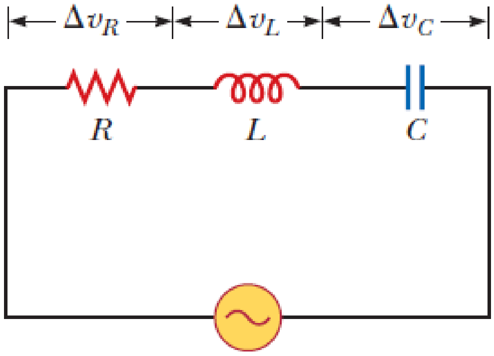
33.5 The RLC series circuit

In the previous sections we considered individual elements connected to an AC source. Here, we will adopt a circuit that contains a combination of circuit elements: a resistor, an indicator, and a capacitor.



The instantaneous applied voltage is

$$\Delta v = V_{max} \sin \omega t$$

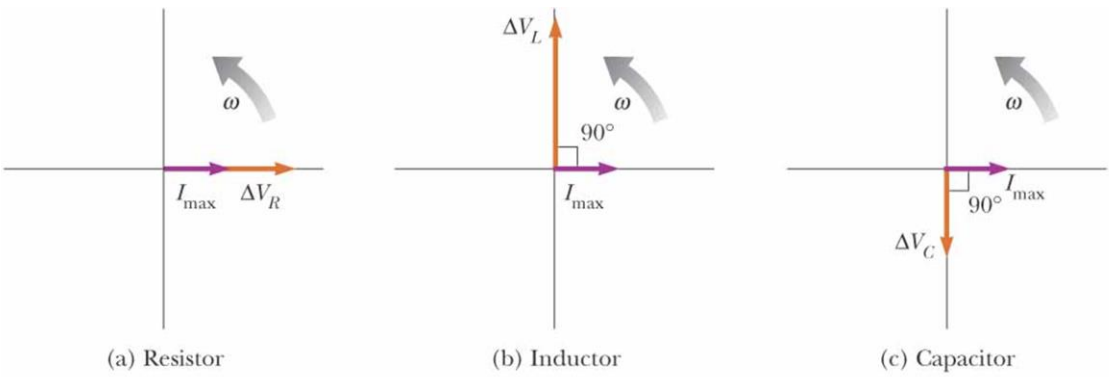
The current also varies as

$$i = I_{max} \sin(\omega t - \phi) \qquad 33.10$$

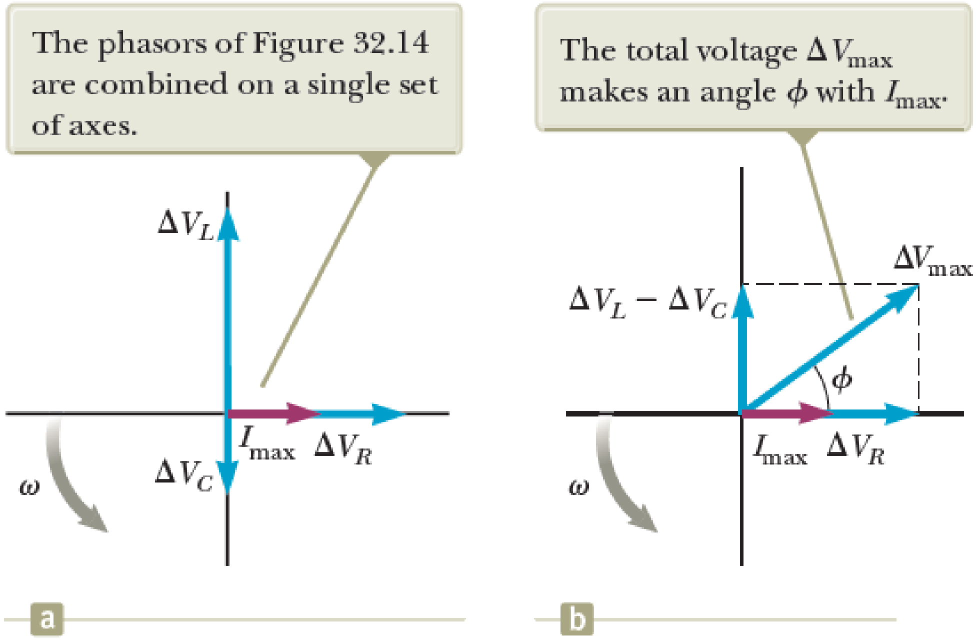
where φ is some phase angle between the current and the applied voltage. Based on our discussion in chapter 28, the current everywhere in the circuit must be the same, while the voltage across each element has a different amplitude and phase.

$$\Delta v_R = I_{max} R = \Delta V_R \sin \omega t$$
$$\Delta v_L = I_{max} X_L \sin\left(\omega t + \frac{\pi}{2}\right) = \Delta V_L \cos \omega t$$
$$\Delta v_C = I_{max} X_C \sin\left(\omega t - \frac{\pi}{2}\right) = -\Delta V_C \cos \omega t$$

Phase relationships between the voltage and current phasors for (a) a resistor, (b) an indicator, and (c) a capacitor connected in series are shown in following figure.



Because the phasors are rotating vectors, the voltage phasors can be combined using vector addition as in the following figure.



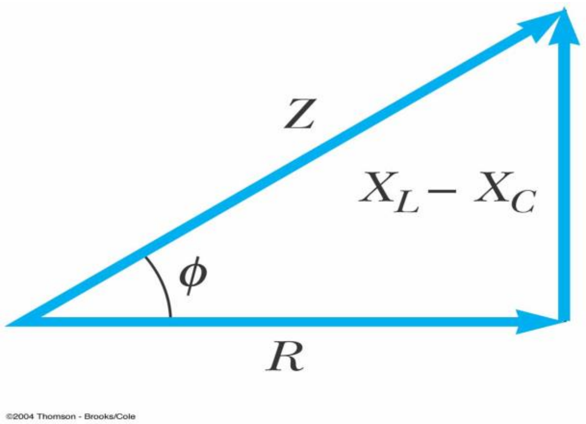
From this figure, we conclude that,

$$\Delta V_{max} = \sqrt{\Delta V_R^2 + (\Delta V_L - \Delta V_C)^2} = \sqrt{(I_{max}R)^2 + (I_{max}X_L - I_{max}X_C)^2}$$
$$\Delta V_{max} = I_{max}\sqrt{R^2 + (X_L - X_C)^2} \qquad 33.12$$
$$I_{max} = \frac{\Delta V_{max}}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\Delta V_{max}}{Z}$$

Where $Z = \sqrt{R^2 + (X_L - X_C)^2}$ and is called the impedance of the circuit.

From the figure, one can also find the phase angel φ,

$$\phi = \tan^{-1}\left(\frac{\Delta V_L - \Delta V_C}{\Delta V_R}\right) = \tan^{-1}\left(\frac{I_{max}X_L - I_{max}X_C}{I_{max}R}\right) = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \qquad 33.13$$



Exercise:

Based on the phase angle in eq. (33.13), discuss the following situations:

1- When $X_L > X_C$
2- When $X_L < X_C$
3- When $X_L = X_C$

⚡ Phase Angle and Frequency in RLC Circuits

In an **RLC series AC circuit**, the **phase angle (φ)** between the current and the voltage depends on the relationship between **inductive reactance (X_L)** and **capacitive reactance (X_C)**:

◆ 1. When $X_L > X_C$

- This occurs at **high frequencies**.
- The **net reactance is inductive**, so:
  - The **current lags** behind the voltage.
  - The **phase angle φ is positive**.
- We say the circuit is **inductive**.

◈ 2. When $X_L < X_C$

- This occurs at **low frequencies**.
- The **net reactance is capacitive**, so:
  - The **current leads** the voltage.
  - The **phase angle φ is negative**.
- The circuit is **capacitive**.

⚖ 3. When $X_L = X_C$

- The **inductive and capacitive effects cancel out**.
- The circuit is **resonant** or **purely resistive**.
- The **phase angle is zero**:
  - **Voltage and current are in phase**.

📘 Summary Table:

$$\text{Inductive reactance: } X_L = \omega L = 2\pi f L$$

$$\text{Capacitive reactance: } X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

Condition	**φ (Phase Angle)**	**Behavior**
$X_L > X_C$	Positive	Inductive: current lags
$X_L < X_C$	Negative	Capacitive: current leads
$X_L = X_C$	Zero	Purely resistive

Example-1:

A series RLC circuit has R = 425 V, L = 1.25 H, and C = 3.50 mF. It is connected to an AC source with f = 60.0 Hz and ΔV_{max} = 150 V.

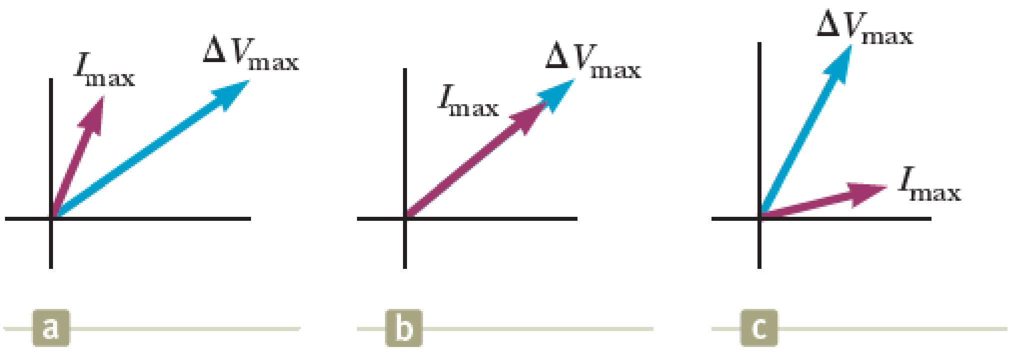
(A) Determine the inductive reactance, the capacitive reactance, and the impedance of the circuit.
(B) Find the maximum current in the circuit.
(C) Find the phase angle between the current and voltage.
(D) Find the maximum voltage across each element.
(E) What replacement value of L should an engineer analyzing the circuit choose such that the current leads the applied voltage by 30^0 rather than 34^0? All other values in the circuit stay the same.

Quiz-1:

Examine the **Figure**, which includes diagrams labeled (a), (b), and (c).
For each part, determine the correct relationship between the inductive reactance X_L and the capacitive reactance X_C.
Label each as one of the following:

- $X_L > X_C$
- $X_L < X_C$
- $X_L = X_C$

Indicate which condition corresponds to each part: (a), (b), and (c).



(a)

- Current **leads** voltage (phasor is ahead in rotation direction).
- ✅ **This means $X_L < X_C$** (capacitive behavior).

(b)

- Current and voltage are **in phase**.
- ✅ **This means $X_L = X_C$** (resonance).

(c)

- Voltage **leads** current.
- ✅ **This means $X_L > X_C$** (inductive behavior).

Quiz-2: In a series RLC circuit, increasing the frequency of the source causes which of the following to decrease?

A. Inductive reactance X_L
B. Capacitive reactance X_C
C. Resonant frequency
D. Total impedance near resonance