26-2 **Resistance**

I- Current density:

If a wire is connected across the terminals of a battery, the conductor is not in static equilibrium. In this case, the conductor has a nonzero electric field, and a current exists in the wire.

Current density (J) is used to study the flow of charge through a cross-section of the conductor at a particular point.

$$J = \frac{I}{A} \qquad (26\text{-}4)$$

The current density J in the conductor is defined as the current per unit area.

Since: $I = nev_d A$

Then, $J = nev_d$

II- Conductivity (α) vs. Resistivity (ρ)

A current density **J** and an electric field **E** are established in a conductor whenever a potential difference is maintained across the conductor. If the potential difference is constant, the current is also constant. *In some materials, the current density is proportional to the electric field:*

$$\mathbf{J} \propto \mathbf{E}$$
$$\mathbf{J} = \sigma \ \mathbf{E} \qquad (26\text{-}5)$$

Where the constant of proportionality σ is called the **conductivity** ($\boldsymbol{\sigma = \frac{1}{\rho}, where\ \rho\ is\ called\ reistivity}$) of the conductor. Materials that obey Equation (26-5) are said to follow Ohm's law.

More specifically, **Ohm’s law states that**;

Ohm’s law states that for many materials, including most metals, the ratio of current density to the electric field remains constant (denoted as σ) and is independent of the electric field generating the current, depending on the material's properties.

Science History Images/Alamy

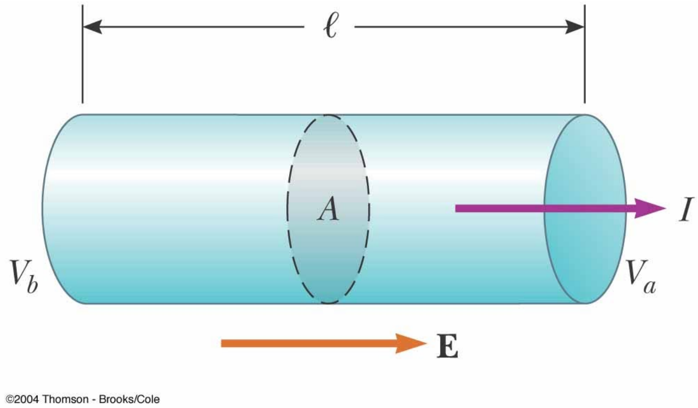
Georg Simon Ohm
German physicist (1789–1854)
Ohm, a high school teacher and later a professor at the University of Munich, formulated the concept of resistance and discovered the proportionalities expressed in Equations 26.6 and 26.7.

Materials that obey Ohm’s law and hence demonstrate this simple relationship between **E** and **J** are said to be ***ohmic***, while materials that do not obey Ohm’s law are said to be non-ohmic.

III- Resistance

Now, let’s explore practical situations where Ohm’s law is not a fundamental law of nature but rather an empirical relationship that holds true only under specific conditions.

Consider a segment of straight wire of uniform cross-sectional area *A and* length *l*



If the field is assumed to be uniform, the potential difference is related to the field through the relationship,

$$\Delta V = E\ l$$

$$\because J = \sigma E$$

$$J = \sigma \frac{V}{l}$$

$$\because J = \frac{I}{A}$$

$$\therefore \frac{I}{A} = \sigma \frac{V}{l}$$

$$V = I\left(\frac{l}{\sigma A}\right)$$

$$V = IR$$

V=IR (26-6)

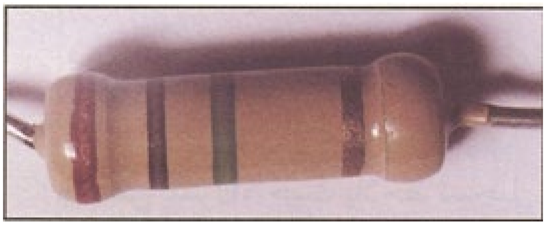
$$R = \frac{\ell}{\sigma A} = \rho \frac{\ell}{A} = \frac{\Delta V}{I}$$

Simulations of Ohm's Law:

https://phet.colorado.edu/en/simulations/ohms-law

R is the resistance of the conductor.

The resistance is defined as the ratio of the potential difference across a conductor to the current flowing through it.



Unit for R: ohm (Ω).

$$\mathbf{1\ \Omega = 1\ V/A}$$

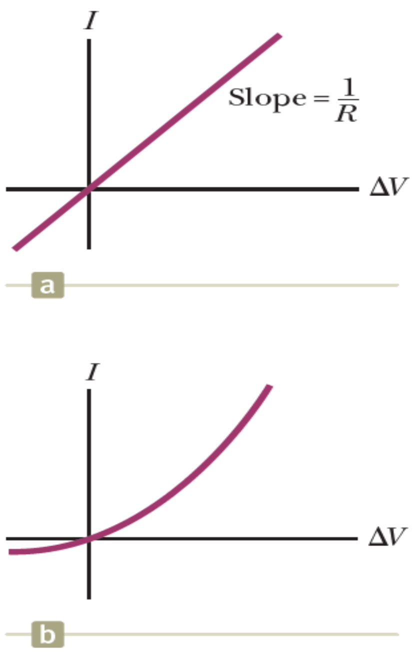
$$\rho(\text{resistivity}) = \frac{1}{\sigma}$$

$$R \equiv \frac{\ell}{\sigma A} \equiv \rho \frac{\ell}{A} \equiv \frac{\Delta V}{I} \qquad (26\text{-}7)$$

Comments:

- Resistors are used in electric circuits to control the flow of current through various parts of the circuit.
- The resistance of a sample of the material depends on the geometry of the sample and the resistivity of the material.
  See the simulation:
  https://phet.colorado.edu/en/simulations/resistance-in-a-wire

- More precisely, the resistance of a given cylindrical conductor, such as a wire, is proportional to its length and inversely proportional to its cross-sectional area. **If the length of a wire is doubled, its resistance doubles. If its cross-sectional area is doubled, its resistance decreases by one-half.**
- Ohmic materials and devices exhibit a linear current–voltage relationship over a broad range of applied voltages (see Figure). The slope of the I-versus-ΔV curve in the linear region yields a value of $1/R$. Nonohmic materials have a nonlinear current–potential difference relationship.



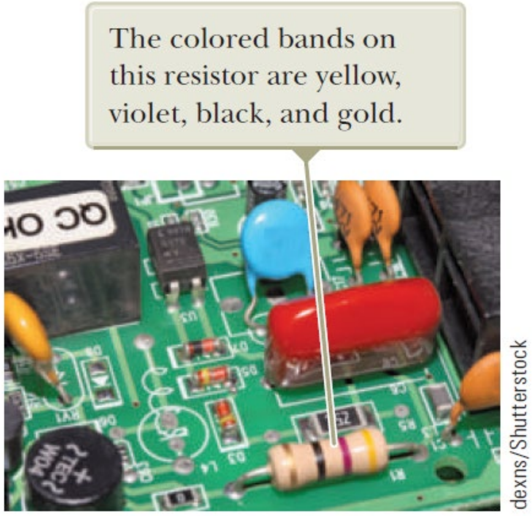
- Resistors come in various types, categorized by their construction, materials, and functionality. some common types are:

  Carbon Composition Resistors: Made from a mixture of carbon and a binder. They are inexpensive but have high noise levels.

  Metal Film Resistors: High precision and low noise are commonly used in circuits requiring accuracy.

  Wire-Wound Resistors: Made by winding metal wire around an insulating core. They are used in high-power applications.

Thick and Thin Film Resistors: These are used in surface-mount devices (SMD) and offer better stability.



- **Values of resistors** in ohms are typically indicated by color coding.

- Every ohmic material has a **characteristic resistivity** that depends on the properties of the material and on temperature.
- Resistivity is a property of **substances**. The resistance of a material depends on its geometry and its resistivity.
- Resistance is a property of an **object**. An ideal conductor would have zero resistivity. An ideal insulator would have infinite resistivity.

TABLE 26.2 Resistivities and Temperature Coefficients of Resistivity for Various Materials

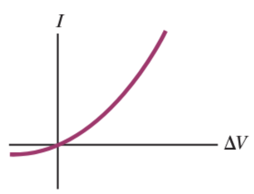
Material	Resistivity[a] ($\Omega \cdot m$)	Temperature Coefficient[b] α [$(°C)^{-1}$]
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Platinum	11×10^{-8}	3.92×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Nichrome[c]	1.00×10^{-6}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon[d]	2.3×10^{3}	-75×10^{-3}
Glass	10^{10} to 10^{14}	
Hard rubber	$\sim 10^{13}$	
Sulfur	10^{15}	
Quartz (fused)	75×10^{16}	

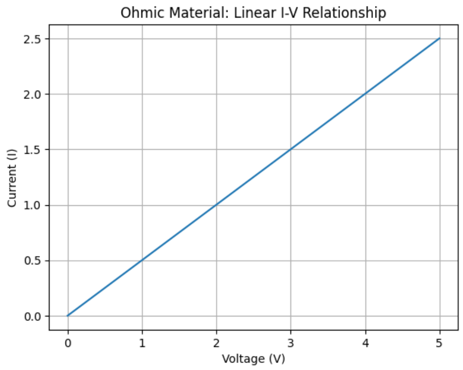
[a] All values at 20°C. All elements in this table are assumed to be free of impurities.
[b] See Section 26.4.
[c] A nickel–chromium alloy commonly used in heating elements. The resistivity of Nichrome varies with composition and ranges between 1.00×10^{-6} and 1.50×10^{-6} $\Omega \cdot m$.
[d] The resistivity of silicon is very sensitive to purity. The value can be changed by several orders of magnitude when it is doped with other atoms.

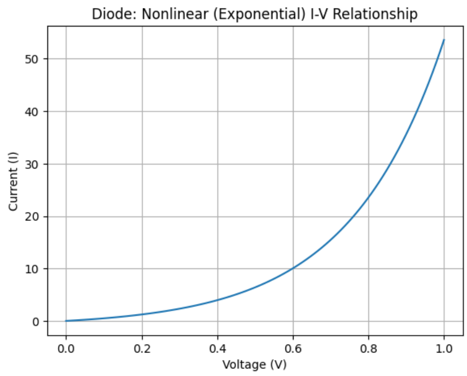
Think about it:

1-- A cylindrical wire has a radius r and length ℓ. If both r and ℓ are doubled, does the resistance of the wire (a) increase, (b) decrease, or (c) remain the same?

2—In the Figure, as the applied voltage increases, does the resistance of the diode (a) increase, (b) decrease, or (c) remain the same?







Ohmic material → Straight-line I–V graph (constant resistance)
Diode → Exponential I–V curve (decreasing resistance with increasing voltage)

Example-1

Calculate the resistance of an aluminum cylinder that is 10.0 cm long and has a cross-sectional area of $2.0\text{x } 10^{-4}$ m^2. Repeat the calculation for a cylinder of the same dimensions and made of glass having a resistivity of $3\text{x}10^{10}$ Ω . ρ(Aluminum)=$2.82\text{x}10^{-8}$ (Ω.m)

Solution:

Use R = ρL/A

For aluminum:
L = 10.0 cm = 0.10 m
$R = (2.82 \times 10^{-8})(0.10) / (2.00 \times 10^{-4})$
$R = 1.41 \times 10^{-5}\ \Omega$

For glass:
$R = (3.0 \times 10^{10})(0.10) / (2.00 \times 10^{-4})$
$R = 1.5 \times 10^{13}\ \Omega$

Think about these two values !!

Example-2

(a) Calculate the resistance per unit length of a 22-gauge Nichrome wire, which has a radius of 0.321 mm. ρ(Nichrome)=$1\text{x}10^{-6}$ (Ω.m)
(b) If a potential difference of 10 V is maintained across a 1.0-m length of the Nichrome wire, what is the current in the wire?

Solution:

(a) $A = \pi r^2$
$r = 0.321\ mm = 3.21 \times 10^{-4}\ m$
$A = \pi(3.21 \times 10^{-4})^2 = 3.24 \times 10^{-7}\ m^2$

$R/L = \rho/A = (1.5 \times 10^{-6}) / (3.24 \times 10^{-7})$
$R/L = 4.6\ \Omega/m$

(b) For 1.0 m: $R = 4.6\ \Omega$
$I = V/R = 10 / 4.6$
$I = 2.2\ A$

Example-3

A 0.900 V potential difference is maintained across a 1.50 m length of tungsten wire with a cross-sectional area of 0.600 mm^2. What is the current in the wire? (Resistivity of tungsten is $5.60 \times 10^{-8}\ \Omega\ .\ m$).

Solution:

$A = 0.600\ mm^2 = 6.00 \times 10^{-7}\ m^2$
$\Delta V = IR$

$R = (\rho L)/A$

$\Delta V = I\rho L/A$

$==> I = \Delta V\ A / (\rho L)$
$I = (0.900)(6.00 \times 10^{-7}) / [(5.60 \times 10^{-8})(1.50)]$
$I = 6.43\ A$

Example-4

A conductor of uniform radius 1.20 cm carries a current of 3.00 A produced by an electric field of 120 V/m. What is the resistivity of the material?

Solution:

$J = I/A$ and $J = E/\rho$
Therefore $\rho = E\,A / I$

$r = 1.20$ cm $= 1.20 \times 10^{-2}$ m
$A = \pi r^2 = \pi(1.20 \times 10^{-2})^2$

$\rho = [\pi(1.20 \times 10^{-2})^2(120)] / 3.00$
$\rho = 0.0181\ \Omega\cdot\text{m}$