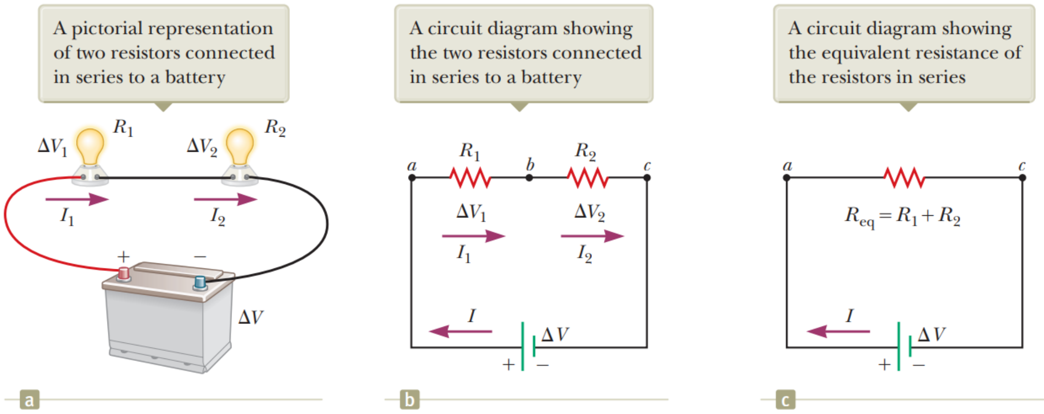
27.2 Resistors in series and parallel:

- **Combining resistors in series:**



When two or more resistors are connected together as are the light- bulbs in the figure, they are said to be in *series.*

In a series connection, all the charges moving through one resistor must also pass through the second resistor.

For a series combination of resistors, the current in the two resistors is the same because any charge that passes through R_1 must also pass through R_2. As a result,

$$I=I_1=I_2 \quad (27.4)$$

However, the voltage differences at each resistor is different,

$$\nabla V = \nabla V_1 + \nabla V_2 \quad (27.5)$$

To find the equivalent resistance, one can do the following:

$$\nabla V = \nabla V_1 + \nabla V_2 = I_1 R_1 + I_2 R_2 \quad (27.6)$$

However, $I=I_1=I_2$

So, $\nabla V = I(R_1 + R_2)$

Then,

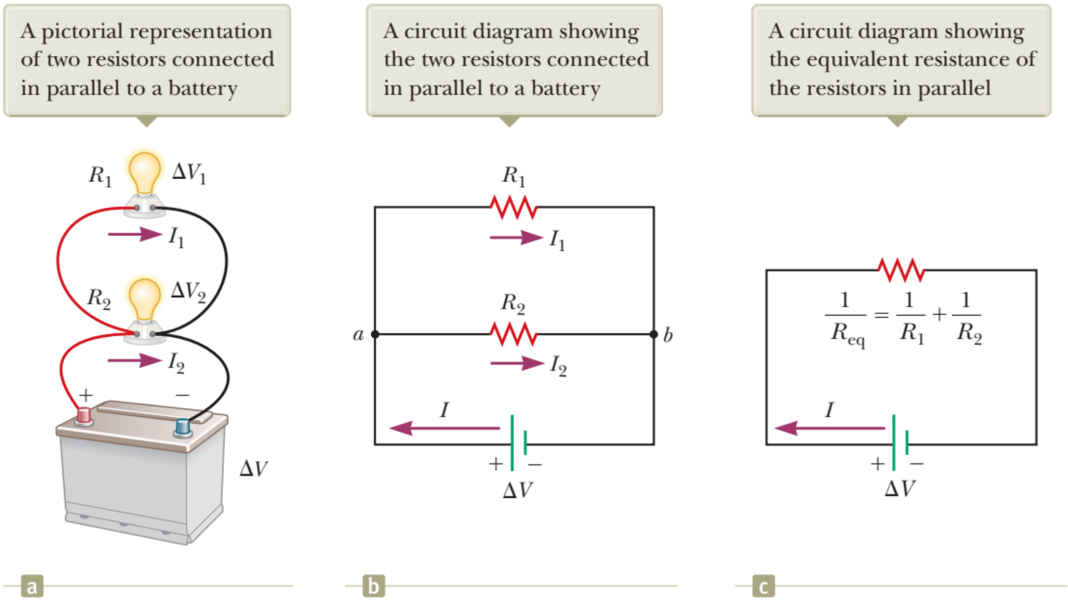
$$IR_{eq} = I(R_1 + R_2)$$
$$R_{eq} = R_1 + R_2 \quad (27.7)$$

The equivalent resistance of three or more resistors connected in series is:

$$R_{eq.} = R_1 + R_2 + R_3 + \cdots$$

This relationship indicates that the equivalent resistance of a series connection of resistors is always greater than any individual resistance.

- **Combining resistors in parallel:**
  On the other hand, one can combine the resistors in parallel, as shown in the figure below.



In this figure, when the current I reaches point a, called a junction, it splits into two parts, with I_1 going through R_1 and I_2 *going through* R_2 . *A junction is any* point in a circuit where a current can split,

$$I = I_1 + I_2 \qquad (27.8)$$

When resistors are connected in parallel, the potential difference across them is the same.

$$\nabla V = \nabla V_1 = \nabla V_2 \qquad (27.9)$$

From eq. (28.2),

$$\frac{\nabla V}{R_{eq}} = \frac{\nabla V_1}{R_1} + \frac{\nabla V_2}{R_2} = \nabla V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad (27.10)$$

An extension of this analysis to three or more resistors in parallel gives:

$$\frac{1}{R_{eq.}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

So, one can say that the equivalent resistance of two or more resistors connected in parallel is always less than the least resistance in the group.

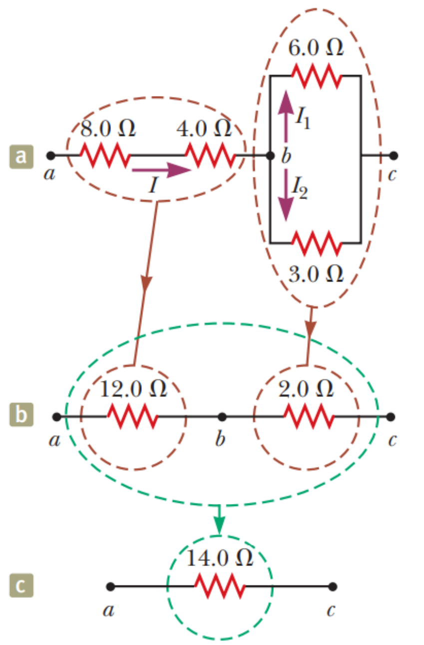
Please note that:

Household circuits are always wired so that the appliances are connected in parallel. Each device operates independently of the others, so if one is switched off, the others remain on. In addition, the devices operate on the same voltage.

Examples (27.4) (27.5)

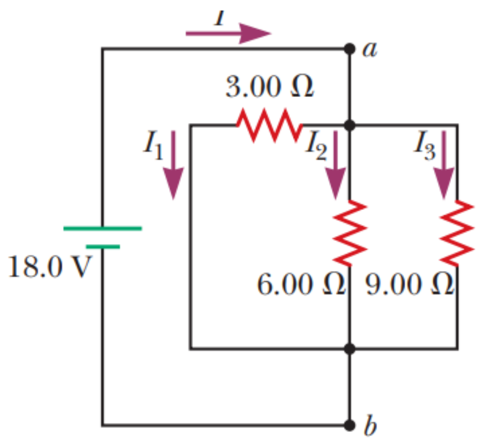
Example-1: Four resistors are connected as shown in the Figure.

(A) Find the equivalent resistance between points a and c.
(B) What is the current in each resistor if a potential difference of 42 V is maintained between a and c?



Example 2: Three resistors are connected as shown in the Figure. A potential difference of 18.0 V is maintained between points a and b.

(A) Calculate the equivalent resistance of the circuit.
(B) Find the current in each resistor.
(C) Calculate the power delivered to each resistor and the total power delivered to the combination of resistors.



Solution

Because the resistors are in parallel, the same voltage 18.0 V appears across each branch.

(A) The equivalent resistor:

$$\frac{1}{R_{\text{equivalent.}}} = \frac{1}{3} + \frac{1}{6} + \frac{1}{9} = \frac{6+3+2}{18} = \frac{11}{18}$$

Then, $R_{\text{equivalent.}} = \frac{18}{11} \approx 1.64\ \Omega$

(B) Currents

$I_1 = V/R_1 = 18.0/3.00 = 6.00$ A

$I_2 = V/R_2 = 18.0/6.00 = 3.00$ A

$I_3 = V/R_3 = 18.0/9.00 = 2.00$ A

Total current: $I = I_1 + I_2 + I_3 = 11.00$ A

(C) Powers

$P_1 = I_1^2R_1 = (6.00)^2 \times 3.00 = 108.0$ W

$P_2 = I_2^2R_2 = (3.00)^2 \times 6.00 = 54.0$ W

$P_3 = I_3^2R_3 = (2.00)^2 \times 9.00 = 36.0$ W

Total power: $P_{total} = P_1 + P_2 + P_3 = 198.0$ W

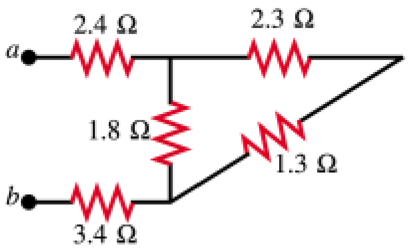
Equivalent Resistance

$R_{eq} = V / I = 18.0 / 11.00 = 1.636\ \Omega$

Example-3:

Question

Determine the equivalent resistance between points a and b (in Ω) for the circuit.



Solution

Let's define the resistances as follows:

- From a to node T: 2.4 Ω resistor.
- From b to node B: 3.4 Ω resistor.
- Between T and B: a 1.8 Ω vertical resistor in parallel with a series path of 2.3 Ω (top branch) and 1.3 Ω (diagonal).

1) Combine the elements between nodes T and B.

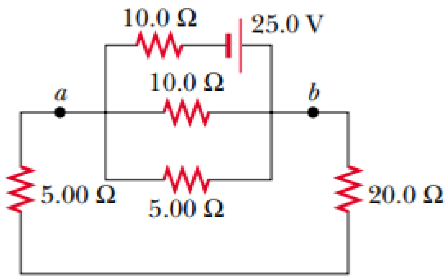
Series path: 2.3 Ω + 1.3 Ω = 3.6 Ω.

Parallel with 1.8 Ω: R_{TB} = (1.8 × 3.6) / (1.8 + 3.6) = 1.20 Ω.

2) The resistors 2.4 Ω (a→T), R_{TB} (T↔B), and 3.4 Ω (B→b) are in series:

$R_{eq(a–b)}$ = 2.4 + 1.20 + 3.4 = 7.0 Ω.

Example-4:

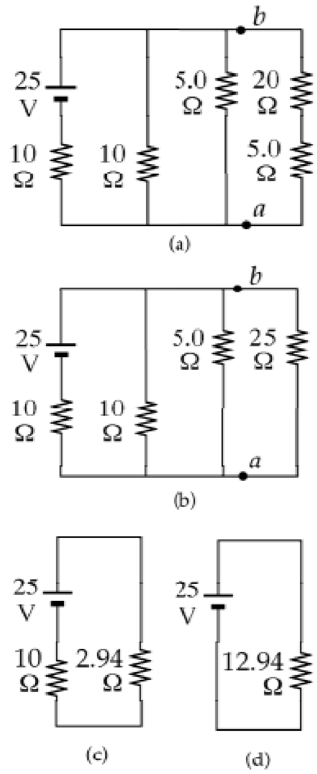


A 25.0 V battery is in series with a 10.0 Ω resistor. Between points a and b, there are three branches in parallel: (1) a 10.0 Ω resistor, (2) a 5.00 Ω resistor, and (3) a series combination of 20.0 Ω and 5.00 Ω (total 25.0 Ω). (a) Find the current in the 20.0 Ω resistor. (b) Find the potential difference between points a and b.

Solution

1) Equivalent of the parallel network between a and b:

$R_{eq(a–b)}$ = 1 / (1/10.0 + 1/5.00 + 1/25.0) = 2.94 Ω.



2) Total resistance seen by the battery:

R_{total} = 10.0 Ω + R_{eq} = 10.0 + 2.94 = 12.94 Ω.

3) Battery current:

$I_{battery}$ = V / R_{total} = 25.0 / 12.94 = 1.93 A.

4) Voltage across the a–b network:

V_{ab} = $I_{battery}$ × R_{eq} = 1.93 × 2.94 = 5.68 V.

5) Current in the right branch (20.0 Ω in series with 5.00 Ω → 25.0 Ω):

I_{branch} = V_{ab} / 25.0 = 5.68 / 25.0 = 0.227 A ≈ 227 mA.

This is the current through the 20.0 Ω resistor (same as the branch current).

Final Answers

(a) Current through 20.0 Ω resistor: 0.227 A (≈ 227 mA).

(b) Potential difference between a and b: 5.68 V.