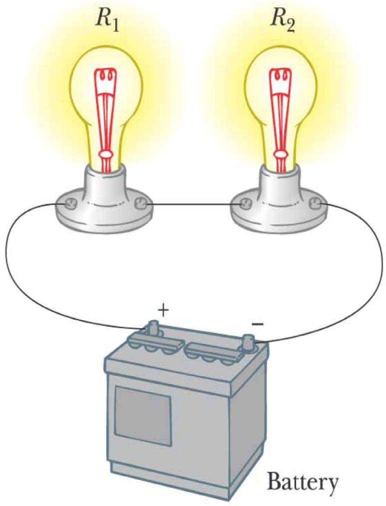
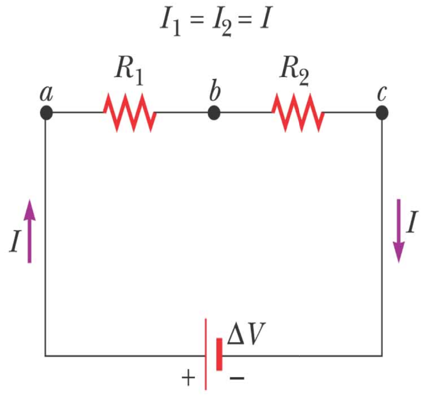
28.2 resistors in series and parallel:

- **Combining resistors in series:**



(a)



(b)

When two or more resistors are connected together as are the light- bulbs in the figure, they are said to be in *series.*

In a series connection, all the charges moving through one resistor must also pass through the second resistor.

For a series combination of resistors, the current in the two resistors are the same because any charge passes through R_1 must also pass through R_2. As a result,

$$I=I_1=I_2 \qquad (28.4)$$

However, the voltage differences at each resistor is different,

$$\nabla V = \nabla V_1 + \nabla V_2 \qquad (28.5)$$

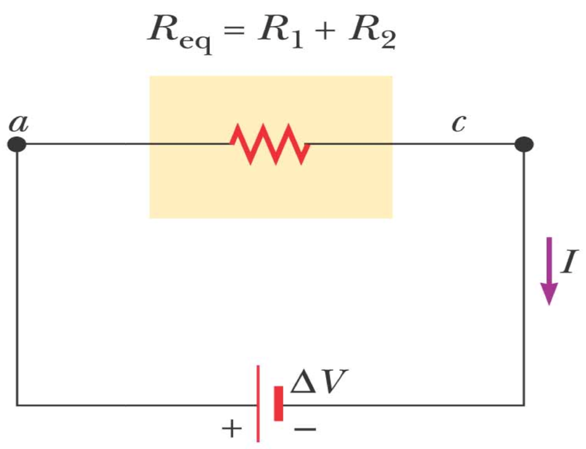
To find the equivalent resistance, one can do the following:

$$\nabla V = \nabla V_1 + \nabla V_2 = I_1 R_1 + I_2 R_2 \qquad (28.6)$$

However, $I=I_1=I_2$

So, $\nabla V = I(R_1 + R_2)$

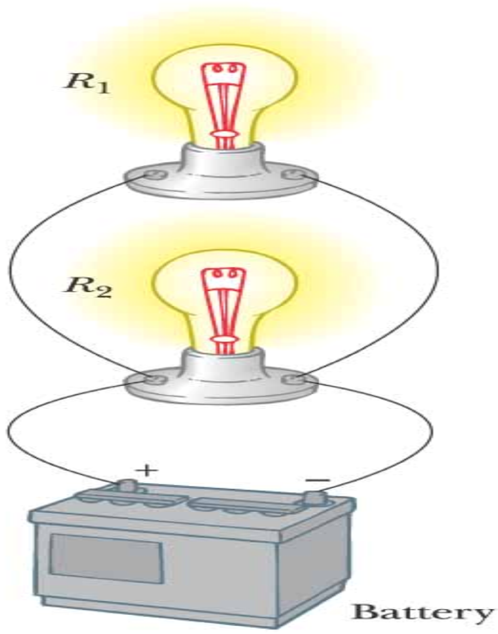
Then,
$$IR_{eq} = I(R_1 + R_2)$$
$$R_{eq} = R_1 + R_2 \qquad (28.7)$$



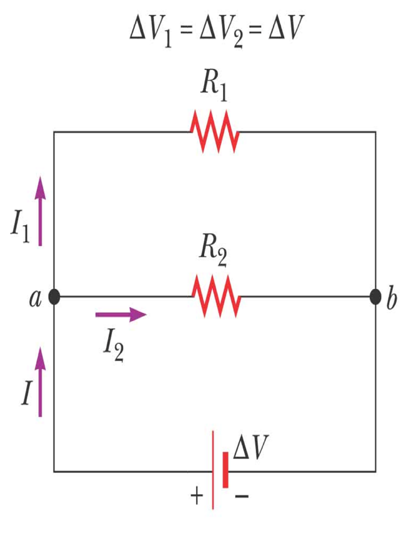
(c)

This relationship indicates that the equivalent resistance of a series connection of resistors is always greater than any individual resistance.

- **Combining resistors in parallel:**
  On the otherwise, one can combine the resistors in parallel as shown in the figure below.



(a)



(b)

In this figure, When the current *I reaches point a, called a junction, it splits into* two parts, with I_1 *going through* R_1 *and* I_2 *going through* R_2 *. A junction is any* point in a circuit where a current can split,

$$I=I_2+I_2 \qquad (28.8)$$

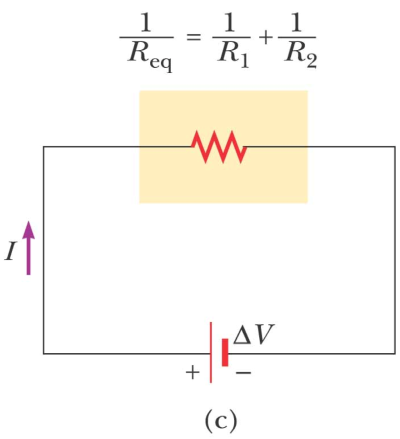
When the resistors are connected in parallel, the potential difference is the same.

$$\nabla V = \nabla V_1 = \nabla V_2 \qquad (28.9)$$

From eq. (28.2),

$$\begin{aligned} \frac{\nabla V}{R_{eq}} &= \frac{\nabla V_1}{R_1} + \frac{\nabla V_2}{R_2} = \nabla V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \\ \frac{1}{R_{eq}} &= \frac{1}{R_1} + \frac{1}{R_2} \end{aligned} \qquad (28.10)$$

So, one can say that the equivalent resistance of two or more resistors connected in parallel is always less than the least resistance in the group.



Please note that:

Household circuits are always wired such that the appliances are connected in parallel. Each device operates independently of the others so that if one is switched off, the others remain on. In addition, the devices operate on the same voltage.

Examples (28.3) (28.4) (28.5)

a- Find the equivalent resistance between points a and c.

b- What is the current in each resistor if a potential differences of 42 V is maintained between a and c?

(A) Find the equivalent resistance between points a and

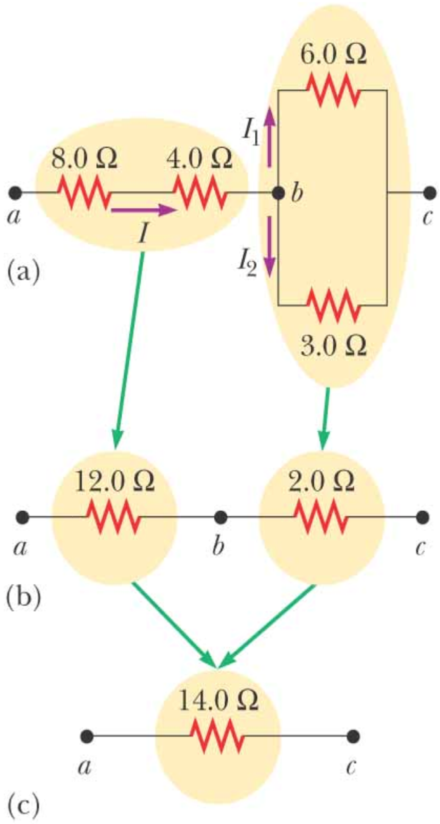
Solution The combination of resistors can be reduced i steps, as shown in Figure 28.9. The 8.0-Ω and 4.0-Ω resistoı are in series; thus, the equivalent resistance between a and is 12.0 Ω (see Eq. 28.5). The 6.0-Ω and 3.0-Ω resistors are i parallel, so from Equation 28.7 we find that the equivaler resistance from b to c is 2.0 Ω. Hence, the equivalent resi tance from a to c is 14.0 Ω.

(B) What is the current in each resistor if a potential diffe ence of 42 V is maintained between a and c?

Solution The currents in the 8.0-Ω and 4.0-Ω resistors aı the same because they are in series. In addition, this is th same as the current that would exist in the 14.0-Ω equivaler resistor subject to the 42-V potential difference. Therefore using Equation 27.8 ($R = \Delta V/I$) and the result from paı (A), we obtain

$$I = \frac{\Delta V_{ac}}{R_{eq}} = \frac{42\text{ V}}{14.0\ \Omega} = 3.0\text{ A}$$

This is the current in the 8.0-Ω and 4.0-Ω resistors. Whe this 3.0-A current enters the junction at b, however, it split with part passing through the 6.0-Ω resistor (I_1) and paı through the 3.0-Ω resistor (I_2). Because the potential diffe ence is ΔV_{bc} across each of these parallel resistors, we se that $(6.0\ \Omega)I_1 = (3.0\ \Omega)I_2$, or $I_2 = 2I_1$. Using this result an the fact that $I_1 + I_2 = 3.0$ A, we find that $I_1 = 1.0$ A an



$I_2 = 2.0$ A. We could have guessed this at the start by noting that the current in the 3.0-Ω resistor has to be twice that in the 6.0-Ω resistor, in view of their relative resistances and the fact that the same voltage is applied to each of them.

As a final check of our results, note that $\Delta V_{bc} = (6.0\ \Omega)I_1 = (3.0\ \Omega)I_2 = 6.0$ V and $\Delta V_{ab} = (12.0\ \Omega)I = 36$ V; therefore, $\Delta V_{ac} = \Delta V_{ab} + \Delta V_{bc} = 42$ V, as it must.