Chapter 28

Magnetic field

28.1 Analysis Model: Particle in a Field (Magnetic)

1. Introduction to Magnetic Fields

- Magnetic effects, like a bar magnet attracting a paper clip, occur **at a distance** without physical contact.
- A **magnetic field** exists in regions around moving electric charges and magnetic materials.
- Unlike electric fields, which originate from charges, magnetic fields are generated by **moving charges** and **permanent magnets**.

  See the attachment file for A Brief History of Magnetism

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2. Magnetic Poles and Their Properties

- Any source of a magnetic field has **two poles**:
  - **North Pole (N)**
  - **South Pole (S)**
- If a bar magnet is **suspended freely**, it aligns itself so that its **north pole points toward Earth's geographic North Pole**.
- Magnetic poles exert **attractive and repulsive forces** similar to electric charges:
  - **Like poles repel** (N-N or S-S).
  - **Opposite poles attract** (N-S).
- However, **magnetic monopoles do not exist**—every magnet always has both a north and a south pole.
  - **If you cut a magnet in half, each piece still has a north and a south pole.**

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3. Representation of Magnetic Fields

- Magnetic field lines, similar to electric field lines, represent magnetic fields.
- **Direction of field lines:**
  - **Outside a magnet:** From **north** to **south**.
  - **Inside a magnet:** From **south** to **north** (forming closed loops).
- The **strength of the magnetic field** is indicated by the **density of field lines** (closer lines = stronger field).

📌 **Example:**

- A **compass needle** aligns itself with the magnetic field because the needle's north pole points in the field direction.

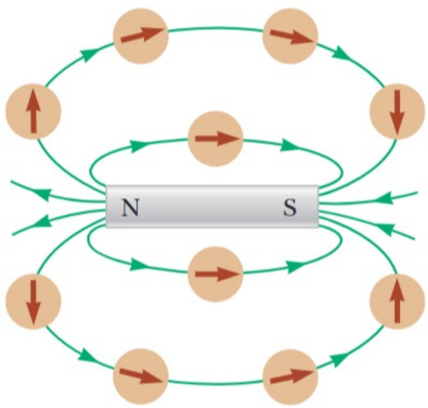

Figure 28.1 Compass needles can be used to trace the magnetic field lines in the region outside a bar magnet.

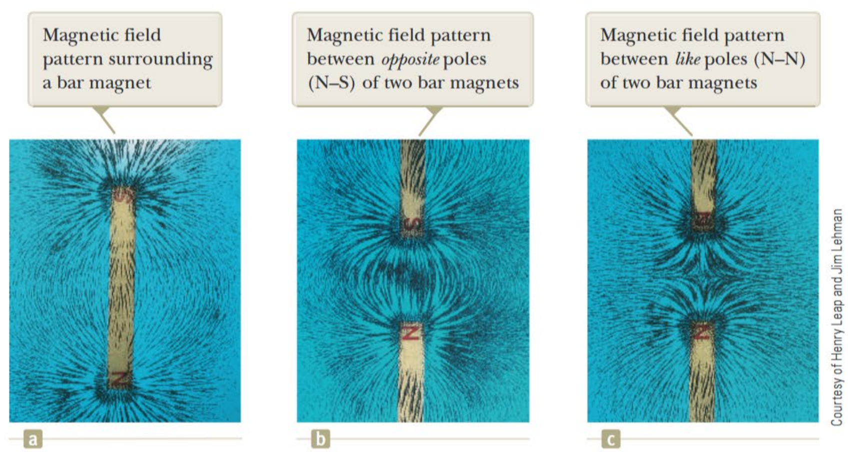

Figure 28.2 Magnetic field patterns can be displayed with iron filings sprinkled on paper near magnets.

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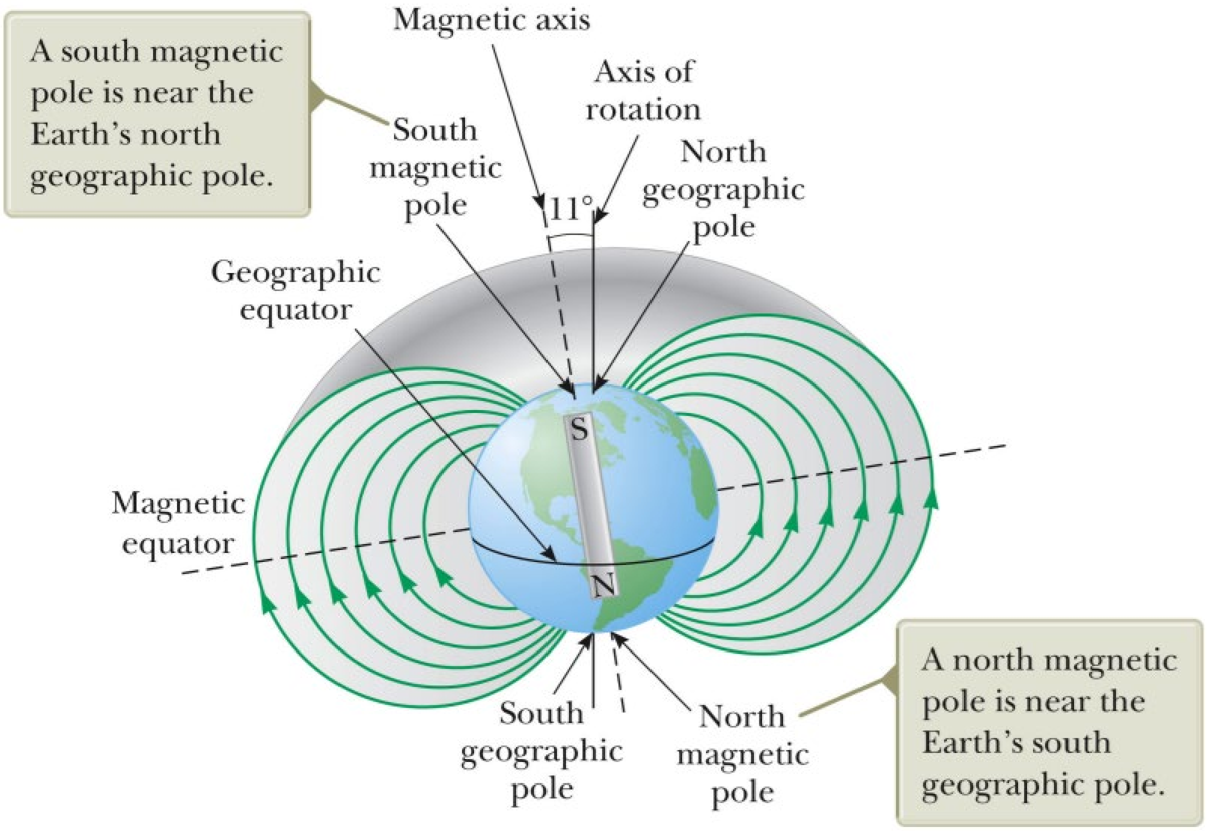
4. Earth's Magnetic Field

- Earth behaves like a **giant bar magnet** with a magnetic field similar to that of a **dipole magnet**.
- However, the Earth's **geomagnetic poles** are opposite to the geographic poles:
  - **The north geomagnetic pole is near the south geographic pole.**
  - **The south geomagnetic pole is near the north geographic pole.**
- The Earth's magnetic field protects us from **solar winds** and plays a key role in navigation.



5. Magnetic forces

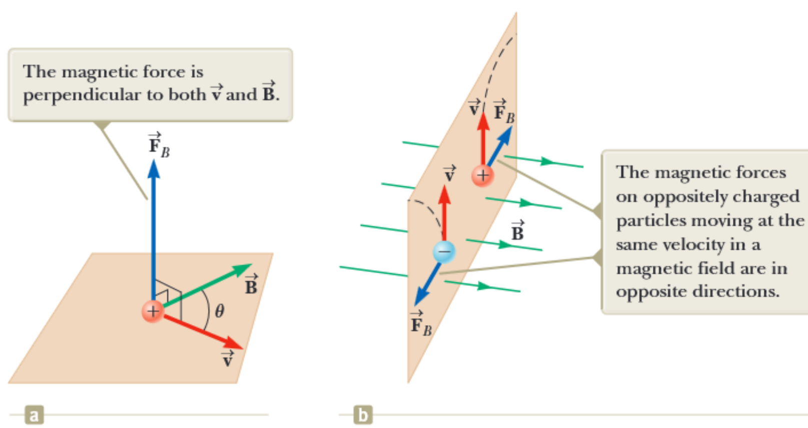


Figure 28.4 (a) The direction of the magnetic force $\vec{F}_B$ acting on a charged particle moving with a velocity $\vec{v}$ in the presence of a magnetic field $\vec{B}$. (b) Magnetic forces on positive and negative charges. A uniform magnetic field is represented by uniformly spaced magnetic field lines. At any point, the magnetic field vector $\vec{B}$ is parallel to the field line. The dashed lines show the paths of the particles, which are investigated in Section 28.2.

We can define a magnetic field **B** at a point in space in terms of the magnetic force F_B that the field exerts on a test object, which we use a charged particle moving with velocity v.

Assuming that no electric **(E)** or gravitational **(g)** fields are present at the location of the test object.

The magnitude $\boldsymbol{F_B}$ *of the magnetic force exerted on the particle is proportional* to the charge ***q*** *and to the speed* **v** *of the particle.*

The magnitude and direction of F_B *depend on the velocity of the particle v and* on the magnitude and direction of the magnetic field **B**.

• When a charged particle moves parallel to the magnetic field vector (i.e., $\theta = 0$) , the magnetic force acting on the particle is zero.

• When the particle's velocity vector makes any angle with the magnetic field, the magnetic force acts in a direction perpendicular to both **v** and **B**; F_B *is perpendicular to the plane formed by v and B*

• The magnetic force exerted on a positive charge is in the direction opposite to the direction of the magnetic force exerted on a negative charge moving in the same direction.

• The magnitude of the magnetic force exerted on the moving particle is proportional to $\sin_\theta$, where is the angle the particle's velocity vector makes with the direction of B.

We can summarize these observations by writing the magnetic force in the form

$$\vec{F}_B = q\ \vec{v} x \vec{B} = q|\vec{v}||\vec{B}|\ \sin\theta \qquad 29.1$$

From this Equation, we see that the SI unit of magnetic field is the newton per coulomb-meter per second, which is called the tesla (T):

$$1\,T = 1\,\frac{N}{C.\frac{m}{s}}$$

Because a coulomb per second is defined to be an ampere,

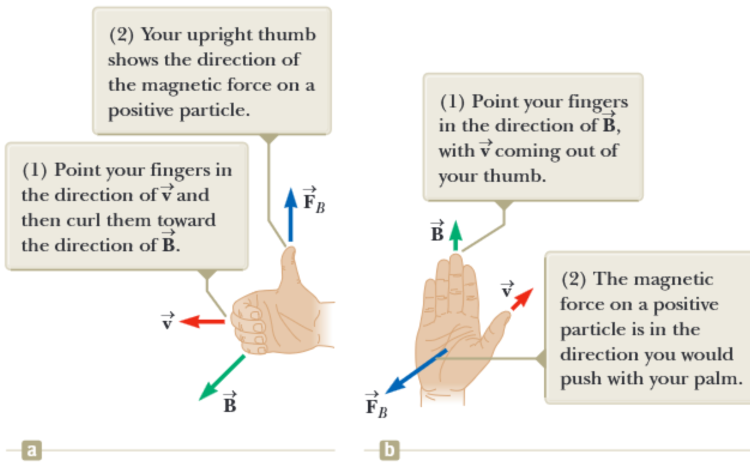
$$1\,T = 1\,\frac{N}{A.m}$$

A non-SI magnetic-field unit in common use, called the gauss (G), is related to the Tesla through the conversion 1 T = 10^4 G.

- ➢ The kinetic energy of a charged particle moving through a magnetic field cannot be altered by the magnetic field alone. The field can alter the direction of the velocity vector, but it cannot change the speed or kinetic energy of the particle.

- The magnetic force **changes the direction of motion** but **not the speed** of the particle.

Figure 28.5 Two right-hand rules for determining the direction of the magnetic force $\vec{F}_B = q\vec{v} \times \vec{B}$ acting on a particle with charge q moving with a velocity $\vec{v}$ in a magnetic field $\vec{B}$. (a) In this rule, the magnetic force is in the direction in which your thumb points. (b) In this rule, the magnetic force is in the direction of your palm, as if you are pushing the particle with your hand.



Please remember that:

Unit vector cross product relations:

$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$

$\hat{\imath} \times \hat{\jmath} = -\hat{\jmath} \times \hat{\imath} = \hat{k}$

$\hat{\jmath} \times \hat{k} = -\hat{k} \times \hat{\jmath} = \hat{\imath}$

$\hat{k} \times \hat{\imath} = -\hat{\imath} \times \hat{k} = \hat{\jmath}$

Vector cross product using determinant form:

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} \hat{\mathbf{i}} - \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} \hat{\mathbf{j}} + \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix} \hat{\mathbf{k}}$$

$$\mathbf{A} \times \mathbf{B} = (A_yB_z - A_zB_y)\hat{\mathbf{i}} - (A_xB_z - A_zB_x)\hat{\mathbf{j}} + (A_xB_y - A_yB_x)\hat{\mathbf{k}}$$

Right-Hand Rule (RHR-1) for Magnetic Force

To find the **direction** of the force:

1. **Point your right-hand fingers in the direction of velocity v**.
2. **Curl your fingers toward B** (magnetic field direction).
3. **Your thumb points toward the force** for a **positive charge**.
4. **The force is in the opposite direction for a negative charge (e.g., electrons).**

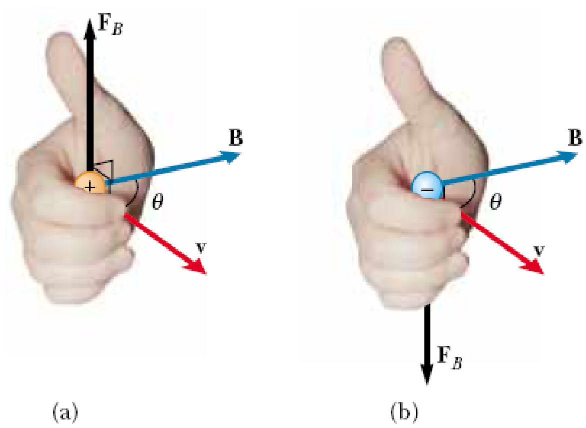
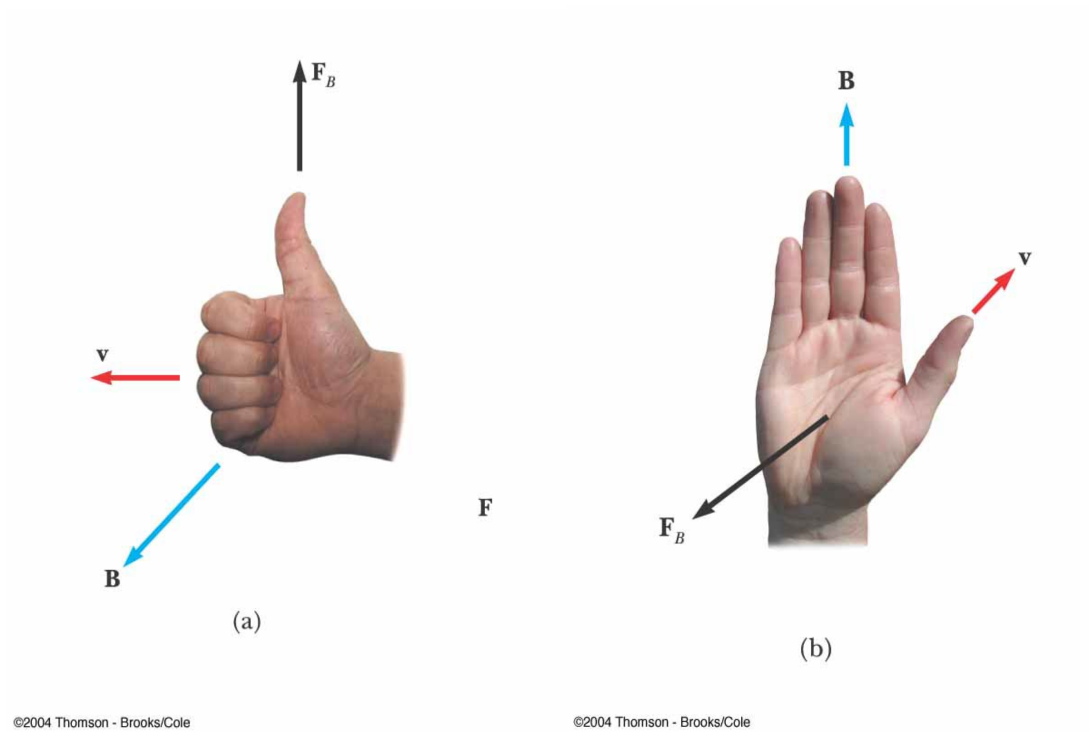
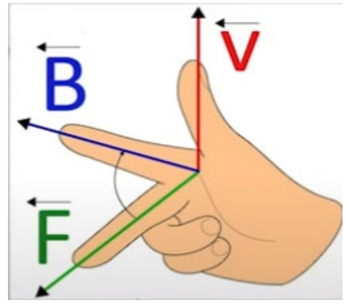


Figure 29.4 The right-hand rule for determining the direction of the magnetic force $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$ acting on a particle with charge q moving with a velocity **v** in a magnetic field **B**. The direction of $\mathbf{v} \times \mathbf{B}$ is the direction in which the thumb points. (a) If q is positive, $\mathbf{F}_B$ is upward. (b) If q is negative, $\mathbf{F}_B$ is downward, antiparallel to the direction in which the thumb points.

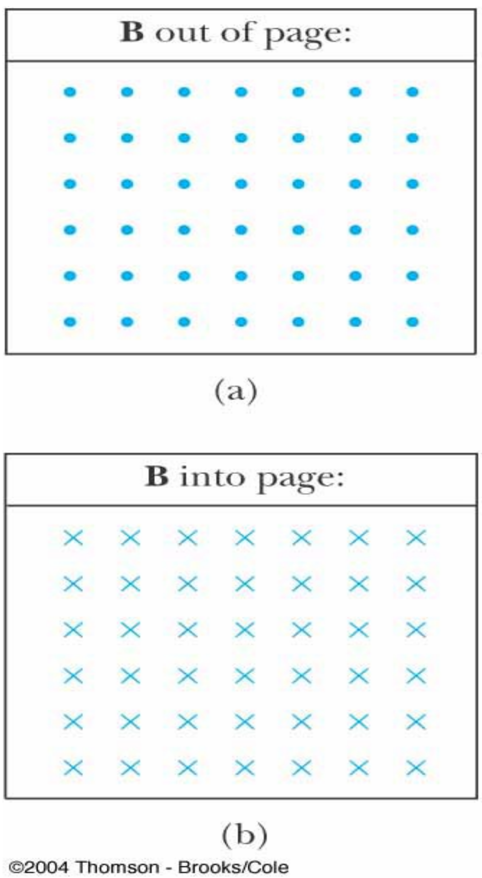


Also, another way to figure out the direction of the magnetic force.

- **Index finger (Blue)** → Magnetic field **B**
- **Middle finger (Red)** → Velocity **V**
- **Thumb (Green)** → Force **F**

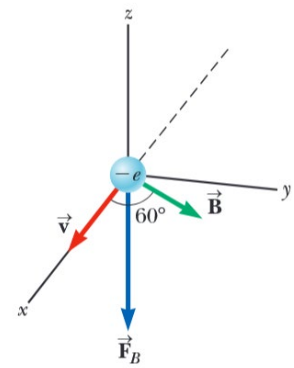


- If the particle is **negative (like an electron)**, the **force reverses** direction (opposite to your thumb).



Example 1:

An electron moves through space as a cosmic ray with a speed of $8.0x10^6$ m/s along the x-axis (see the Figure). At its location, the magnetic field of the Earth has a magnitude of 0.050 mT, and is directed at an angle of 60^0 to the x-axis, lying in the xy plane. Calculate the magnetic force on the electron.



Solution:

Given:

• $v = 8.0\times10^6$ m/s (along +x)

• $B = 0.050$ mT $= 5.0\times10^{-5}$ T

• $\theta = 60°$ (angle between v and B)

• $q = -1.602\times10^{-19}$ C (for electron)

The magnitude of the magnetic force is given by:

$F = |q|\, v\, B \sin\theta$

Substitute the values:

$F = (1.602\times10^{-19})(8.0\times10^6)(5.0\times10^{-5})\sin60°$

$vB = 8.0\times10^6 \times 5.0\times10^{-5} = 400$

$F = (1.602\times10^{-19})(400)(0.866) \approx 5.5\times10^{-17}$ N

Direction: The magnetic force is given by $F = q\,(v \times B)$.

Since v is along +x and B lies in the xy-plane at 60° to x, $(v \times B)$ points along +z. Because the charge of the electron is negative, the force direction is reversed (−z).

Final Answer:

$F = 5.5\times10^{-17}$ N, directed along the −z-axis.

Example2:

An electron moving along the positive *x-axis perpendicular* to a magnetic field experiences a magnetic deflection in the negative *y direction. What is the direction of* the magnetic field?

Solution:

The magnetic force on a moving charge is given by:

$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$

Because the electron has a negative charge ($q < 0$), the direction of the magnetic force is opposite to the direction of ($\mathbf{v} \times \mathbf{B}$).

Step 1: Determine (v × B) direction

• **v** is along +x

• The observed force **F** is along −y

→ Therefore, ($\mathbf{v} \times \mathbf{B}$) must be along +y (opposite of −y because q is negative).

Step 2: Apply the right-hand rule

Using the right-hand rule for ($\mathbf{v} \times \mathbf{B}$):

• **v** (+x) × **B** (?) = +y

• This is true if **B** points along +z.

Example 3:

A proton moves perpendicular to a uniform magnetic field B at 1.0×10^{7} m/s and experiences an acceleration of 2.0×10^{13} m/s^2 in the + x direction when its velocity is in the +z direction. Determine the magnitude and direction of the field.

Solution:

Given:
• Velocity, $v = 1.0\times10^{7}$ m/s (along +z)
• Acceleration, $a = 2.0\times10^{13}$ m/s^2 (along +x)
• Charge of proton, $q = 1.602\times10^{-19}$ C
• Mass of proton, $m = 1.67\times10^{-27}$ kg
The magnetic force on the proton provides the centripetal acceleration:
$F = q(v \times B) = m\cdot a$

Step 1: Magnitude of B

Since $v \perp B$, we use:
$qvB = ma \;\rightarrow\; B = (m\cdot a)/(q\cdot v)$
Substitute the known values:
$B = [(1.67\times10^{-27})(2.0\times10^{13})] / [(1.602\times10^{-19})(1.0\times10^{7})]$
$B = (3.34\times10^{-14}) / (1.602\times10^{-12}) = 0.0208$ T
→ $B = 2.1\times10^{-2}$ T

Step 2: Direction of B

The proton (positive charge) has velocity along +z and acceleration (force) along +x. Using the right-hand rule for $v \times B \rightarrow F$, point fingers along +z and curl toward +x; the thumb points along −y. Hence, the magnetic field points in the −y direction.

Final Answer:

Magnitude: $B = 2.1\times10^{-2}$ T
Direction: −y direction