

26-4 Resistance and Temperature

1. Introduction

- Electrical resistance (**R**) is a measure of how much a material **opposes or resists the flow of electric current**.
- The resistance of a material **changes with temperature** due to variations in atomic vibrations and electron scattering.
- This temperature dependence is crucial in designing electrical circuits, power transmission lines, and electronic sensors.

2. Resistivity and Temperature Dependence

- **Resistivity (ρ)** is an intrinsic property of a material that determines how much it resists current flow.
- The relationship between resistivity and temperature is given by:

$$\rho = \rho_0 [1 + \alpha(T - T_0)]$$

where:

- ρ = resistivity at temperature T ($\Omega \cdot \text{m}$)
- ρ_0 = resistivity at reference temperature T_0 (usually 20°C or 0°C)
- α = **temperature coefficient of resistivity** ($^\circ\text{C}^{-1}$)
- T, T_0 = temperatures in $^\circ\text{C}$

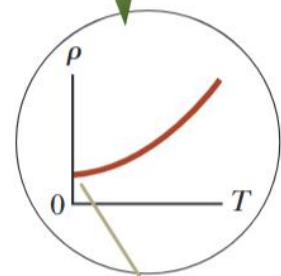
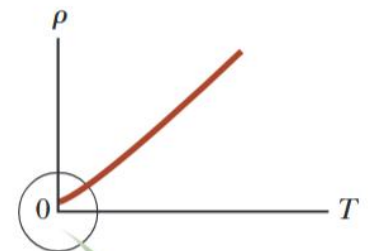
Also, we can calculate α as following”

$$\alpha = \frac{\Delta\rho/\rho_0}{\Delta T}$$

Where: $\Delta\rho = \rho - \rho_0$; $\Delta T = T - T_0$

Understanding α :

- **For metals:** $\alpha > 0$, meaning resistivity **increases** with temperature.
- **For semiconductors and insulators:** $\alpha < 0$, meaning resistivity **decreases** with temperature.



As T approaches absolute zero, the resistivity approaches a nonzero value.

Material	Resistivity at 20°C ($\Omega \cdot \text{m}$)	Temperature Coefficient (α) ($^{\circ}\text{C}^{-1}$)
Copper (Cu)	1.68×10^{-8}	3.9×10^{-3}
Aluminum (Al)	2.65×10^{-8}	4.0×10^{-3}
Silver (Ag)	1.59×10^{-8}	3.8×10^{-3}
Silicon (Si)	6.4×10^{26}	-75×10^{-3}

3. Derivation of Resistance-Temperature Relationship

We know that **resistance** is related to **resistivity** by:

$$R = \rho \frac{l}{A}$$

where:

- R = resistance (Ω)
- ρ = resistivity ($\Omega \cdot \text{m}$)
- l = conductor length (m)
- A = cross-sectional area (m^2)

Substituting ρ :

Using the equation for resistivity:

$$R = [\rho_0(1 + \alpha(T - T_0))] \frac{l}{A}$$

Since l and A are constant, we simplify:

$$R = [R_0(1 + \alpha(T - T_0))]$$

where:

- $R_0 = \rho_0 \frac{l}{A}$ is the resistance at the reference temperature T_0 .
- This equation shows that **resistance increases linearly with temperature** for most conductors.

4. Superconductors and Thermistors

A. Superconductors

- Some materials exhibit **zero resistance** below a critical temperature (T_c).

- This is called **superconductivity** and is used in **MRI machines, maglev trains, and quantum computing**.

B. Thermistors

- **Thermistors** are semiconductor devices with a **high negative α** .
- Used in **temperature sensors** since their resistance changes significantly with temperature.

Problem 1: Change in Resistance with Temperature

Given: A **copper wire** has a resistance of **10.0 Ω** at **20°C**. Find its resistance at **100°C**.
(Given: $\alpha(\text{Cu})=3.9 \times 10^{-3} \text{ }^\circ\text{C}^{-1}$). (**Answer:** 13.12 Ω)

Problem 2: Change in Resistivity

Given: A silver wire has a resistivity of **1.59 $\times 10^{-8} \Omega \cdot \text{m}$** at **20°C**. Find its resistivity at **100°C**.
(Given: $\alpha(\text{Ag})=3.8 \times 10^{-3} \text{ }^\circ\text{C}^{-1}$). (**Answer:** 2.7 $\times 10^{-8} \Omega \cdot \text{m}$)

Problem 3:

A resistance thermometer, which measures temperature by the change in resistance of a conductor, is made from platinum and has a resistance of 50 Ω at 20 $^\circ\text{C}$. When immersed in a vessel containing melting indium, its resistance increases to 76.8 Ω . Calculate the melting point of the indium. Assume the temperature dependence of resistance for platinum is linear in this range with $\alpha = 3.92 \times 10^{-3} \text{ (}^\circ\text{C)}^{-1}$.

Solution

For a metal resistance thermometer, the resistance varies with temperature approximately linearly:

$$R = R_0 [1 + \alpha (T - T_0)].$$

Solve for the temperature rise $\Delta T = T - T_0$:

$$\Delta T = (R - R_0) / (\alpha R_0).$$

Insert the values: $R = 76.8 \Omega$, $R_0 = 50 \Omega$, $\alpha = 3.92 \times 10^{-3} \text{ (}^\circ\text{C)}^{-1}$.

$$\Delta T = (76.8 - 50) / (3.92 \times 10^{-3} \times 50) = 136.735 \text{ }^\circ\text{C} \approx 137 \text{ }^\circ\text{C}.$$

Therefore, $T = T_0 + \Delta T = 20 \text{ }^\circ\text{C} + 136.735 \text{ }^\circ\text{C} = 156.735 \text{ }^\circ\text{C} \approx 157 \text{ }^\circ\text{C}$.

Melting point of indium $\approx 157 \text{ }^\circ\text{C}$.

6. Applications of Temperature-Dependent Resistance

- ✓ **Electrical Wiring:** High-voltage power lines use materials with low α to reduce energy losses.
- ✓ **Temperature Sensors:** Thermistors in medical devices and thermostats.
- ✓ **Superconductors:** Used in MRI machines, fusion reactors, and particle accelerators.
- ✓ **Electronic Circuits:** Some components (e.g., resistors) are designed to minimize temperature effects.

Option:

Example: Plotting Resistance vs. Temperature

This script calculates and plots the resistance of **copper** as a function of temperature using the equation:

Python Code:

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import numpy as np

import matplotlib.pyplot as plt

# Given values

R0 = 10 # Resistance at reference temperature ( $\Omega$ )

T0 = 20 # Reference temperature ( $^{\circ}\text{C}$ )

alpha_copper = 3.9e-3 # Temperature coefficient for copper ( $^{\circ}\text{C}^{-1}$ )

# Define temperature range

T = np.linspace(-50, 200, 100) # From  $-50^{\circ}\text{C}$  to  $200^{\circ}\text{C}$ 

# Calculate resistance at each temperature

R = R0 * (1 + alpha_copper * (T - T0))

# Plot the graph

plt.figure(figsize=(8, 5))

plt.plot(T, R, label="Copper", color='b')

plt.axvline(x=T0, color='r', linestyle='--', label="Reference Temp ( $20^{\circ}\text{C}$ )")

# Labels and title

plt.xlabel("Temperature ( $^{\circ}\text{C}$ )")

plt.ylabel("Resistance ( $\Omega$ )")

plt.title("Resistance vs. Temperature for Copper")

plt.legend()

plt.grid(True)

plt.show()
```