in the reverse direction through a source of emf. The potential energy increases whenever the charge passes through a battery from the negative terminal to the positive terminal.

Rule 2:

The sum of the potential difference across all electric elements around any closed circuit loop must equal zero,

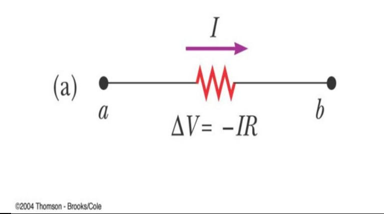
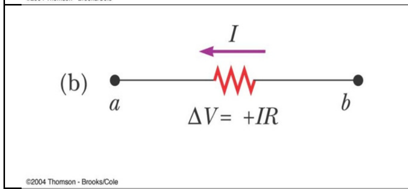
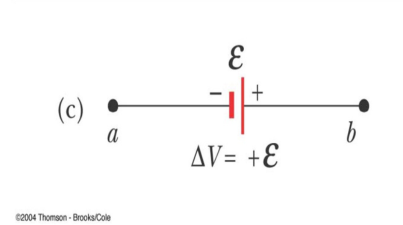
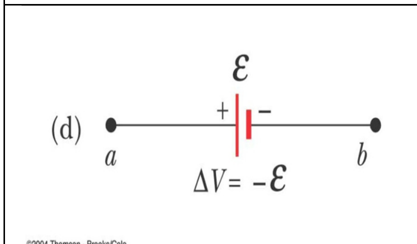
$$\sum \nabla V = 0$$

Let us imagine moving a charge around the loop. When the charge returns to the starting point, the charge–circuit system must have the same energy as when the charge started from it.

The sum of the increases in energy in some circuit elements must equal the sum of the decreases in energy in other elements.

The potential energy decreases whenever the charge moves through a potential drop *IR* across a resistor or whenever it moves in the reverse direction through a source of emf. The potential energy increases when-ever the charge passes through a battery from the negative terminal to the positive terminal.

Each circuit element is traversed from left to right →

<table>
<tr><td>
</td><td>• Because charges move from the high-potential end of a resistor to the low potential end, if a resistor is traversed in the direction of the current, the change in potential *V across the resistor is - IR*</td></tr>
<tr><td>
</td><td>• If a resistor is traversed in the direction *opposite the current, the change in potential V across the resistor is + IR*</td></tr>
<tr><td>
</td><td>• If a source of emf (assumed to have zero internal resistance) is traversed in the direction of the emf (from - to+), the change in potential *V is +ε.*
The emf of the battery increases the electric potential as we move through it in this direction.</td></tr>
<tr><td>
</td><td>• If a source of emf (assumed to have zero internal resistance) is traversed in the direction opposite the emf (from + to -), the change in potential *V is - ε.*
In this case the emf of the battery reduces the electric potential as we move through it.</td></tr>
</table>

You may follow the following steps when wanting to solve problems based on Kirchhoff's rules:

1) Identify all of the junctions or branch points in the circuit.
2) Identify the current loops that exist in the circuit. Choose any loop and apply the loop rule.

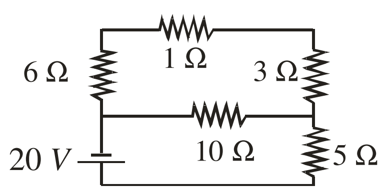
Remember that (+) to (-) is a potential drop while (-) to (+) is a potential gain. Write equations for each loop. Remember that the sums of the potential drops and gains must be zero.

3) Reapply the loop rule as needed. For each unknown current you will need to write an equation. The fewer terms in which you express unknown currents, the fewer equations you have to write.
4) Solve the equations to determine the unknown currents.

Examples 28.7 and 28.8

Example:

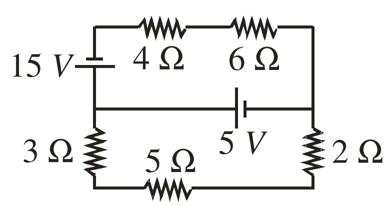
Find the electric current passing through R = 10 Ω?



I= 1 A

Example:

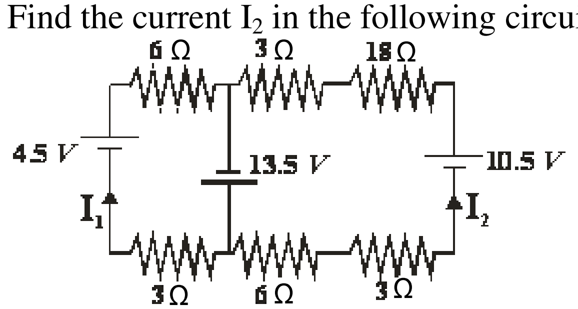
Find the electric current passing through the resistor of 3 Ω.



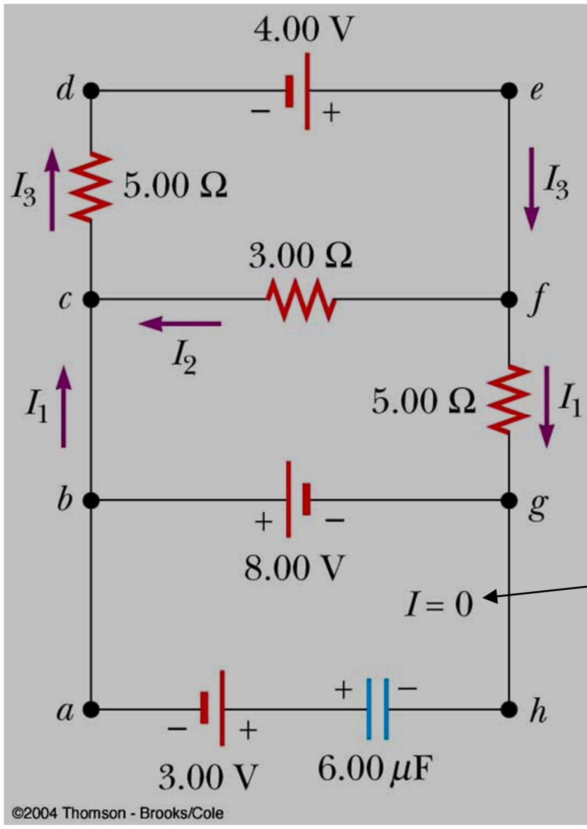
I=0.5A

Example:

Find the current I_2 in the following circuit.



RC CIRCUITE:
Example:



Please note that at steady state condition:

I=0 in the abgha closed loop

Example 28.8 A Single-Loop Circuit

A single-loop circuit contains two resistors and two batteries, as shown in Figure 28.16. (Neglect the internal resistances of the batteries.)

(A) Find the current in the circuit.

Solution We do not need Kirchhoff's rules to analyze this simple circuit, but let us use them anyway just to see how they are applied. There are no junctions in this single-loop circuit; thus, the current is the same in all elements. Let us assume that the current is clockwise, as shown in Figure 28.16. Traversing the circuit in the clockwise direction, starting at a, we see that $a \to b$ represents a potential difference of $+\mathcal{E}_1$, $b \to c$ represents a potential difference of $-IR_1$, $c \to d$ represents a potential difference of $-\mathcal{E}_2$, and

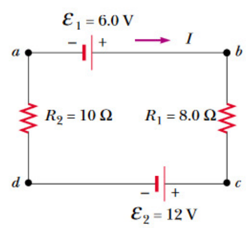


Figure 28.16 (Example 28.8) A series circuit containing two batteries and two resistors, where the polarities of the batteries are in opposition.

$d \to a$ represents a potential difference of $-IR_2$. Applying Kirchhoff's loop rule gives

$$\sum \Delta V = 0$$

$$\mathcal{E}_1 - IR_1 - \mathcal{E}_2 - IR_2 = 0$$

Solving for I and using the values given in Figure 28.16, we obtain

$$(1) \qquad I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2} = \frac{6.0\text{ V} - 12\text{ V}}{8.0\ \Omega + 10\ \Omega} = \boxed{-0.33\text{ A}}$$

The negative sign for I indicates that the direction of the current is opposite the assumed direction. Notice that the emfs in the numerator subtract because the batteries have opposite polarities in Figure 28.16. In the denominator, the resistances add because the two resistors are in series.

(B) What power is delivered to each resistor? What power is delivered by the 12-V battery?

Solution Using Equation 27.23,

$$\mathcal{P}_1 = I^2R_1 = (0.33\text{ A})^2(8.0\ \Omega) = \boxed{0.87\text{ W}}$$

$$\mathcal{P}_2 = I^2R_2 = (0.33\text{ A})^2(10\ \Omega) = \boxed{1.1\text{ W}}$$

Hence, the total power delivered to the resistors is $\mathcal{P}_1 + \mathcal{P}_2 = 2.0$ W.

The 12-V battery delivers power $I\mathcal{E}_2 = 4.0$ W. Half of this power is delivered to the two resistors, as we just calculated. The other half is delivered to the 6-V battery, which is being charged by the 12-V battery. If we had included the internal resistances of the batteries in our analysis, some of the power would appear as internal energy in the batteries; as a result, we would have found that less power was being delivered to the 6-V battery.

What If? **What if the polarity of the 12.0-V battery were reversed? How would this affect the circuit?**

Answer While we could repeat the Kirchhoff's rules calculation, let us examine Equation (1) and modify it accordingly. Because the polarities of the two batteries are now in the same direction, the signs of $\mathcal{E}_1$ and $\mathcal{E}_2$ are the same and Equation (1) becomes

$$I = \frac{\mathcal{E}_1 + \mathcal{E}_2}{R_1 + R_2} = \frac{6.0\text{ V} + 12\text{ V}}{8.0\ \Omega + 10\ \Omega} = 1.0\text{ A}$$

The new powers delivered to the resistors are

$$\mathcal{P}_1 = I^2R_1 = (1.0\text{ A})^2(8.0\ \Omega) = 8.0\text{ W}$$

$$\mathcal{P}_2 = I^2R_2 = (1.0\text{ A})^2(10\ \Omega) = 10\text{ W}$$

This totals 18 W, nine times as much as in the original circuit, in which the batteries were opposing each other.

Example 28.9 Applying Kirchhoff's Rules

Interactive

Find the currents I_1, I_2, and I_3 in the circuit shown in Figure 28.17.

Solution Conceptualize by noting that we cannot simplify the circuit by the rules of adding resistances in series and in parallel. (If the 10.0-V battery were taken away, we could reduce the remaining circuit with series and parallel combinations.) Thus, we categorize this problem as one in which we must use Kirchhoff's rules. To analyze the circuit, we arbitrarily choose the directions of the currents as labeled in Figure 28.17. Applying Kirchhoff's junction rule to junction c gives

$$(1) \qquad I_1 + I_2 = I_3$$

We now have one equation with three unknowns—I_1, I_2, and I_3. There are three loops in the circuit—*abcda, befcb,* and *aefda.* We therefore need only two loop equations to determine the unknown currents. (The third loop equation would give no new information.) Applying Kirchhoff's loop rule to loops *abcda* and *befcb* and traversing these loops clockwise, we obtain the expressions

$$(2) \qquad abcda \quad 10.0\text{ V} - (6.0\ \Omega)I_1 - (2.0\ \Omega)I_3 = 0$$

$$(3) \quad befcb \quad -14.0\text{ V} + (6.0\ \Omega)I_1 - 10.0\text{ V} - (4.0\ \Omega)\ I_2 = 0$$

Note that in loop *befcb* we obtain a positive value when traversing the 6.0-Ω resistor because our direction of travel is opposite the assumed direction of I_1. Expressions (1), (2), and (3) represent three independent equations with three unknowns. Substituting Equation (1) into Equation (2) gives

$$10.0\text{ V} - (6.0\ \Omega)I_1 - (2.0\ \Omega)\ (I_1 + I_2) = 0$$

$$(4) \qquad 10.0\text{ V} = (8.0\ \Omega)I_1 + (2.0\ \Omega)I_2$$

Dividing each term in Equation (3) by 2 and rearranging gives

$$(5) \qquad -12.0\text{ V} = -(3.0\ \Omega)I_1 + (2.0\ \Omega)I_2$$

Subtracting Equation (5) from Equation (4) eliminates I_2, giving

$$22.0\text{ V} = (11.0\ \Omega)I_1$$

$$I_1 = \boxed{2.0\text{ A}}$$

Using this value of I_1 in Equation (5) gives a value for I_2:

$$(2.0\ \Omega)I_2 = (3.0\ \Omega)I_1 - 12.0\text{ V}$$
$$= (3.0\ \Omega)(2.0\text{ A}) - 12.0\text{ V} = -6.0\text{ V}$$

$$I_2 = \boxed{-3.0\text{ A}}$$

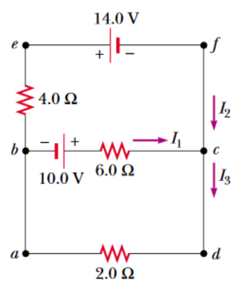


Figure 28.17 (Example 28.9) A circuit containing different branches.

Finally,

$$I_3 = I_1 + I_2 = \boxed{-1.0\text{ A}}$$

To finalize the problem, note that I_2 and I_3 are both negative. This indicates only that the currents are opposite the direction we chose for them. However, the numerical values are correct. What would have happened had we left the current directions as labeled in Figure 28.17 but traversed the loops in the opposite direction?

Example 28.10 A Multiloop Circuit

(A) Under steady-state conditions, find the unknown currents I_1, I_2, and I_3 in the multiloop circuit shown in Figure 28.18.

Solution First note that because the capacitor represents an open circuit, there is no current between *g* and *b* along path *ghab* under steady-state conditions. Therefore, when the charges associated with I_1 reach point *g*, they all go toward point *b* through the 8.00-V battery; hence, $I_{gb} = I_1$. Labeling the currents as shown in Figure 28.18 and applying Equation 28.9 to junction *c*, we obtain

$$(1) \qquad I_1 + I_2 = I_3$$

Equation 28.10 applied to loops *defcd* and *cfgbc*, traversed clockwise, gives

$$(2) \qquad defcd \quad 4.00\text{ V} - (3.00\ \Omega)I_2 - (5.00\ \Omega)I_3 = 0$$

$$(3) \qquad cfgbc \quad (3.00\ \Omega)I_2 - (5.00\ \Omega)I_1 + 8.00\text{ V} = 0$$

From Equation (1) we see that $I_1 = I_3 - I_2$, which, when substituted into Equation (3), gives

$$(4) \qquad (8.00\ \Omega)I_2 - (5.00\ \Omega)I_3 + 8.00\text{ V} = 0$$

Subtracting Equation (4) from Equation (2), we eliminate I_3 and find that

$$I_2 = -\frac{4.00\text{ V}}{11.0\ \Omega} = \boxed{-0.364\text{ A}}$$

Because our value for I_2 is negative, we conclude that the direction of I_2 is from *c* to *f* in the 3.00-Ω resistor. Despite this interpretation of the direction, however, we must continue to use this negative value for I_2 in subsequent calculations because our equations were established with our original choice of direction.

Using $I_2 = -0.364$ A in Equations (3) and (1) gives

$$I_1 = \boxed{1.38\text{ A}} \qquad I_3 = \boxed{1.02\text{ A}}$$

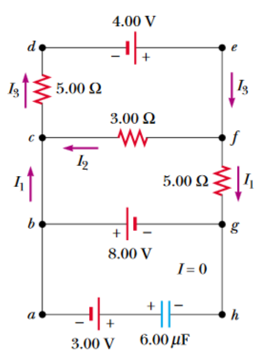


Figure 28.18 (Example 28.10) A multiloop circuit. Kirchhoff's loop rule can be applied to *any* closed loop, including the one containing the capacitor.

(B) What is the charge on the capacitor?

Solution We can apply Kirchhoff's loop rule to loop *bghab* (or any other loop that contains the capacitor) to find the potential difference ΔV_{cap} across the capacitor. We use this potential difference in the loop equation without reference to a sign convention because the charge on the capacitor depends only on the magnitude of the potential difference. Moving clockwise around this loop, we obtain

$$-8.00\text{ V} + \Delta V_{cap} - 3.00\text{ V} = 0$$

$$\Delta V_{cap} = 11.0\text{ V}$$

Because $Q = C\,\Delta V_{cap}$ (see Eq. 26.1), the charge on the capacitor is

$$Q = (6.00\ \mu\text{F})(11.0\text{ V}) = \boxed{66.0\ \mu\text{C}}$$

Why is the left side of the capacitor positively charged?