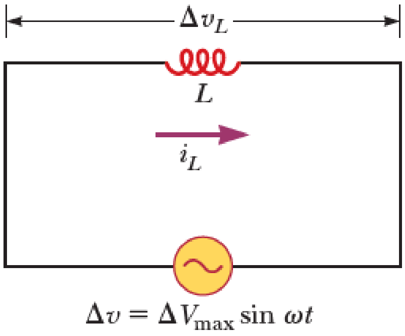
33.3 Inductors in AC circuit



A simple AC circuit in the figure in front contains an inductor and an AC source.

Similar to our treatment in the previous section, we find that:

$$\Delta v - L\frac{di_L}{dt} = 0 \qquad 33.6$$

$$V_{max} \sin \omega t = L\frac{di_L}{dt}$$

$$\Rightarrow i_L = \frac{V_{max}}{L}\int \sin \omega t \; dt = -\frac{V_{max}}{\omega L}\cos \omega t$$

Using $\cos \omega t = -\sin\left(\omega t - \frac{\pi}{2}\right)$

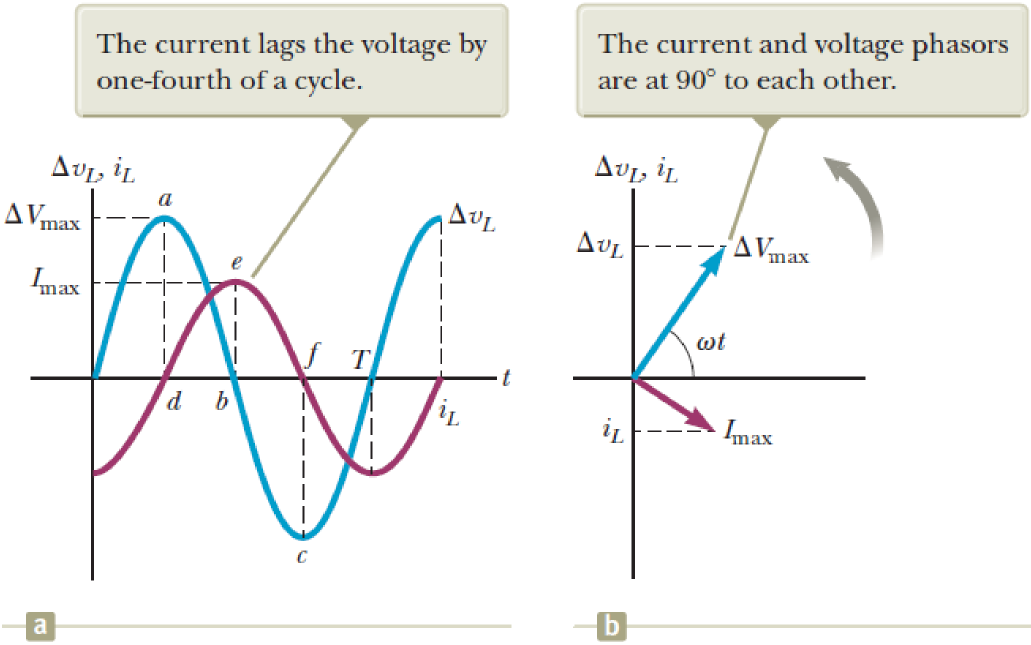
$$i_L = I_{max} \sin\left(\omega t - \frac{\pi}{2}\right) \qquad 33.7$$

Where $I_{max} = \frac{V_{max}}{\omega L} = \frac{V_{max}}{X_L}$

Also, X_L is the inductive reactance, $X_L = \omega L$.

Plot of the instantaneous current i_L and instantaneous voltage Dv_L across the inductor as functions of time is shown in the figure below.

It shows that the instantaneous current and the instantaneous voltage across the circuit is out of phase by $\frac{\pi}{2}\text{rad} = 90^0$.



Remarks about the points a,b,c,d,e, and f are discussed

- when the current i_L in the inductor is a maximum (point *b* in Fig. (a)), the voltage across the inductor is zero (point *d*).
- Note that the voltage reaches its maximum value one quarter of a period before the current reaches its maximum value. Thus, we see that: for a sinusoidal applied voltage, the current in an inductor always lags behind the voltage across the inductor by 90° (one-quarter cycle in time).
- From the phasor diagram (Fig. (b)), the phasors are at 90° to one another, representing the 90° phase difference between current and voltage.

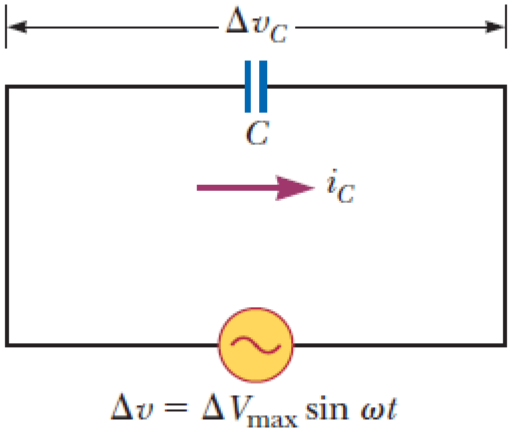
The phasor diagram for the inductive circuit shows that the current lags behind the voltage by 90°.

Example-1:

In a purely inductive AC circuit, L = 25.0 mH and the rms voltage is 150 V. Calculate the inductive reactance and rms current in the circuit if the frequency is 60.0 Hz.

33.4 Capacitor in AC circuit

In the figure in front, an AC circuit connects to a capacitor. By using Kirchhoff's law, we find



$$\Delta v + \Delta v_C = 0$$

$$\Delta v - \frac{q}{C} = 0 \qquad 33.8$$

$$q = \Delta v C = C V_{max} \sin \omega t$$

We know that $I = \frac{dq}{dt}$. Then,

$$\frac{dq}{dt} = \frac{d}{dt} C V_{max} \sin \omega t = \omega C V_{max} \cos \omega t$$

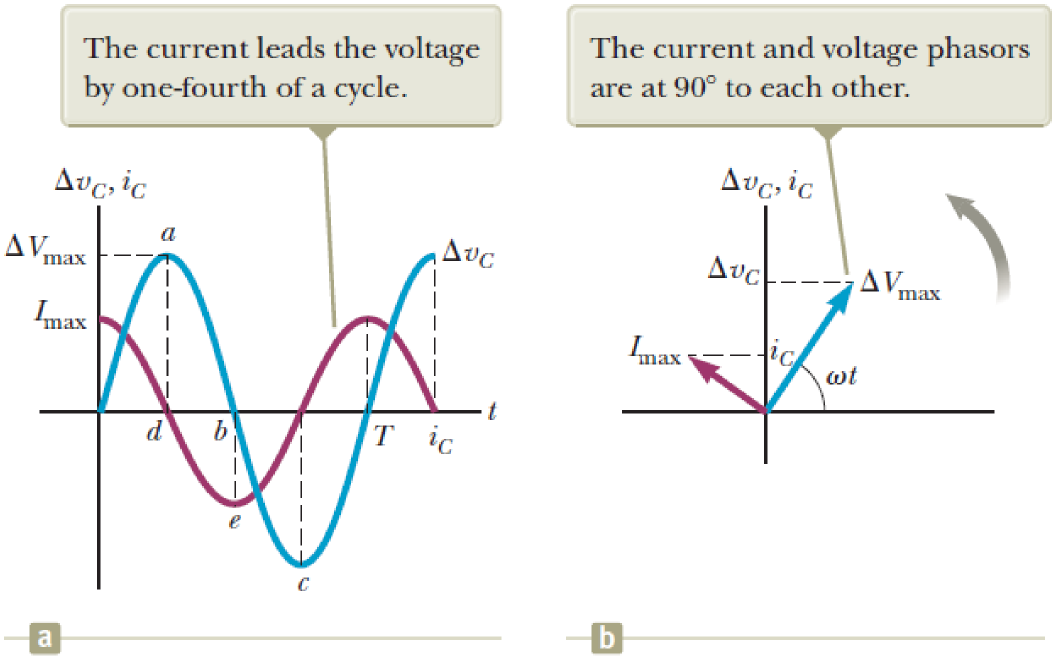
$$\Rightarrow i_C = \omega C V_{max} \cos \omega t = I_{max} \sin(\omega t + \frac{\pi}{2}) \qquad 33.9$$

Where $I_{max} = \omega C V_{max} = \frac{V_{max}}{X_C}$

$X_C = (\frac{1}{\omega C})$ and is called the capacitive reactance.

Plot of the instantaneous current i_C and the instantaneous voltage Δv_C across the capacitor as functions of time is shown in the figure below.

The current leads the voltage by one-fourth of a cycle.



- At **point a** in Figure a, the **voltage across the capacitor is at its maximum**, which corresponds to the **maximum charge** on the capacitor.

- At this same instant (**point d**), the **current is zero**.

- At **points such as e**, the **current reaches its maximum magnitude**—this occurs when the **charge on the capacitor is zero**, and it begins to **recharge with opposite polarity**.

- When the **charge is zero**, the **voltage across the capacitor is also zero** (**point b**).

- Similar to inductors, **phasor diagrams** can be used to represent the **current and voltage** in a capacitor.

- The phasor diagram in **Figure b** shows that for a **sinusoidally applied voltage**, the **current leads the voltage across a capacitor by 90°**.

Phasor diagram for the capacitive circuit shows that the current and the voltage phasors are at 90° to each other.

Exampl-2: An 8.00-mF capacitor is connected to the terminals of a 60.0-Hz AC source whose rms voltage is 150 V. Find the capacitive reactance and the rms current in the circuit.

Quick Quiz 32.3

In the AC circuit shown in **Figure 32.11**, the **frequency of the AC source** is varied while keeping the **voltage amplitude constant**.
At which frequency does the lightbulb glow the brightest?

(a) At high frequencies
(b) At low frequencies
(c) The brightness remains the same at all frequencies

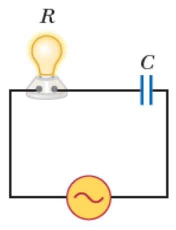


Figure 32.11 (Quick Quiz 32.3)

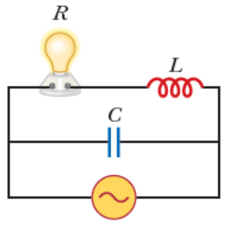


Figure 32.12 (Quick Quiz 32.4)

Quick Quiz 32.4

Refer to the AC circuit in **Figure 32.12**. The **frequency of the AC source** is adjusted, but the **voltage amplitude is held constant**.
When does the lightbulb reach its maximum brightness?

(a) At high frequencies
(b) At low frequencies
(c) The brightness is unaffected by frequency