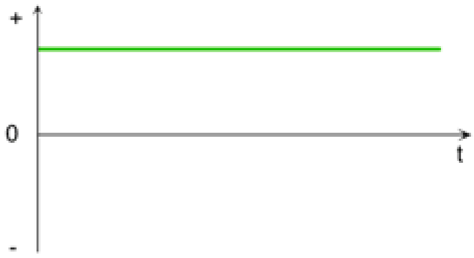
Chapter 33

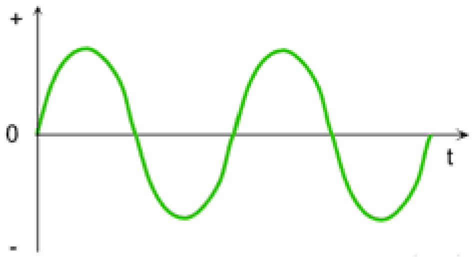
Alternating-current circuits

Alternating Current (AC) is a type of electrical current, in which the direction of the flow of electrons switches back and forth at regular intervals or cycles. Current flowing in power lines and normal household electricity that comes from a wall outlet is alternating current. The standard current used in the U.S. is 60 cycles per second (i.e. a frequency of 60 Hz); in Europe and most other parts of the world it is 50 cycles per second (i.e. a frequency of 50 Hz.).

Direct Current



Alternating current



Introduction to AC and DC

Attached.

In this chapter, we will learn the behavior of the voltage and current in RLC circuits with AC.

Impedance Values and Phase Angles for Various Circuit-Element Combinations[a]

Circuit Elements	Impedance Z	Phase Angle ϕ
R	R	$0°$
C	X_C	$-90°$
L	X_L	$+90°$
R C	$\sqrt{R^2 + X_C^2}$	Negative, between $-90°$ and $0°$
R L	$\sqrt{R^2 + X_L^2}$	Positive, between $0°$ and $90°$
R L C	$\sqrt{R^2 + (X_L - X_C)^2}$	Negative if $X_C > X_L$ Positive if $X_C < X_L$

❖ AC Voltage Source

An AC circuit consists of circuit elements and a power source that provides an alternating voltage Δv. This time-varying voltage from the source is described by

$$\Delta v = V_{max} \sin \omega t$$

33.1

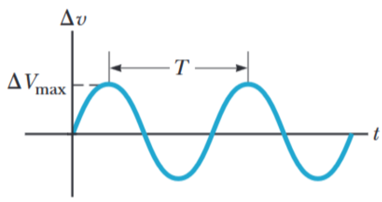


Figure 32.1 The voltage supplied by an AC source is sinusoidal with a period *T*.

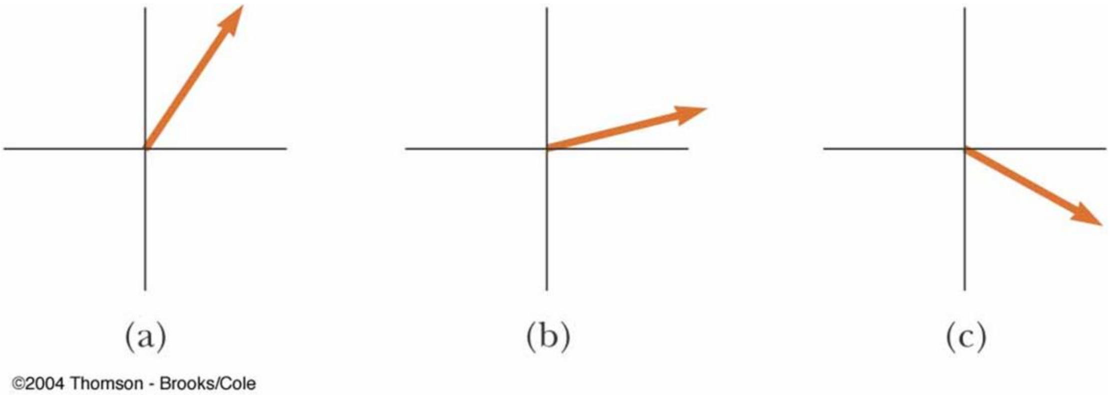
Where V_{max} represents the maximum output voltage of the source, or the voltage amplitude. As shown in the figure at the top of the AC, the voltage is positive during one-half of the cycle and negative during the other half. The angular frequency of the AC voltage is given by

$$\omega = 2\pi f = 2\pi \frac{1}{T} \qquad 33.2$$

Where f is the frequency of the source and T is the period.

- ➢ To simplify our analysis of circuits containing two or more elements, we use graphical constructions called ***phasor diagrams***.
- ➢ *In these constructions, alternating (sinusoidal)* quantities, such as current and voltage, are represented by rotating vectors called **phasors**.
- ➢ The length of the phasor represents the amplitude (maximum value) of the quantity, and the projection of the phasor onto the vertical axis represents the instantaneous value of the quantity.

- As we shall see, a phasor diagram greatly simplifies matters when we must combine several sinusoidally varying currents or voltages that have different phases.



33.2 Resistors in an AC circuit

In the figure in front, a simple AC circuit contains a resistor and an AC source.

At any instant, the algebraic sum of the voltages around a closed loop in a circuit must be zero (Kirchhoff's loop rule).

$$\Delta v - i_R R = 0 \qquad 33.3$$

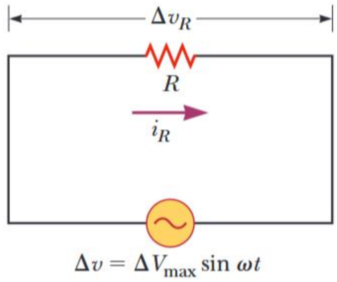


Figure 32.2 A circuit consisting of a resistor of resistance R connected to an AC source, designated by the symbol ~. At the moment depicted in the figure, the current is to the right in the resistor.

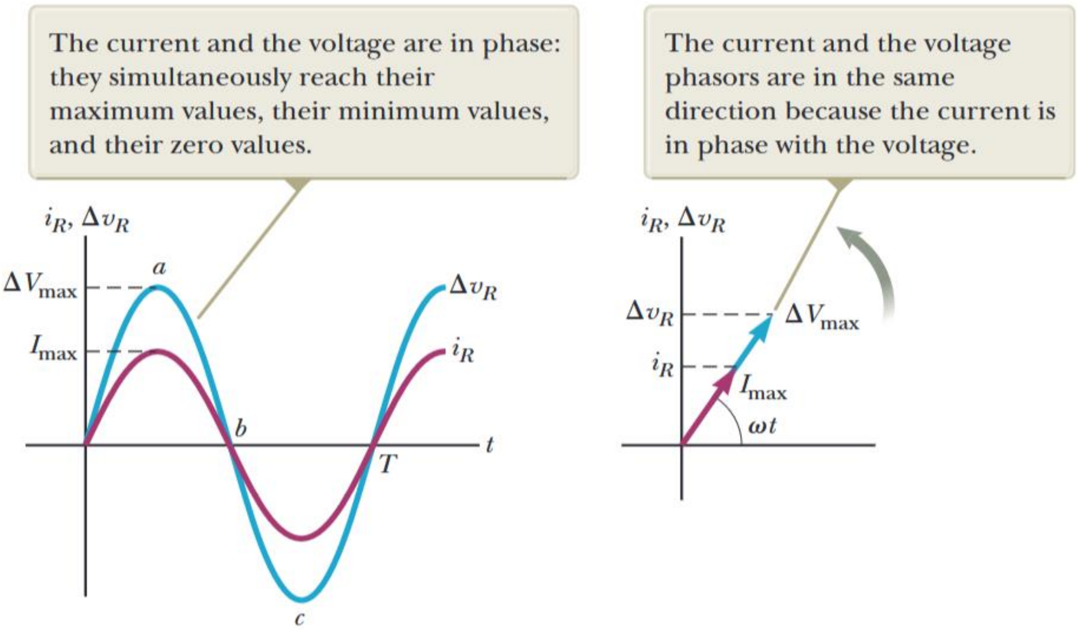
Substituting eq. (33.1) in eq. (33.2), we get

$$V_{max} \sin \omega t = i_R R$$

$$\Rightarrow i_R = \frac{V_{max}}{R} \sin \omega t = I_{max} \sin \omega t \qquad 33.4$$

Where the maximum current, $I_{max} = \frac{V_{max}}{R}$.

Plot of the instantaneous current i_R and instantaneous voltage Δv_R across a resistor as functions of time is shown in the figure below.



- **The current is in phase with the voltage, which means that the current is zero when the voltage is zero, maximum when the voltage is maximum, and minimum when the voltage is minimum.**
- **At time $t = T$, one cycle of the time-varying voltage and current has been completed.**
- **Also, one need to notice that:**

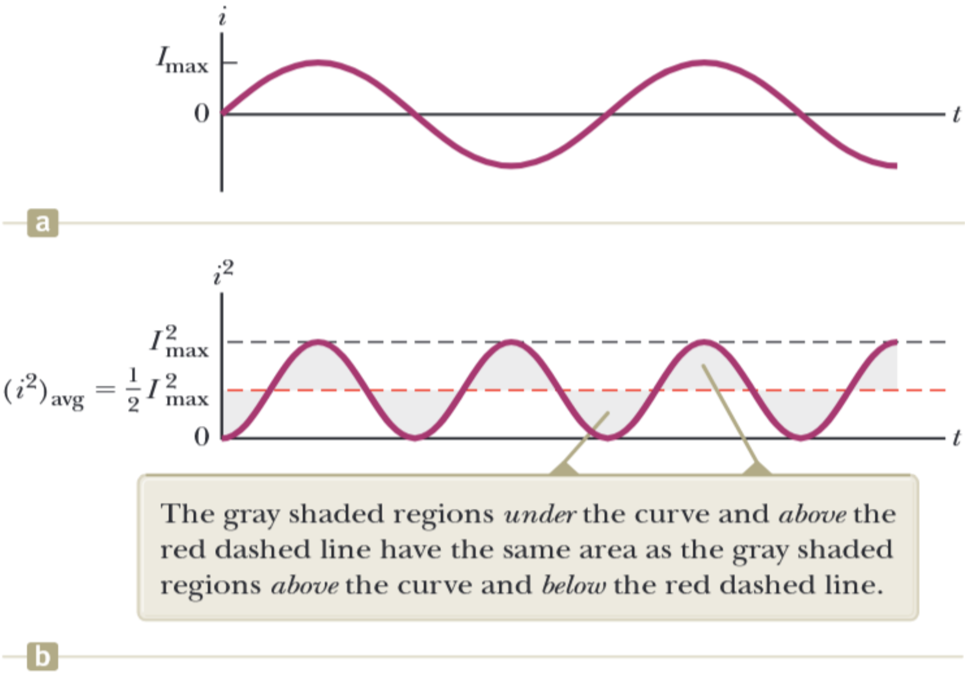
  $I_{max} > V_{max}$ if R < 1

  $I_{max} < V_{max}$ if R > 1

<u>Phasor diagram for the resistive circuit showing that the current is in phase with the voltage.</u>

RMS current:

In the following figure, the current squared in a resistor as a function of time.



Notice that the gray shaded regions *under* the curve and *above* the dashed line for $\frac{I^2_{max}}{2}$ have the same area as the gray shaded regions *above* the curve and *below* the dashed line for $\frac{I^2_{max}}{2}$. Thus, the average value of i^2 is $\frac{I^2_{max}}{2}$.

What is of importance in an ac circuit is an average value of current, referred to as the **rms (root mean square) current**. $I_{rms} = \sqrt{(i^2)_{ave}}$

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = 0.707 \ I_{max} \qquad 33.5$$

The average power delivered to a resistor that carries an alternating current is

$$P_{ave} = I^2_{rms} R$$

Similarly,

$$\Delta V_{rms} = \frac{V_{max}}{\sqrt{2}} = 0.707 \ V_{max}$$

Example-1:

The voltage output of an AC source is given by the expression Δv= 200 sin ωt, where Δv is in volts. Find the rms current in the circuit when this source is connected to a 47.0-Ω resistor.

Example-2

An AC source of v(t)=120 sin (100πt) V is connected across a 20 Ω resistor.
Find:

1. **The current as a function of time**
2. **The peak and RMS current**
3. **The average power dissipated**

Solution:

1. Voltage: $v(t) = 120\sin(100\pi t)$

   Ohm's law:

$$i(t) = \frac{v(t)}{R} = \frac{120\sin(100\pi t)}{20} = 6\sin(100\pi t)\ \mathrm{A}$$

2. Peak current: $I_{\max} = 6\ \mathrm{A}$

   RMS current:

$$I_{\mathrm{rms}} = \frac{I_{\max}}{\sqrt{2}} = \frac{6}{\sqrt{2}} \approx 4.24\ \mathrm{A}$$

3. Average power:

$$P_{\mathrm{avg}} = I_{\mathrm{rms}}^2 R = (4.24)^2 \times 20 \approx 359.3\ \mathrm{W}$$