27.3 Kirchhoff's rules:

As we discussed, we can analyze simple circuits using the expression V=IR *and the rules for series and parallel combinations of resistors.* However, it is not often possible to reduce a circuit to a single loop.

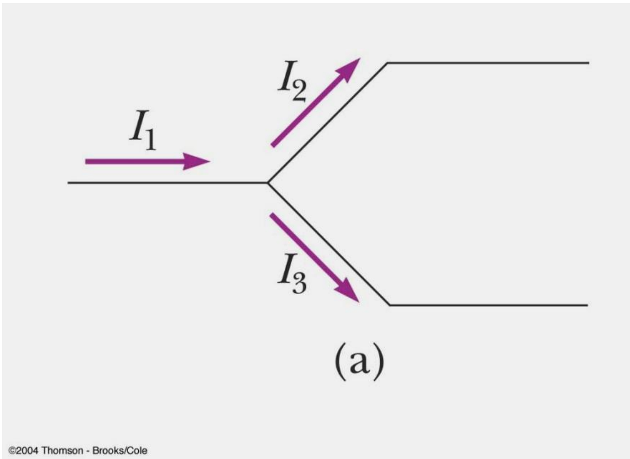
-The procedure for analyzing more complex circuits is greatly simplified if we use two principles called Kirchhoff's rules:

Rule 1:

The sum of electric current entering any junction in a circuit must equal the sum of electric current leaving that junction as given by:

$$\sum I_{in} = \sum I_{out}$$

$$I_1 = I_2 + I_3$$

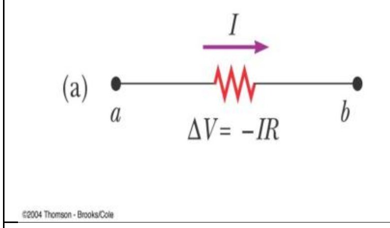
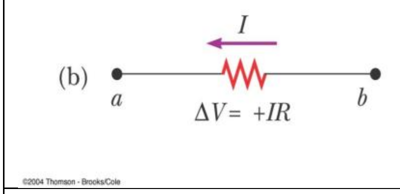
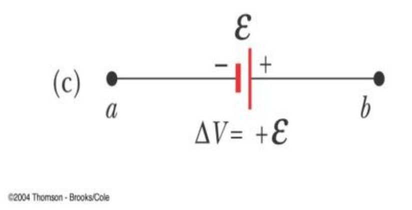
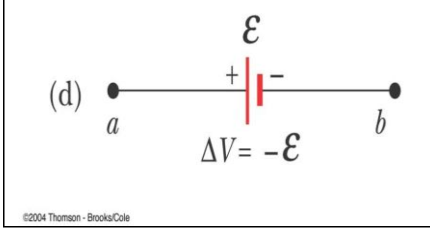


Rule 2:

The sum of the potential difference across all electric elements around any closed circuit loop must equal zero,

$$\sum \nabla V = 0$$

Assuming each circuit element is traversed from left to right →

	• Because charges move from the high-potential end of a resistor to the low-potential end, if a resistor is traversed in the direction of the current, the change in potential *V across the resistor is -IR*
	• If a resistor is traversed in the direction *opposite the current, the change in potential V across the resistor is +IR*
	• If a source of emf (assumed to have zero internal resistance) is traversed in the direction of the emf (from – to +), the change in potential *V is* +ε. The emf of the battery increases the electric potential as we move through it in this direction.
	• If a source of emf (assumed to have zero internal resistance) is traversed in the direction opposite the emf (from + to -), the change in potential *V is* - ε. In this case, the emf of the battery reduces the electric potential as we move through it.

You may follow the following steps when wanting to solve problems based on Kirchhoff's rules:

1) Identify all of the junctions or branch points in the circuit.
2) Identify the current loops that exist in the circuit. Choose any loop and apply the loop rule.

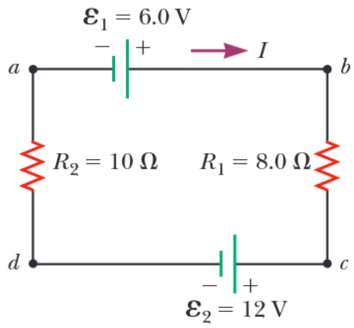
Remember that (+) to (-) is a potential drop while (-) to (+) is a potential gain. Write equations for each loop. Remember that the sums of the potential drops and gains must be zero.

3) Reapply the loop rule as needed. For each unknown current, you will need to write an equation. The fewer terms in which you express unknown currents, the fewer equations you have to write.
4) Solve the equations to determine the unknown currents.

Examples:

Example-1

A single-loop circuit consists of two resistors and two batteries, as shown in the Figure. (Neglect the internal resistances of the batteries.) Find the current in the circuit.



Solution:

Given

• $R_1 = 8.0\ \Omega$ (top-right resistor)

• $R_2 = 10\ \Omega$ (left resistor)

• $\varepsilon_1 = 6.0$ V (top battery)

• $\varepsilon_2 = 12$ V (bottom battery)

• The two batteries have opposite polarities as drawn in the figure.

Approach (Kirchhoff's Loop Rule)

Assume a clockwise current I (our sign convention guess). Traverse the loop clockwise, starting at point a. The potential changes encountered are:

• **Across R_1:** $-I R_1$ (drop in direction of assumed current)

• **Across R_2:** $-I R_2$ (drop in direction of assumed current)

• **Across ε_1 (from − to + terminal):** $+\varepsilon_1$

• **Across ε_2 (from + to − terminal because its polarity opposes ε_1):** $-\varepsilon_2$

Apply KVL

The sum of potential changes around the loop is zero:

$$0 = +\varepsilon_1 - I R_1 - \varepsilon_2 - I R_2$$

$$\Rightarrow \varepsilon_1 - \varepsilon_2 - I (R_1 + R_2) = 0$$

$$\Rightarrow I = (\varepsilon_1 - \varepsilon_2) / (R_1 + R_2)$$

Substitute Numbers

$$I = (6.0\ \text{V} - 12\ \text{V}) / (8.0\ \Omega + 10\ \Omega)$$

$$I = (-6.0\ \text{V}) / (18.0\ \Omega) = -0.33\ \text{A}$$ (to two significant figures)

Interpretation

The negative sign indicates that the actual current direction is opposite to the assumed clockwise direction. **Hence, the current has a magnitude of 0.33 A and flows counterclockwise.**

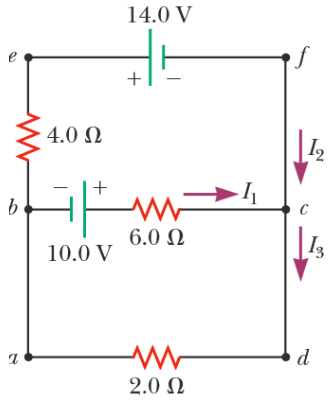
What if the polarity of the 12.0-V battery were reversed? How would that affect the circuit?

With the 12.0 V battery reversed, both battery emfs aid each other. Repeating KVL gives:

$I = (\varepsilon_1 + \varepsilon_2) / (R_1 + R_2) = (6.0\ V + 12.0\ V) / (8.0\ \Omega + 10\ \Omega) = 18.0\ V / 18.0\ \Omega = 1.0\ A$ (clockwise).

Example-2

Find the currents I_1 , I_2 , and I_3 in the circuit shown in the Figure.



Solution:

Note on Traversal Direction

Before applying Kirchhoff's Voltage Law (KVL), we must select a direction for traversing each loop—either clockwise or counterclockwise. This choice is entirely arbitrary, but it must be used consistently when assigning voltage rises and drops throughout that loop. If a current value later appears negative, it simply indicates that the actual current flows opposite to the assumed direction. The magnitude remains correct, and the negative sign only reflects direction reversal.

Kirchhoff's Rules – Worked Solution

Equations from KCL and KVL

Junction (node c): (1) $I_1 + I_2 - I_3 = 0 \;\Rightarrow\; I_3 = I_1 + I_2$.

Loop abcda: (2) $10.0\ V - (6.0\ \Omega)\ I_1 - (2.0\ \Omega)\ I_3 = 0$

Loop befc b: (3) $-24.0\ V + (6.0\ \Omega)\ I_1 - (4.0\ \Omega)\ I_2 = 0$

Elimination Steps

Substitute (1) into (2): (4) $10.0\ V - (8.0\ \Omega)\ I_1 - (2.0\ \Omega)\ I_2 = 0$

Multiply (3) by 4 and (4) by 3:

(5) $-96.0\ V + (24.0\ \Omega)\ I_1 - (16.0\ \Omega)\ I_2 = 0$

(6) $30.0\ V - (24.0\ \Omega)\ I_1 - (6.0\ \Omega)\ I_2 = 0$

Add (5) and (6): $-66.0\ V - (22.0\ \Omega)\ I_2 = 0 \;\Rightarrow\; I_2 = -3.0\ A$

Use (3) with $I_2 = -3.0\ A$: $-24.0 + 6\ I_1 - 4(-3) = 0 \Rightarrow I_1 = 2.0\ A$

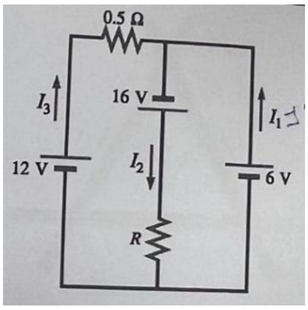
Use (1): $I_3 = I_1 + I_2 = 2.0 - 3.0 = -1.0$ A

Final Currents

$I_1 = 2.0$ A, $I_2 = -3.0$ A, $I_3 = -1.0$ A (negative means opposite the initial arrow).

Example-3:

For the circuit shown in the figure, the current I_1=10 A. Find the current I_2 (in amperes).



Solution:

Given: Right-branch current **I_1 = 10 A (upward)**; top resistor **0.5 Ω**. Find **I_2** (downward).

Step 1 – Junction (current) relation

At the top-right junction, the middle-branch current equals the sum of the currents entering from the right and along the top resistor:

$I_2 = I_1 + I_3$ … (eq. 1)

Step 2 – KVL in the left loop

Traverse the left loop clockwise (up through 12 V, right across the 0.5 Ω from right→left, down through 16 V, then back along the bottom). Using rises as + and drops as − gives:

$$+12 - 0.5 I_3 + 16 - I_2 R = 0$$

$$\Leftrightarrow \quad I_2 R + 0.5 I_3 = 28 \qquad \text{... (eq. 2)}$$

Step 3 – KVL in the right loop

Traverse the right loop clockwise (up through 6 V, left across the 0.5 Ω, down through 16 V, then back along the bottom). With the same sign convention:

$$-6 + I_2 R - 16 = 0 \Leftrightarrow I_2 R = 22 \qquad \text{... (eq. 3)}$$

Solve using the same order

From (eq. 3): $I_2 R = 22$.

Substitute into (eq. 2): $22 + 0.5 I_3 = 28 \Rightarrow 0.5 I_3 = 6 \Rightarrow$ **$I_3 = 12$ A**

Use (eq. 1) with $I_1 = 10$ A: $I_2 = I_1 + I_3 = 10 + 12 =$ **22 A**

If you wish to back out R using (eq. 3): $R = (I_2 R) / I_2 = 22 / 22 =$ **1 Ω**

Example-4:

Find the equivalent resistance R_{eq} of the resistors between points **a** and **b** in the circuit shown in the figure.