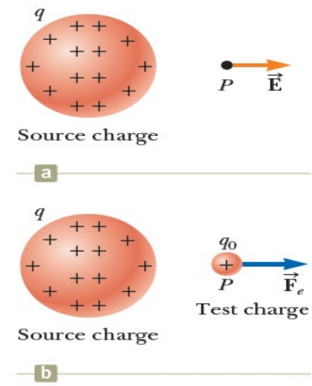
22.4 Analysis Model: Particle in a Field (Electric)

Concept of electric field

- An electric field represents the region around a charged object where other charges (around the source charge) experience a force due to its influence.
- A charged particle, with charge q, produces an electric field in the region of space around it.
- A small test charge q_0 is placed near the source charge to detect and measure the presence and direction of the electric field.



Definition of the electric field:

***The electric field E** at a point in space is defined as the electric force **F_e** acting on a positive test charge **q_0** placed at that point divided by the magnitude of the test charge:*

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q_0}$$

As we discussed in the previous section, the electric force due to point charges can be written as:

$$F = K_e \frac{|q_1||q_2|}{r^2}\hat{r}$$

So, one can re-write the electric force between q and q_0 as follows:

$$\vec{F} = K_e \frac{q\, q_0}{r^2}\,\hat{r}$$

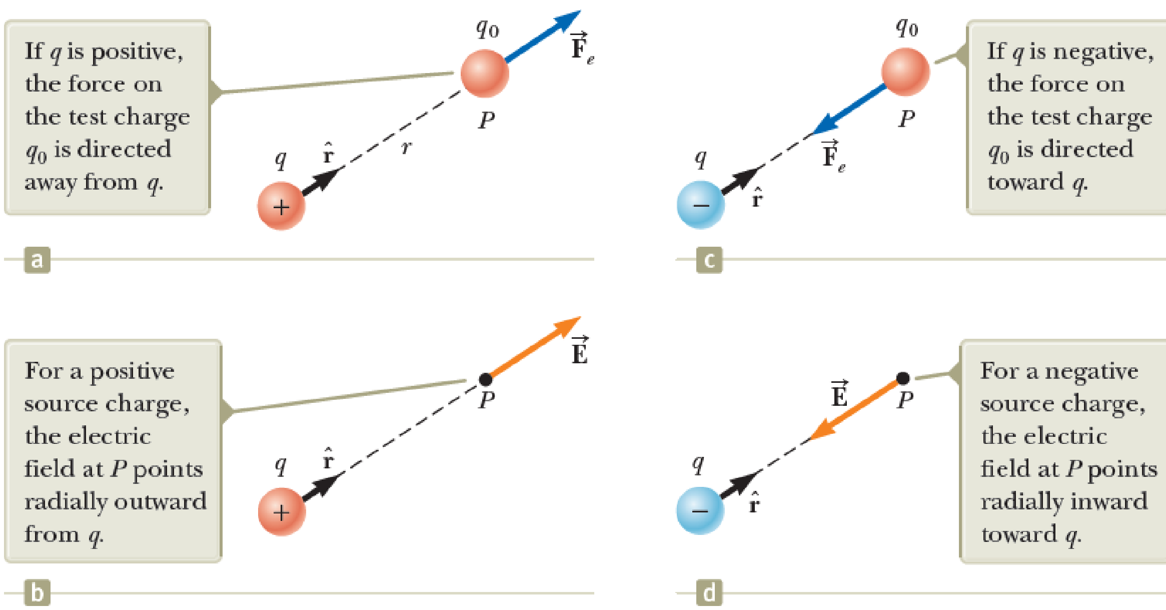
The electric field can also be derived:

$$\vec{E} = \frac{\vec{F}}{q_0} = K_e \frac{q\, q_0}{q_0 r^2}\,\hat{r} = K_e \frac{q}{r^2}\,\hat{r}$$

The vector $\vec{E}$ the SI units of (newtons per coulomb) (N/C).

<u>The direction of electric force and electric field:</u>

<u>Remember:</u> $\vec{F} = q\,\vec{E}$

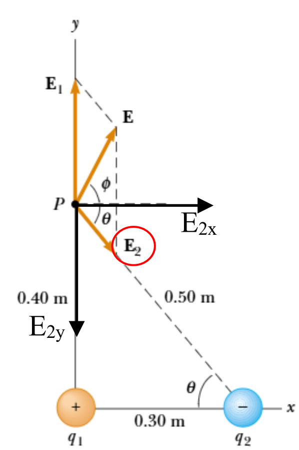


At any point P, the total electric field due to a group of charges equals the vector sum of the electric fields of the individual charges.

$$\boldsymbol{E} = k_e \sum_i \frac{q_i}{r_i^2}\,\hat{\boldsymbol{r}}_i$$

Example-1:

A charge q_1= 7 μC is located at the origin and second charge q_2= - 5 μC is located on the x axis, 0.30 m from the origin. Find the electric field at the point P which has the coordinates (0, 0.40) m.



Solution:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = |\vec{E}_1|(+\hat{j}) + |\vec{E}_2|(+\hat{\imath}) + |\vec{E}_2|(-\hat{j})$$

$$= E_{1y} + E_{2x} - E_{2y}$$

1- $\vec{E}_1 = |\vec{E}_1|\,\hat{r} = K\frac{q_1}{r^2}\,\hat{r} = 9x10^9\frac{(7x10^{-6})}{(0.4)^2}\,(+\hat{j})$

$$= 3.9\ x\ 10^5\ \hat{j}\ N/C$$

2- $\vec{E}_2 = |\vec{E}_2|\,\hat{r} = E_{2x}\hat{\imath} + E_{2y}\hat{j} = |\vec{E}_2|\cos\theta\ \hat{\imath} - |\vec{E}_2|\sin\theta\,\hat{j}$

$r^2 = (0.3)^2 + (0.4)^2 = 0.25$ $\qquad r = 0.5\,m$

$\cos\theta = \frac{0.3}{0.5}$ $\qquad \sin\theta = \frac{0.4}{0.5}$

$$|\vec{E}_2| = K\frac{q_2}{r^2} = 9x10^9\frac{(5x10^{-6})}{(0.5)^2} = 1.8\ x\ 10^5\ N/C$$

$$\vec{E}_2 == |\vec{E}_2|\,(\cos\theta\ \hat{\imath} - \sin\theta\,\hat{\imath}) = 1.8x10^5\ (\frac{0.3}{0.5}\hat{\imath} - \frac{0.4}{0.5}\hat{j})$$

3- $\vec{E} = \vec{E}_1 + \vec{E}_2$

$$= (3.9\ x\ 10^5)\,\hat{j} + (1.1\ x\ 10^5)\,\hat{\imath} - (1.44\ x\ 10^5)\,\hat{j}$$

$$= (1.1\,\hat{\imath} + 2.44\,\hat{j})x\ 10^5$$

This is the coordinate of the electric field at P point.

4- $|\vec{E}| = \sqrt{\vec{E}_1^2 + \vec{E}_2^2} = \sqrt{(1.1x10^5)^2 + (2.44x10^5)^2}$

$$= 2.7\ x\ 10^5 N/C$$

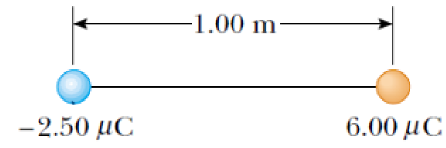
This is the magnitude of the electric field at point P.

5- $\tan\emptyset = \frac{E_y}{E_x} = \frac{2.44\ x\ 10^5}{1.1\ x\ 10^5} = 2.22$

$$\emptyset = tan^{-1}(2.22) = 65.73^0$$

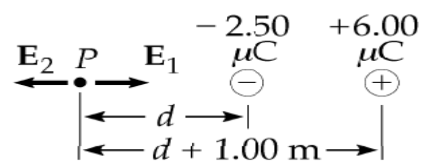
ϕ represents the direction of the electric field at point P.

Example-2:



In this figure, determine the point where the electric field is zero.

Solution:



$$\vec{E}_P = \vec{E}_1 + \vec{E}_2$$

$$\vec{E}_P = 0 =====> \vec{E}_1 = \vec{E}_2$$

$$K\,\frac{q_1}{d^2} = K\,\frac{q_2}{(1+d)^2} =====> q_1(1+d)^2 = q_2 d^2$$

$$\sqrt{\frac{q_1}{q_2}}(1+d) = \pm\, d ====> d\left(\sqrt{\frac{q_1}{q_2}} \mp 1\right) = -\sqrt{\frac{q_1}{q_2}}$$

Please consider this step carefully, especially when given q_1=q_2.

$$d = 1.8\,m \quad from\; q_1$$