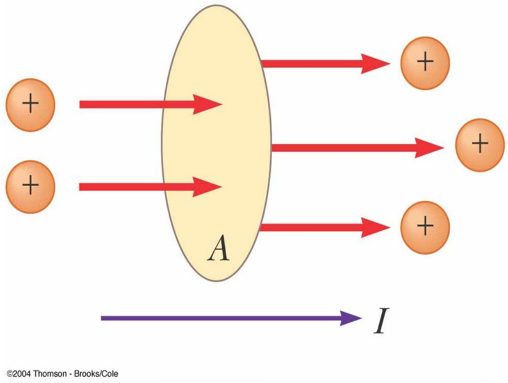
Chapter 26

Current and Resistance

26-1 **Electric Current**

Consider a system of electric charges in motion. A current is said to exist whenever there is a net flow of charge through some region.



The current is the rate at which charge flows through this surface:

$$I = \frac{\Delta q}{\Delta t} \qquad (1\text{-}27)$$

- The unit of current is Ampere [A].

$$\frac{Coulomb}{\sec ond} = Ampere \ [C/s = A]$$

- That is, 1 A of current is equivalent to 1 C of charge passing through the surface area in 1 s.

- The direction of the current is defined as the direction of the flow of positive charge, which is opposite to the direction of the flow of electrons.

- It is common to refer to a moving charge (positive or negative) as a mobile charge carrier. For example, the mobile charge carriers in a metal are electrons.

See the simulation: [Flow of electric current | electron direction #short #shorts #animation #physics - YouTube]

Microscopic View of Electric Current

The net motion of charge carriers is responsible for the flow of electric current. In an isolated conductor, we know there is no net motion of electric charge. However, individual electrons move quite a bit due to their thermal energy. There is no net charge transfer because thermal motion is random, and as many electrons move to the left, an equal number move to the right.

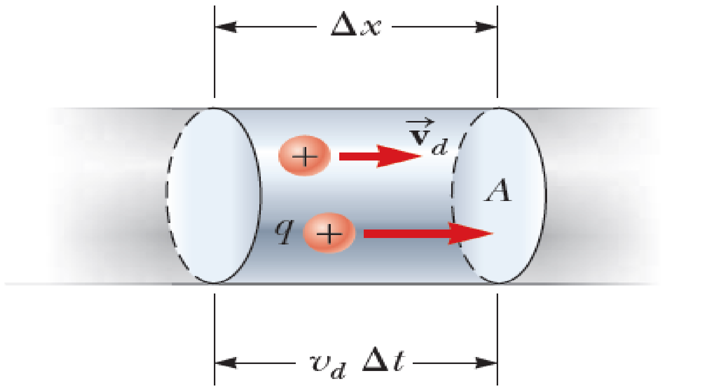


Figure 26.2 A segment of a uniform conductor of cross-sectional area *A*.

One can apply an electric field (or a potential) to cause a net charge transfer. For example, a battery supplies the electric field (or potential difference) necessary for charges to move in an electric circuit.

It is interesting to determine how quickly the electrons responsible for the current move in a conductor.

If n = **number of charge carriers per volume** ($n = \frac{N}{V}$); then the total number of charge, q, of the carriers in a length of wire l long with cross-sectional area A is:

$$N = nV = n.(A.\Delta x) = n.(A.\Delta x).\frac{\Delta t}{\Delta t} = n.A.(\frac{\Delta x}{\Delta t}).\Delta t = n.A.\mathrm{v}_d.\Delta t \qquad (2\text{-}27)$$

Then, the total charge can be written as:

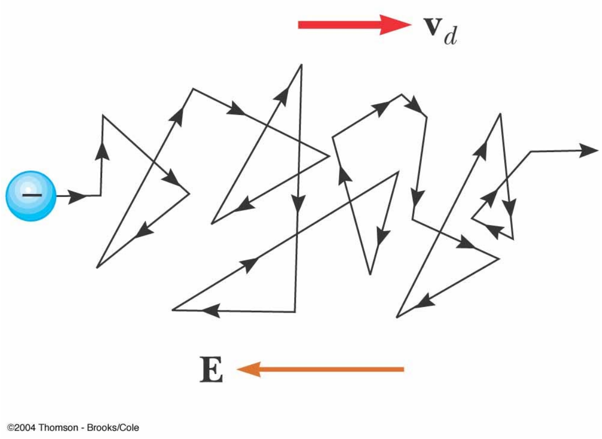
$$\Delta q = N.e \tag{3-27}$$

Finally, we can give an expression of the electric current:

$$I = \frac{\Delta q}{\Delta t} = n\, v_d e\, A$$

Where v_d is called the drift velocity, it represents the average speed of the charge carriers.

(Carriers move in a zigzag fashion due to collision with atoms.)



Example 26-1

The 12-gauge copper wire in a typical residential building has a cross-sectional area of 3.31x 10^{-6} m^2. It carries a constant current of 10.0 A. What is the drift speed of the electrons in the wire? Assume each copper atom contributes one free electron to the current. The density of copper is 8.92 g/cm^3.

Solution:

1) Electron number density (1 free e^- per atom):

$n = N_A\, \rho / M$

$n = (6.02 \times 10^{23}\ mol^{-1}) \times (8920\ kg \cdot m^{-3}) / (0.0635\ kg \cdot mol^{-1}) = 8.456 \times 10^{28}\ m^{-3}$

2) Use the drift-speed relation $v_d = I / (n\, q\, A)$:

$v_d = 10.0\ A / (8.45610^{28}\ m^{-3} \times 1.60 \times 10^{-19}\ C \times 3.31 \times 10^{-6}\ m^2\ )$

$v_d = 2.233 \times 10^{-4}$ m/s

Result

The drift speed is:

$v_d \approx 2.23 \times 10^{-4}$ m/s

Note

This small value (~10^{-4} m/s) illustrates that conduction electrons move very slowly on average; the nearly instantaneous circuit response comes from the electric field propagating at a much higher speed.

Example 26-2

The measured beam current is 30.0 μA in a particular cathode ray tube. How many electrons strike the tube screen every 40.0 s?

Solution:

Given

$I = 30.0\ \mu A = 30.0 \times 10^{-6}$ A

$\Delta t = 40.0$ s

$e = 1.60 \times 10^{-19}$ C

Equations

$I = \Delta Q / \Delta t \;\Rightarrow\; \Delta Q = I\,\Delta t$

Number of electrons: $N = \Delta Q / e$

Solution

1) Compute the total charge arriving in Δt:

$\Delta Q = I\,\Delta t = (30.0 \times 10^{-6}\ A)(40.0\ s) = 1.20 \times 10^{-3}$ C

2) Convert charge to number of electrons:

$N = \Delta Q / e = (1.20 \times 10^{-3}\ C) / (1.60 \times 10^{-19}\ C)$

$= 7.50 \times 10^{15}$ electrons (strike the screen every 40.0 s).

Example 26-3

An aluminum wire having a cross-sectional area of 4.00×10^{-6} m^2 carries a current of 5.00 A. Find the drift speed of the electrons in the wire. The density of aluminum is 2.70 g/cm^3. Assume that each atom supplies one conduction electron. (The aluminum (Al) molar mass is 27 g/mol).

Solution:

We use the relation:

$I = nqAv_d$

Step 1: Determine the number density (n) of conduction electrons.

Molar mass of aluminum = 27.0 g/mol
Avogadro's number $N_A = 6.02 \times 10^{23}$ atoms/mol
Density $\rho = 2.70$ g/cm³

For 1 mole (27.0 g):
Volume $V = m/\rho = 27.0 / 2.70 = 10.0\ \text{cm}^3 = 1.0 \times 10^{-5}\ \text{m}^3$

Number density:
$n = N_A / V = (6.02 \times 10^{23}) / (1.0 \times 10^{-5})$
$n = 6.02 \times 10^{28}$ electrons/m³

Step 2: Calculate the drift speed.

$v_d = I / (nqA)$

$I = 5.00$ A
$q = 1.60 \times 10^{-19}$ C
$A = 4.00 \times 10^{-6}\ \text{m}^2$

$v_d = 5.00 / [(6.02 \times 10^{28})(1.60 \times 10^{-19})(4.00 \times 10^{-6})]$

$v_d = 1.30 \times 10^{-4}$ m/s

Final Answer:

The drift speed of the electrons is 1.30×10^{-4} m/s.

Example 26-4

A high-voltage transmission line with a diameter of 2.00 cm and a length of 200 km carries a steady current of 1000 A. If the conductor is copper wire with a free charge density of 8.49×10^{28} electrons/m³, how long does it take one electron to travel the full length of the line?

Solution:

Given

Diameter D = 2.00 cm ⇒ radius $r = 1.00 \times 10^{-2}$ m

Length L = 200 km = 2.00×10^{5} m

Current I = 1000 A

Free-electron density $n = 8.49 \times 10^{28}\ \text{m}^{-3}$

Electron charge $q = 1.60 \times 10^{-19}$ C

Solution

1) Cross-sectional area of the wire: $A = \pi r^2$

$A = \pi(1.00 \times 10^{-2}\ \text{m})^2 = 3.1416\times 10^{-4}\ \text{m}^2$

2) Drift speed using $I = n\, q\, A\, v_d \Rightarrow v_d = I/(n\, q\, A)$

$v_d = 1000\ \text{A} / [\ (8.49 \times 10^{28}\ \text{m}^{-3})(1.60 \times 10^{-19}\ \text{C})(3.1416\times 10^{-4}\ \text{m}^2)\ ]$

$v_d = 2.343\times 10^{-4}\ \text{m/s} \approx 0.234\ \text{mm/s}$

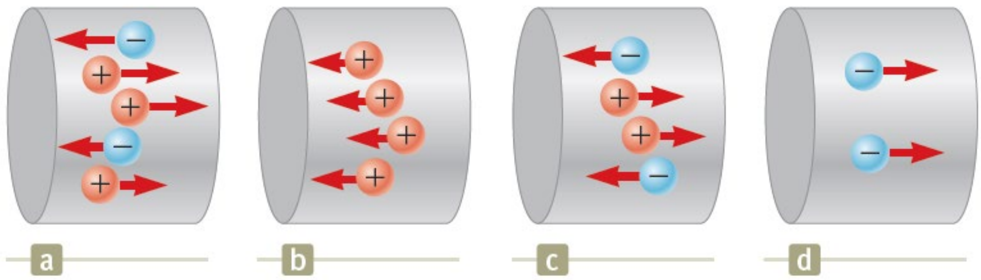
3) Time to travel the full length: $t = L / v_d$

$t = (2.00 \times 10^5\ \text{m}) / (2.34310^{-4}\ \text{m/s}) = 8.53510^8\ \text{s}$

Convert to years: $t \approx 27.06$ years

Example- 26-5

Consider positive and negative charges of equal magnitude moving horizontally through the four regions shown in the following Figure. Rank the current in these four regions from highest to lowest.



COMMENTS:

Drift velocity (v_d) is the slow, average velocity at which free electrons move through a conductor due to an applied electric field. While individual electrons experience rapid random motion, their overall net movement in a specific direction—caused by the field—constitutes drift velocity.

!!! Drift Velocity and the Speed of Electricity Transfer!!!

- Drift velocity is very slow (typically around 0.1 mm/s in household wiring).
- However, electricity (energy transfer) happens almost instantly when a switch is turned on.
- This is because the electric field propagates close to the speed of light in the wire (~50–99% of c, depending on the material).
- The electric field pushes electrons all along the wire simultaneously, like a chain reaction in a pipe full of water—when you push electrons at one end, others respond immediately at the other end.

Notes:

i- The number of moles ($n^\backslash$) is given by:

$$n^\backslash = \frac{mass\ of\ substance\ (in\ grams)}{molar\ mass\ (\frac{g}{mol})}$$

Where:

- Mass of substance = Given mass in grams.
- Molar mass = Mass of one mole of the element (found in the periodic table).

ii- The Total Number of Atoms

Use Avogadro's number ($N_A = 6.022 \times 10^{23}$ atoms/mol) to find the number of atoms:

Number of atoms = $n^\backslash \times N_A$

Where:

- $n^{\backslash}$ = Number of moles (from I).
- N_A = Avogadro's number.

iii- The Number of Electrons per Atom

Each element has a specific atomic number (Z), which tells you the number of electrons in a neutral atom.

i.e., Total number of electrons = Number of atoms x Z

where:

- Z = Atomic number (from the periodic table).

In previous problems 1 and 3, for simplicity, we assume that each atom contributes one electron.