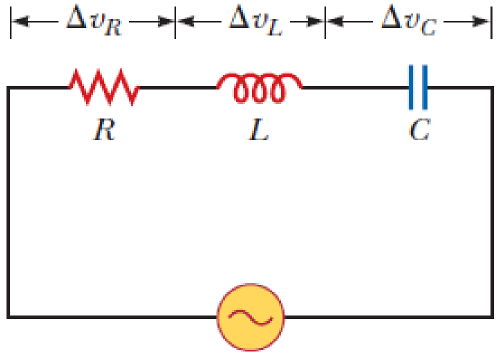
32.5 The RLC series circuit



In the previous sections, we considered individual elements connected to an AC source. Here, we will adopt a circuit that contains a combination of circuit elements: a resistor, an indicator, and a capacitor.

The instantaneous applied voltage is

$$\Delta v = V_{max} \sin \omega t$$

The current also varies as

$$i = I_{max} \sin(\omega t - \phi) \qquad 32.10$$

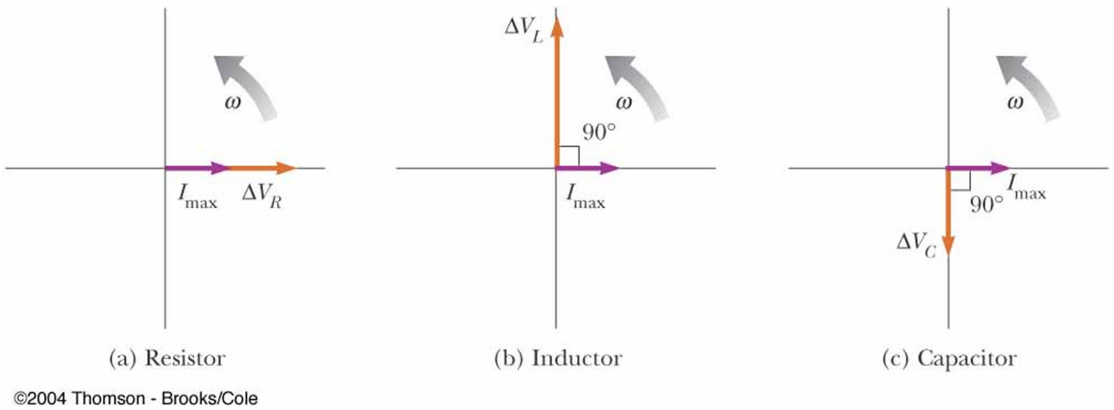
where φ is some phase angle between the current and the applied voltage. Based on our discussion in chapter 27, the current everywhere in the circuit must be the same, while the voltage across each element has a different amplitude and phase.

$$\Delta v_R = I_{max} R = \Delta V_R \sin \omega t$$

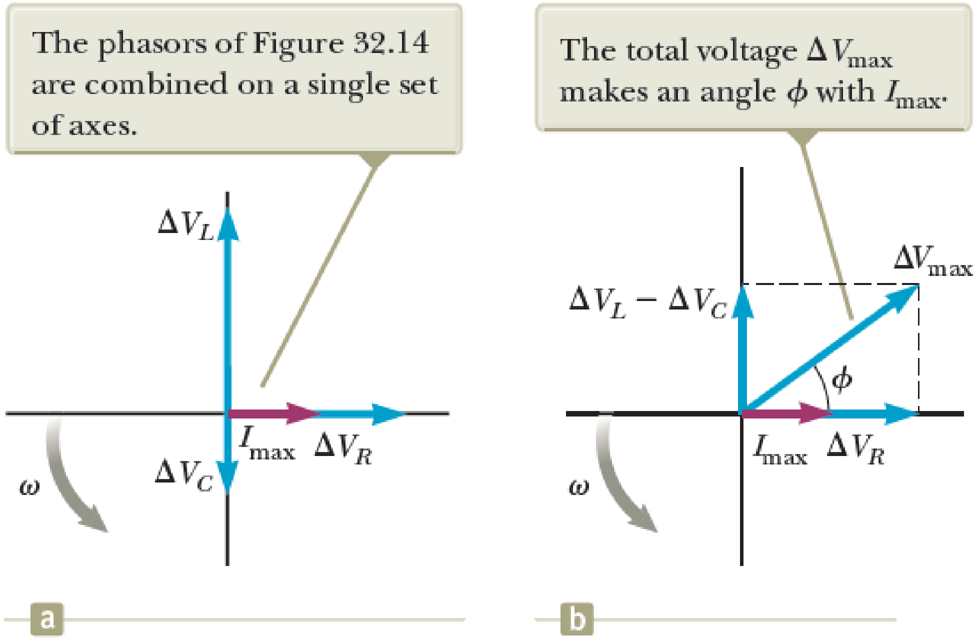
$$\Delta v_L = I_{max} X_L \sin\left(\omega t + \frac{\pi}{2}\right) = \Delta V_L \cos \omega t$$

$$\Delta v_C = I_{max} X_C \sin\left(\omega t - \frac{\pi}{2}\right) = -\Delta V_C \cos \omega t$$

Phase relationships between the voltage and current phasors for (a) a resistor, (b) an indicator, and (c) a capacitor connected in series are shown in the following figure.



Because the phasors are rotating vectors, the voltage phasors can be combined using vector addition as in the following figure.



From this figure, we conclude that,

$$\Delta V_{max} = \sqrt{\Delta V_R^2 + (\Delta V_L - \Delta V_C)^2} = \sqrt{(I_{max}R)^2 + (I_{max}X_L - I_{max}X_C)^2}$$

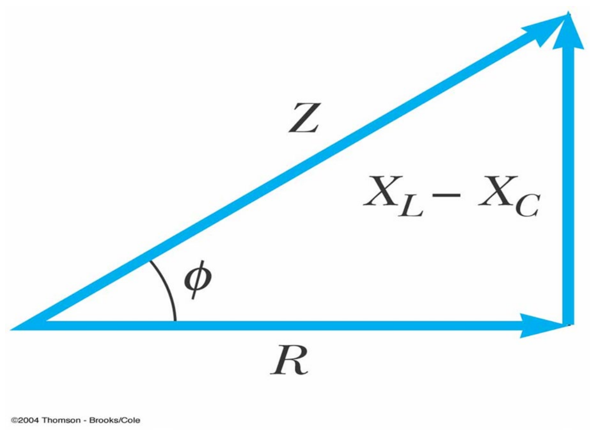
$$\Delta V_{max} = I_{max}\sqrt{R^2 + (X_L - X_C)^2} \qquad 32.12$$

$$I_{max} = \frac{\Delta V_{max}}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\Delta V_{max}}{Z}$$

Where $Z = \sqrt{R^2 + (X_L - X_C)^2}$ and is called the impedance of the circuit.

From the figure, one can also find the phase angle φ,

$$\phi = \tan^{-1}\left(\frac{\Delta V_L - \Delta V_C}{\Delta V_R}\right) = \tan^{-1}\left(\frac{I_{max}X_L - I_{max}X_C}{I_{max}R}\right) = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \qquad 32.13$$



Exercise:

Based on the phase angle in eq. (32.13), discuss the following situations:

1- When $X_L > X_C$
2- When $X_L < X_C$
3- When $X_L = X_C$

⚡ Phase Angle and Frequency in RLC Circuits

In an **RLC series AC circuit**, the **phase angle (φ)** between the current and the voltage depends on the relationship between **inductive reactance (X_L)** and **capacitive reactance (X_C)**:

◆ 1. When $X_L > X_C$

- This occurs at **high frequencies**.
- The **net reactance is inductive**, so:
  - The **current lags** behind the voltage.
  - The **phase angle φ is positive**.
- We say the circuit is **inductive**.

◈ 2. When $X_L < X_C$

- This occurs at **low frequencies**.
- The **net reactance is capacitive**, so:
  - The **current leads** the voltage.
  - The **phase angle φ is negative**.
- The circuit is **capacitive**.

⚖️ 3. When $X_L = X_C$

- The **inductive and capacitive effects cancel out**.
- The circuit is **resonant** or **purely resistive**.
- The **phase angle is zero**:
  - **Voltage and current are in phase**.

📓 Summary Table:

$$\text{Inductive reactance: } X_L = \omega L = 2\pi f L$$

$$\text{Capacitive reactance: } X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

Condition	**φ (Phase Angle)**	**Behavior**
$X_L > X_C$	Positive	Inductive: current lags
$X_L < X_C$	Negative	Capacitive: current leads
$X_L = X_C$	Zero	Purely resistive

Quick Quiz 32.3

In the AC circuit shown in **Figure 32.11**, the **AC source frequency** is varied while the voltage amplitude is held **constant**.
At which frequency does the lightbulb glow the brightest?

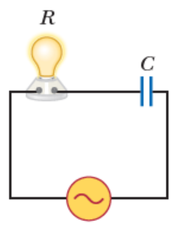


Figure 32.11 (Quick Quiz 32.3)

(a) At high frequencies
(b) At low frequencies
(c) The brightness remains the same at all frequencies

Quick Quiz 32.4

Refer to the AC circuit in **Figure 32.12**. The **AC source frequency is adjusted, while** the **voltage amplitude is held constant**.
When does the lightbulb reach its maximum brightness?

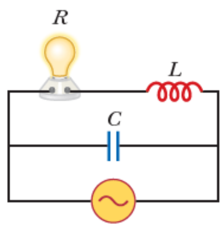


Figure 32.12 (Quick Quiz 32.4)

(a) At high frequencies
(b) At low frequencies
(c) The brightness is unaffected by frequency

Example-1:

A series RLC circuit has R = 425 Ω, L = 1.25 H, and C = 3.50 mF. It is connected to an AC source with f = 60.0 Hz and ΔV_{max} = 150 V.

(A) Determine the inductive reactance, the capacitive reactance, and the impedance of the circuit.
(B) Find the maximum current in the circuit.
(C) Find the phase angle between the current and voltage.

(D) Find the maximum voltage across each element.
(E) What replacement value of L should an engineer analyzing the circuit choose such that the current leads the applied voltage by 30^0 rather than 34^0? All other values in the circuit stay the same.

Solution:

(a) Reactances and impedance:

$X_L = \omega L = (377\ s^{-1})(1.25\ H) = 471\ \Omega$

$X_C = 1/(\omega C) = 1\ /\ [(377\ s^{-1})(3.50\times10^{-6}\ F)] = 758\ \Omega$

$Z = \sqrt{(R^2 + (X\_L - X\_C)^2)} = \sqrt{(425^2 + (471 - 758)^2)} = 513\ \Omega$

(b) Maximum current:

$I_{max} = \Delta V_{max}\ /\ Z = 150\ V\ /\ 513\ \Omega = 0.293\ A$

(c) Phase angle (current relative to source voltage):

$\varphi = \tan^{-1}[(X_L - X_C)/R] = \tan^{-1}[(471 - 758)/425] = -34.0°$

Since $X_C > X_L$, φ is negative and the circuit is more capacitive; the current leads the voltage by $|\varphi|$.

(d) Maximum and instantaneous voltages across each element:

$\Delta V_R(max) = I_{max}\ R = (0.293\ A)(425\ \Omega) = 124\ V$

$\Delta V_L(max) = I_{max}\ X_L = (0.293\ A)(471\ \Omega) = 138\ V$

$\Delta V_C(max) = I_{max}\ X_C = (0.293\ A)(758\ \Omega) = 222\ V$

Choosing i(t) as the reference sinusoid and $\omega = 377\ s^{-1}$:

$v_R(t) = (124\ V)\ \sin(377\ t)$

$v_L(t) = (138\ V)\ \cos(377\ t)$

$v_C(t) = (-221\ V)\ \cos(377\ t)$

Example-2:

A sinusoidal voltage source

$\Delta v(t) = (40.0\ V)\ \sin(100t)$

is applied to a series RLC circuit with the following parameters:

$L = 160\ mH$
$C = 99.0\ \mu F$
$R = 68.0\ \Omega$

(a) What is the impedance of the circuit?
(b) What is the maximum current?

(c) Determine the numerical values for I_{max}, ω, and φ in the current equation:
$i(t) = I_{max} \sin(\omega t - \varphi)$.

Solution

(a) Reactances and impedance:

$X_L = \omega L = 100 \times 160\times10^{-3} = 16.0\ \Omega$

$X_C = 1/(\omega C) = 1 / [100 \times 99.0\times10^{-6}] = 101.0\ \Omega$

$Z = \sqrt{[R^2 + (X_L - X_C)^2]} = \sqrt{[(68.0)^2 + (16.0 - 101.0)^2]} = 109.0\ \Omega$

(b) Maximum current:

$I_{max} = \Delta V_{max} / Z = 40.0 / 109.0 = 0.367$ A

(c) Phase constant and current expression:

$\varphi = \tan^{-1}[(X_L - X_C)/R] = \tan^{-1}[(16.0 - 101.0)/68.0] = -51.3°$

Since $X_C > X_L$, the circuit is capacitive and the current leads the voltage.

Final current expression:

$i(t) = 0.367 \sin(100t + 51.3°)$ A

Example – 3:

An AC source with amplitude $\Delta V_{max} = 150$ V and frequency f = 50.0 Hz is connected between points a and d in a series RLC circuit.

Components:

- Resistor between a and b: R = 40.0 Ω
- Inductor between b and c: L = 185 mH
- Capacitor between c and d: C = 65.0 μF

Calculate the maximum voltages between:

a. a and b
b. b and c
c. c and d
d. b and d

Solution

Step 1 – Reactances

The angular frequency is $\omega = 2\pi f = 2\pi(50.0) \approx 314$ rad/s.

Capacitive reactance: $X_C = 1 / (\omega C) \approx 49.0\ \Omega$
Inductive reactance: $X_L = \omega L \approx 58.1\ \Omega$

Step 2 – Impedance and maximum current

The impedance of the series RLC circuit is:
$Z = \sqrt{[R^2 + (X_L - X_C)^2]} \approx 41.0\ \Omega$

Maximum current:
$I_{max} = \Delta V_{max} / Z = 150 / 41.0 \approx 3.66$ A

(a) Voltage between a and b

This is the voltage across the resistor:
$\Delta V_R = I_{max} \cdot R = (3.66)(40.0) \approx$ **146 V**

(b) Voltage between b and c

This is the voltage across the inductor:
$\Delta V_L = I_{max} \cdot X_L = (3.66)(58.1) \approx$ **212 V**

(c) Voltage between c and d

This is the voltage across the capacitor:
$\Delta V_C = I_{max} \cdot X_C = (3.66)(49.0) \approx$ **179 V**

(d) Voltage between b and d

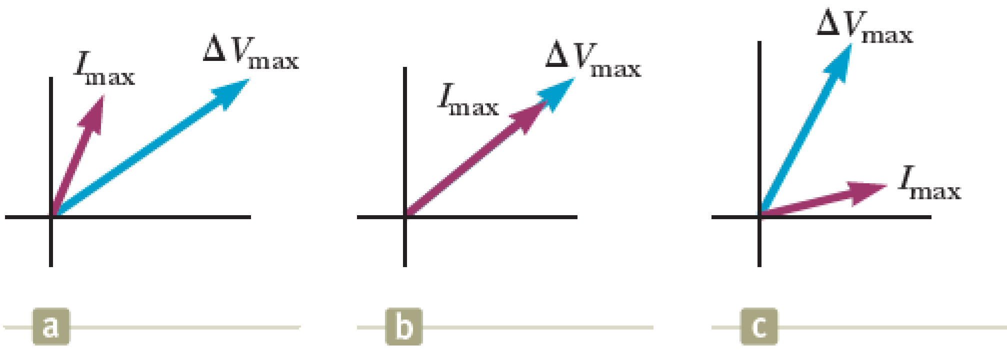
Between b and d we span the inductor and capacitor. Their voltages are out of phase, so the net maximum is the difference:
$\Delta V_{bd} = |\Delta V_L - \Delta V_C| = |212 - 179| \approx$ **33.4 V**

Quiz-1:

Examine the **Figure**, which includes diagrams labeled (a), (b), and (c).
For each part, determine the correct relationship between the inductive reactance X_L and the capacitive reactance X_C.
Label each as one of the following:

- $X_L > X_C$
- $X_L < X_C$
- $X_L = X_C$

Indicate which condition corresponds to each part: (a), (b), and (c).



(a)

- Current **leads** voltage (phasor is ahead in rotation direction).
- ✅ **This means $X_L < X_C$** (capacitive behavior).

(b)

- Current and voltage are **in phase**.
- ✅ **This means $X_L = X_C$** (resonance).

(c)

- Voltage **leads** current.
- ✅ **This means $X_L > X_C$** (inductive behavior).

Quiz-2: In a series RLC circuit, increasing the frequency of the source causes which of the following to decrease?

A. Inductive reactance X_L
B. Capacitive reactance X_C
C. Resonant frequency
D. Total impedance near resonance