27-4 Resistance and Temperature

1. Introduction

- Electrical resistance (**R**) is a measure of how much a material **opposes or resists the flow of electric current**.
- The resistance of a material **changes with temperature** due to variations in atomic vibrations and electron scattering.
- This temperature dependence is crucial in designing electrical circuits, power transmission lines, and electronic sensors.

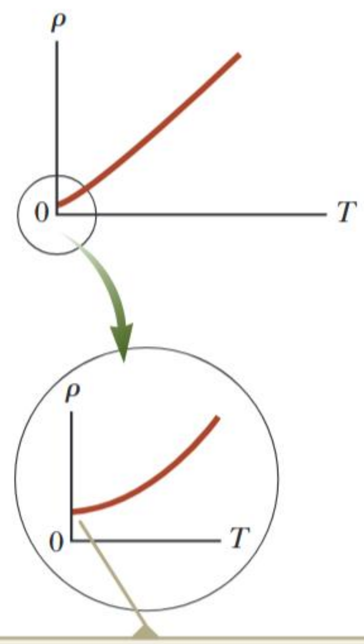
2. Resistivity and Temperature Dependence

- **Resistivity (ρ)** is an intrinsic property of a material that determines how much it resists current flow.
- The relationship between resistivity and temperature is given by:

$$\rho=\rho_0[1+\alpha(T-T_0)]$$

where:

- ρ = resistivity at temperature T (**Ω·m**)
- ρ_0 = resistivity at reference temperature T_0 (usually **20°C or 0°C**)
- α = **temperature coefficient of resistivity** (**°C^{-1}**)
- T, T_0= temperatures in **°C**



As T approaches absolute zero, the resistivity approaches a nonzero value.

Also, we can calculate α as following"

$$\alpha = \frac{\Delta\rho/\rho_0}{\Delta T}$$

Where: $\Delta\rho = \rho - \rho_0$; $\Delta T = T - T_0$

Understanding α:

- **For metals**: $\alpha>0$, meaning resistivity **increases** with temperature.
- **For semiconductors and insulators**: $\alpha<0$, meaning resistivity **decreases** with temperature.

Material	Resistivity at 20°C (Ω·m)	Temperature Coefficient (α) (°C⁻¹)
Copper (Cu)	1.68×10^{-8}	3.9×10^{-3}
Aluminum (Al)	2.65×10^{-8}	4.0×10^{-3}
Silver (Ag)	1.59×10^{-8}	3.8×10^{-3}
Silicon (Si)	6.4×10^{26}	-75×10^{-3}

3. Derivation of Resistance-Temperature Relationship

We know that **resistance** is related to **resistivity** by:

$$R = \rho \frac{l}{A}$$

where:

- R = resistance (**Ω**)
- ρ = resistivity (**Ω·m**)
- *l* = conductor length (**m**)
- A = cross-sectional area (**m²**)

Substituting ρ:

Using the equation for resistivity:

$$R = [\rho_0\big(1 + \alpha(T - T_0)\big)]\frac{l}{A}$$

Since ***l* and A are constant**, we simplify:

$$R = \big[R_0\big(1 + \alpha(T - T_0)\big)\big]$$

where:

- $R_0 = \rho_0 \frac{l}{A}$ is the resistance at the reference temperature T_0.
- This equation shows that **resistance increases linearly with temperature** for most conductors.

4. Superconductors and Thermistors

A. Superconductors

- Some materials exhibit **zero resistance** below a critical temperature (T_c).

- This is called **superconductivity** and is used in **MRI machines, maglev trains, and quantum computing**.

B. Thermistors

- **Thermistors** are semiconductor devices with a **high negative α**.
- Used in **temperature sensors** since their resistance changes significantly with temperature.

Problem 1: Change in Resistance with Temperature

Given: A **copper wire** has a resistance of **10.0 Ω** at **20°C**. Find its resistance at **100°C**. (Given: $\alpha(Cu)=3.9\times10^{-3}$ °C).

Problem 2: Change in Resistivity

Given: A silver wire has a resistivity of **1.59×10^{-8} Ω·m** at **20°C**. Find its resistivity at **100°C**. (Given: $\alpha(Ag)=3.8\times10^{-3}$ °C).

Problem 3:

A resistance thermometer, which measures temperature by the change in resistance of a conductor, is made from platinum and has a resistance of 50 Ω at 20 °C. When immersed in a vessel containing melting indium, its resistance increases to 76.8 Ω. Calculate the melting point of the indium. Assume the temperature dependence of resistance for platinum is linear in this range with $\alpha = 3.92 \times 10^{-3}\ (°C)^{-1}$.

Solution

For a metal resistance thermometer, the resistance varies with temperature approximately linearly:

$R = R_0 [1 + \alpha (T - T_0)]$.

Solve for the temperature rise $\Delta T = T - T_0$:

$\Delta T = (R - R_0) / (\alpha R_0)$.

Insert the values: $R = 76.8\ \Omega$, $R_0 = 50\ \Omega$, $\alpha = 3.92 \times 10^{-3}\ (°C)^{-1}$.

$\Delta T = (76.8 - 50) / (3.92 \times 10^{-3} \times 50) = 136.735\ °C \approx 137\ °C$.

Therefore, $T = T_0 + \Delta T = 20\ °C + 136.735\ °C = 156.735\ °C \approx 157\ °C$.

Melting point of indium ≈ 157 °C.

6. Applications of Temperature-Dependent Resistance

☑ **Electrical Wiring:** High-voltage power lines use materials with low α to reduce energy losses.
☑ **Temperature Sensors:** Thermistors in medical devices and thermostats.
☑ **Superconductors:** Used in MRI machines, fusion reactors, and particle accelerators.
☑ **Electronic Circuits:** Some components (e.g., resistors) are designed to minimize temperature effects.

Option:

Example: Plotting Resistance vs. Temperature

This script calculates and plots the resistance of **copper** as a function of temperature using the equation:

Python Code:

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```
import numpy as np

import matplotlib.pyplot as plt

# Given values

R0 = 10  # Resistance at reference temperature (Ω)

T0 = 20  # Reference temperature (°C)

alpha_copper = 3.9e-3  # Temperature coefficient for copper (°C⁻¹)

# Define temperature range

T = np.linspace(-50, 200, 100)  # From -50°C to 200°C

# Calculate resistance at each temperature

R = R0 * (1 + alpha_copper * (T - T0))

# Plot the graph

plt.figure(figsize=(8, 5))

plt.plot(T, R, label="Copper", color='b')

plt.axvline(x=T0, color='r', linestyle='--', label="Reference Temp (20°C)")

# Labels and title

plt.xlabel("Temperature (°C)")

plt.ylabel("Resistance (Ω)")

plt.title("Resistance vs. Temperature for Copper")

plt.legend()

plt.grid(True)

plt.show()
```