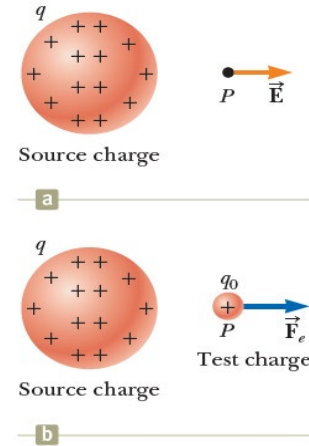


22.4 Analysis Model: Particle in a Field (Electric)

➤ Concept of electric field

- An electric field represents the region around a charged object where other charges (around the source charge) experience a force due to its influence.
- A charged particle, with charge q , produces an electric field in the region of space around it.
- A small test charge q_0 is placed near the source charge to detect and measure the presence and direction of the electric field.



➤ Definition of the electric field:

The electric field E at a point in space is defined as the electric force F_e acting on a positive test charge q_0 placed at that point divided by the magnitude of the test charge:

$$\vec{E} = \frac{\vec{F}}{q_0}$$

As we discussed in the previous section, the electric force due to point charges can be written as:

$$\vec{F} = K_e \frac{|q_1||q_2|}{r^2} \hat{r}$$

So, one can rewrite the electric force between q and q_0 as follows:

$$\vec{F} = K_e \frac{q q_0}{r^2} \hat{r}$$

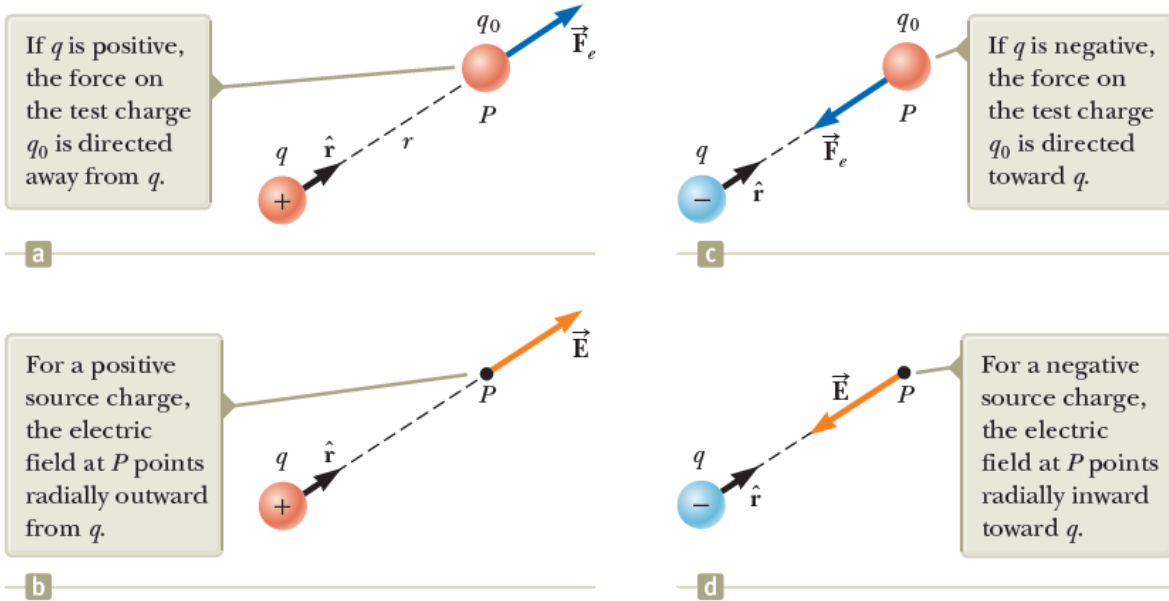
The electric field can also be derived:

$$\vec{E} = \frac{\vec{F}}{q_0} = K_e \frac{q q_0}{q_0 r^2} \hat{r} = K_e \frac{q}{r^2} \hat{r}$$

The SI unit of electric field is the newton per coulomb (N/C).

➤ The direction of electric force and electric field:

Remember: $\vec{F} = q \vec{E}$



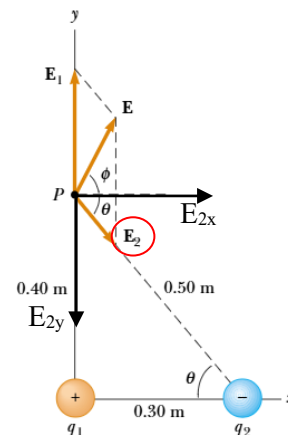
At any point P , the total electric field due to a group of charges equals the vector sum of the electric fields of the individual charges.

$$\vec{E} = k_e \sum_i \frac{q_i}{r_i^2} \hat{r}_i$$

Example-1:

A charge $q_1 = 7 \mu\text{C}$ is located at the origin and second charge $q_2 = -5 \mu\text{C}$ is located on the x axis, 0.30 m from the origin. Find the electric field at the point P which has the coordinates (0, 0.40) m.

Solution:



$$\vec{E} = \vec{E}_1 + \vec{E}_2 = |\vec{E}_1|(+\hat{j}) + |\vec{E}_2|(+\hat{i}) + |\vec{E}_2|(-\hat{j})$$

$$= E_{1y} + E_{2x} - E_{2y}$$

$$1- \vec{E}_1 = |\vec{E}_1| \hat{r} = K \frac{q_1}{r^2} \hat{r} = 9 \times 10^9 \frac{(7 \times 10^{-6})}{(0.4)^2} (+\hat{j})$$

$$= 3.9 \times 10^5 \hat{j} \text{ N/C}$$

$$2- \vec{E}_2 = |\vec{E}_2| \hat{r} = E_{2x}\hat{i} - E_{2y}\hat{j} = |\vec{E}_2| \cos \theta \hat{i} - |\vec{E}_2| \sin \theta \hat{j}$$

$$r^2 = (0.3)^2 + (0.4)^2 = 0.25 \quad r = 0.5 \text{ m}$$

$$\cos \theta = \frac{0.3}{0.5} \quad \sin \theta = \frac{0.4}{0.5}$$

$$|\vec{E}_2| = K \frac{q_2}{r^2} = 9 \times 10^9 \frac{(5 \times 10^{-6})}{(0.5)^2} = 1.8 \times 10^5 \text{ N/C}$$

$$\vec{E}_2 = |\vec{E}_2| (\cos \theta \hat{i} - \sin \theta \hat{j}) = 1.8 \times 10^5 \left(\frac{0.3}{0.5} \hat{i} - \frac{0.4}{0.5} \hat{j} \right)$$

$$3- \vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= (3.9 \times 10^5) \hat{j} + (1.1 \times 10^5) \hat{i} - (1.44 \times 10^5) \hat{j}$$

$$= (1.1 \hat{i} + 2.44 \hat{j}) \times 10^5$$

This is the coordinate of the electric field at P point.

$$4- |\vec{E}| = \sqrt{\vec{E}_1^2 + \vec{E}_2^2} = \sqrt{(1.1 \times 10^5)^2 + (2.44 \times 10^5)^2}$$

$$= 2.7 \times 10^5 \text{ N/C}$$

This is the magnitude of the electric field at point P.

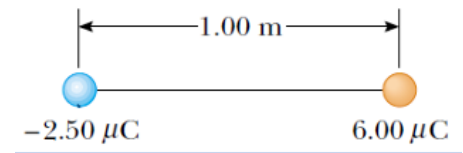
$$5- \tan \phi = \frac{E_y}{E_x} = \frac{2.44 \times 10^5}{1.1 \times 10^5} = 2.22$$

$$\phi = \tan^{-1}(2.22) = 65.73^\circ$$

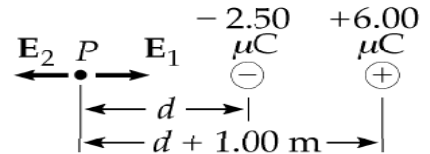
ϕ represents the direction of the electric field at point P.

Example-2:

In this figure, determine the point where the electric field is zero.



Solution:



$$\vec{E}_P = \vec{E}_1 + \vec{E}_2$$

$$\vec{E}_P = \mathbf{0} \implies \vec{E}_1 = \vec{E}_2$$

$$K \frac{q_1}{d^2} = K \frac{q_2}{(1+d)^2} \implies q_1(1+d)^2 = q_2 d^2$$

$$\sqrt{\frac{q_1}{q_2}} (1+d) = \pm d \implies d \left(\sqrt{\frac{q_1}{q_2}} \mp 1 \right) = - \sqrt{\frac{q_1}{q_2}}$$

Please consider this step carefully, especially when given $q_1 = q_2$.

$$d = 1.8 \text{ m} \quad \text{from } q_1$$