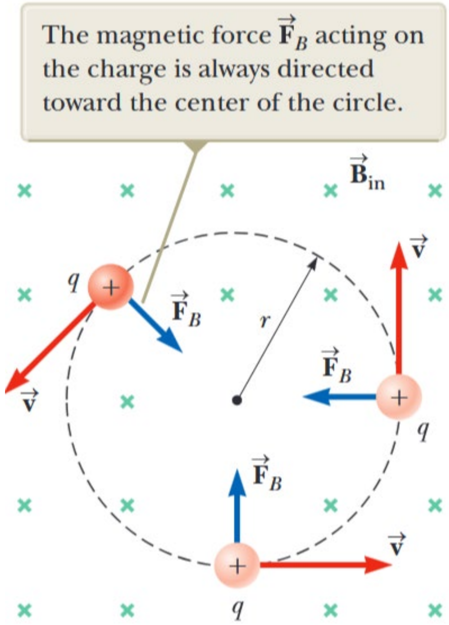
28.2 **Motion of a charged particle in a uniform magnetic field**

The magnetic force acting on a charged particle moving in a **magnetic field is perpendicular to the velocity of** the particle, and consequently, the work done on the particle by the magnetic force is zero.

Because F_B always points toward the circle's center, it changes only the direction of **v** and not its magnitude.

The rotation is counterclockwise for a positive charge. If *q were negative, the rotation would be clockwise.*



$$F_m = F_C$$

$$qvB = m\frac{v^2}{r} \qquad \textbf{28.2}$$

From this equation (28.2), one can calculate **the radius of the path** of a charged particle,

$$r = \frac{mv}{qB}$$

Also, we can calculate **the angular speed**,

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

Cyclotron frequency f (number of revolutions per second):

$$f = \frac{\omega}{2\pi} = \frac{qB}{2\pi m}$$

Moreover, **the time period**,

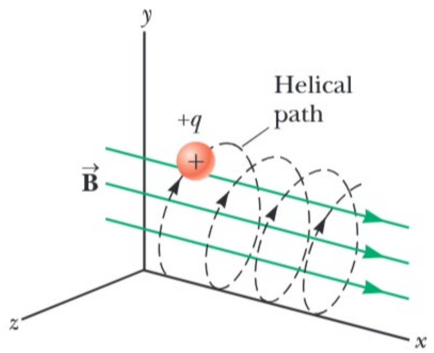
$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

Please remember that the period of the motion T is the time that the particle takes to complete one revolution and is equal to the circumference of the circle divided by the linear speed of the particle.

General Case: Helical Motion

- If the particle's velocity has **both parallel and perpendicular components** to B:

$$v = \sqrt{v_{\perp}^2 + v_{\parallel}^2}$$



➢ The particle moves in a **helix**:
   - Circular motion due to $v_{\perp}$(v_y as shown in the figure).
   - Uniform motion along the magnetic field direction due to $v_{\parallel}$ (v_x as shown in the figure).

➢ **Velocity Parallel to Magnetic Field**

- If the particle moves **parallel** to B (θ=0° or 180°):

F=qvB sin0 = 0

- **No force**, and the particle continues in a straight line with constant velocity.

Comments:

- Magnetic force does **no work** (because it's perpendicular to velocity).
- **Speed remains constant**, only the **direction** changes.
- Radius and frequency depend on the particle's mass, charge, velocity, and magnetic field strength.

Example-1: A proton is moving in a circular orbit of radius 14 cm in a uniform 0.35-T magnetic field perpendicular to the velocity of the proton. Find the speed of the proton.

• $r = 14$ cm $= 0.14$ m
• $B = 0.35$ T ($\perp$ to v)
• $q = +e = 1.602\times10^{-19}$ C, $m_p = 1.673\times10^{-27}$ kg

Solution:

For uniform circular motion under magnetic force: $q\,v\,B = m\,v^2 / r$.

Solve for v: $v = q\,B\,r / m$.

Substitute: $v = (1.602\times10^{-19}\text{ C})(0.35\text{ T})(0.14\text{ m}) / (1.673\times10^{-27}\text{ kg})$.

$v \approx 4.694\text{x}10^{+06}$ m/s $\approx 4.69 \times 10^6$ m/s.

Answer:

v ≈ 4.69e+06 m/s (about 4.69e+03 km/s).

Example-2: If an electron moves with linear velocity 5×10^3 m/s, under a perpendicular magnetic field of 8 T, what is the radius of its angular path?

A. 5 mm   B. 3.6 nm   C. 1.6 nm   D. 1.4 µm

• $v = 5\times10^3$ m/s
• $B = 8$ T ($\perp$ to v)
• $q = -e$ (use $|q|$), $m_e = 9.109\times10^{-31}$ kg

Solution:

Radius in a uniform B: $r = m\,v / (|q|\,B)$.

$r = (9.109\times10^{-31}\text{ kg})(5\times10^3\text{ m/s}) / ((1.602\times10^{-19}\text{ C})(8\text{ T}))$.

$r \approx 3.554^{-09}$ m $\approx 3.55 \times 10^{-9}$ m $= 3.55$ nm.

Example-3: A 20 mC charge moves in a circular orbit, making 20 turns/s. If the magnetic field, perpendicular to the motion, is 3 T, what is the mass of the particle?

A) 4.77×10^{-4} B) 9.54×10^{-4} C) 1.59×10^{-4} D) 0.24×10^{-4}

Given:

- $q = 20$ mC = 0.020 C
- $f = 20\ s^{-1}$ (cyclotron frequency)
- $B = 3$ T

Solution:

Cyclotron frequency: $f = q B / (2\pi m) \Rightarrow m = q B / (2\pi f)$.

$m = (0.020\ C)(3\ T) / (2\pi \cdot 20\ s^{-1})$.

$m \approx 4.775x10^{-04}$ kg $\approx 4.77 \times 10^{-4}$ kg.

Example-4: An electron ($q=1.6\times10^{-19}$ C, $m=9.1\times10^{-31}$ kg) moves perpendicular to a 0.5 T magnetic field with speed 2×10^{6} m/s.

1. Find the radius of the circular path.
2. Find the frequency of revolution.

- $q = 1.6\times10^{-19}$ C
- $m = 9.1\times10^{-31}$ kg
- $B = 0.5$ T
- $v = 2\times10^{6}$ m/s

Solution:

1 Radius of circular path: $r = m v / (q B)$

$r = (9.1\times10^{-31}\ kg \times 2\times10^{6}\ m/s) / ((1.6\times10^{-19}\ C)(0.5\ T))$

$r = 2.275x^{-05}$ m = 0.023 mm ≈ 22750.00 nm

2 Frequency of revolution: $f = q B / (2\pi m)$

$f = (1.6\times10^{-19}\ C \times 0.5\ T) / (2\pi \times 9.1\times10^{-31}\ kg)$

$f = 1.399x10^{+10}$ Hz ≈ 13.99 GHz

Example 5: The time period for one complete revolution of a charged particle in a magnetic field depends on:

A) Speed of the particle
B) Charge of the particle
C) Mass of the particle
D) Both B and C

Example 6: What happens to the radius of a charged particle's path if the magnetic field strength is doubled?

A) Doubled
B) Halved
C) Unchanged
D) Zero

Example 7: If a charged particle enters a magnetic field at some angle other than 90° or 0°, what will be its path?

A) Straight line
B) Circular
C) Helical
D) Random motion

Example 8: A proton and an electron enter a magnetic field perpendicular to their velocities. How will their circular paths compare?

A) Same radius, same direction
B) Different radii, same direction
C) Same radius, opposite directions
D) Different radii, opposite directions