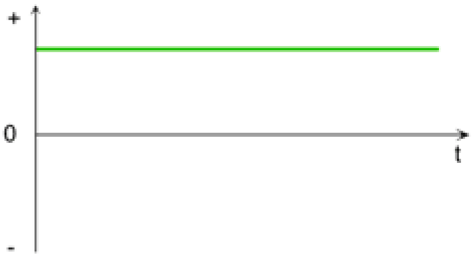
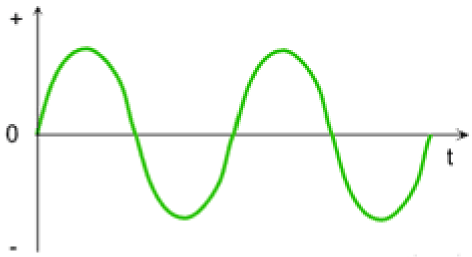
Chapter 32

Alternating-current circuits

Alternating Current (AC) is a type of electrical current in which the direction of the flow of electrons switches back and forth at regular intervals or cycles. The current flowing in power lines and in normal household electricity that comes from a wall outlet is alternating current. The standard current used in the U.S. is 60 cycles per second (i.e., a frequency of 60 Hz); in Europe and most other parts of the world, it is 50 cycles per second (i.e., a frequency of 50 Hz).

Direct Current **Alternating current**





Introduction to AC and DC

- Introduction to AC and DC.pdf

- https://youtu.be/OUcKJuMSSW4?si=QGe_oYzdagqU-1OY

In this chapter, we will learn the behavior of the voltage and current in RLC circuits with AC.

Impedance Values and Phase Angles for Various Circuit-Element Combinations[a]		
Circuit Elements	**Impedance Z**	**Phase Angle ϕ**
R	R	$0°$
C	X_C	$-90°$
L	X_L	$+90°$
R C	$\sqrt{R^2 + X_C^2}$	Negative, between $-90°$ and $0°$
R L	$\sqrt{R^2 + X_L^2}$	Positive, between $0°$ and $90°$
R L C	$\sqrt{R^2 + (X_L - X_C)^2}$	Negative if $X_C > X_L$ Positive if $X_C < X_L$

❖ AC Voltage Source

An AC circuit consists of circuit elements and a power source that provides an alternating voltage Δv. This time-varying voltage from the source is described by

$$\Delta v = V_{max} \sin \omega t \qquad 32.1$$

Where V_{max} represents the maximum output voltage of the source, or the voltage amplitude. As shown in the figure at the top of the AC, the voltage is positive for one half of the cycle and negative for the other half. The angular frequency of the AC voltage is given by

$$\omega = 2\pi f = 2\pi \frac{1}{T} \qquad 32.2$$

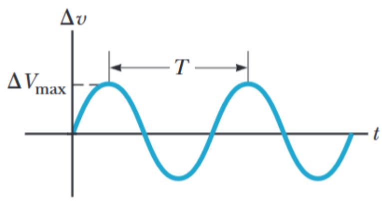
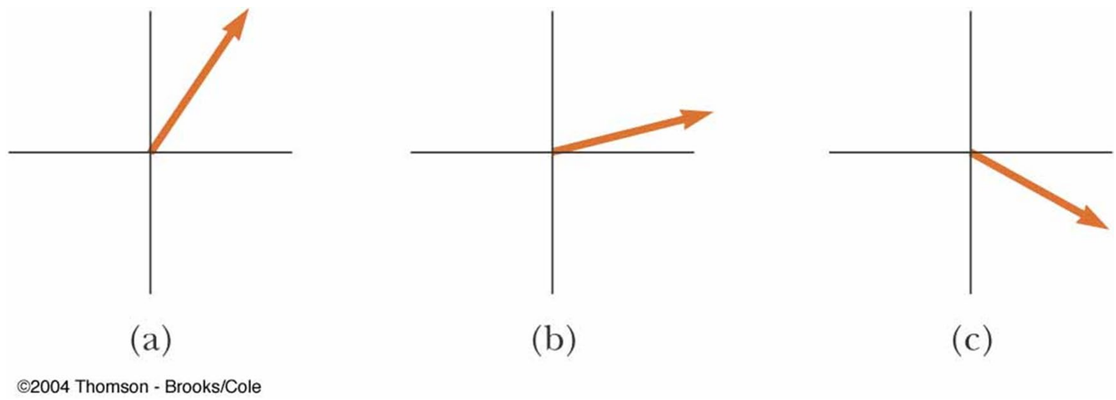


Figure 32.1 The voltage supplied by an AC source is sinusoidal with a period *T*.

Where **f** is the frequency of the source, and **T** is the period.

- To simplify our analysis of circuits containing two or more elements, we use graphical constructions called ***phasor diagrams***.
- *In these constructions, alternating (sinusoidal)* quantities, such as current and voltage, are represented by rotating vectors called **phasors**.
- The length of the phasor represents the amplitude (maximum value) of the quantity, and the projection of the phasor onto the vertical axis represents the instantaneous value of the quantity.
- As we shall see, a phasor diagram greatly simplifies matters when we must combine several sinusoidally varying currents or voltages that have different phases.



Think about it :::

Flickering Light and Frequency

An alternating current (AC) source is connected to a small lamp. Under normal conditions, the AC frequency is 50 Hz, and the light appears steady to the human eye.

Now, suppose the frequency is reduced to 5 Hz.

Question:

What would you observe in the light output, and why? Explain your reasoning in terms of how current, frequency, and human visual perception interact.

Hint:
Think about how quickly the current changes direction at 5 Hz compared with 50 or 60 Hz, and how that affects brightness.

32.2 Resistors in an AC circuit

In the figure in front, a simple AC circuit contains a resistor and an AC source.

At any instant, the algebraic sum of the voltages around a closed loop in a circuit must be zero (Kirchhoff's loop rule).

$$\Delta v - i_R R = 0 \qquad 32.3$$

Substituting eq. (32.1) in eq. (32.2), we get:

$$V_{max} \; \sin \omega \, t = i_R R$$
$$\Rightarrow i_R = \frac{V_{max}}{R} \sin \omega \, t = I_{max} \; \sin \omega \, t \qquad 32.4$$

Where the maximum current: $I_{max} = \frac{V_{max}}{R}$.

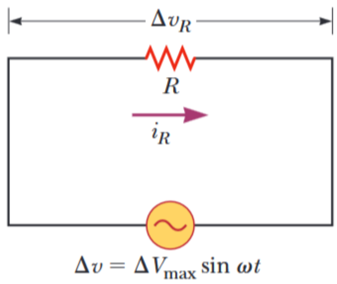
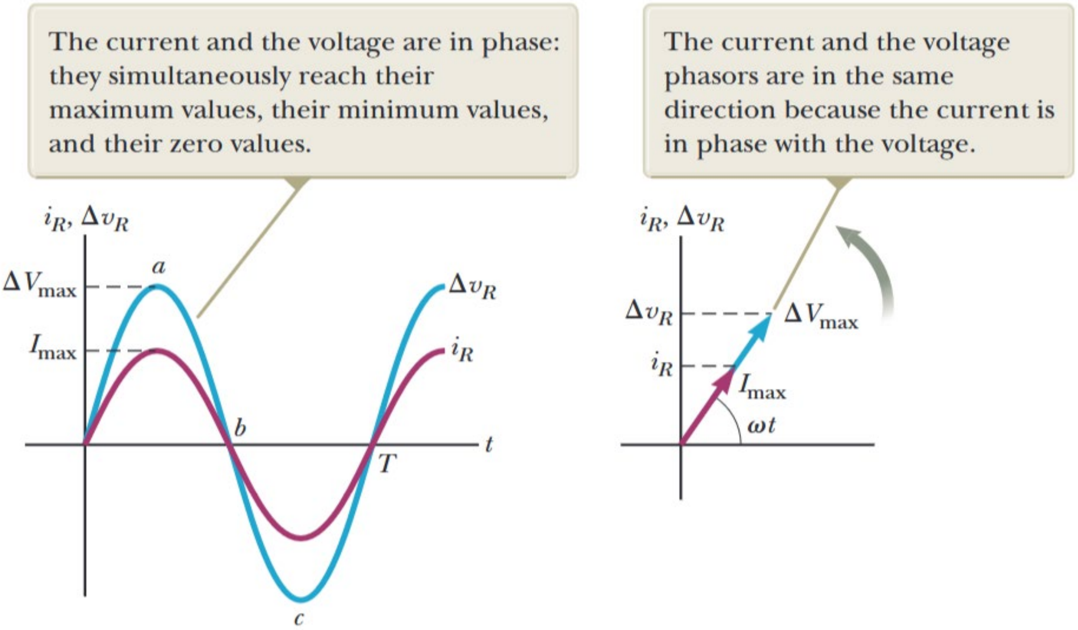


Figure 32.2 A circuit consisting of a resistor of resistance R connected to an AC source, designated by the symbol (~). At the moment depicted in the figure, the current is to the right in the resistor.

Plot of the instantaneous current i_R and instantaneous voltage ΔV_R across a resistor as functions of time is shown in the figure below.



- The current is in phase with the voltage, which means that the current is zero when the voltage is zero, maximum when the voltage is maximum, and minimum when the voltage is minimum.
- **At time $t = T$,** one cycle of the time-varying voltage and current has been completed.
- Also, one needs to notice that:

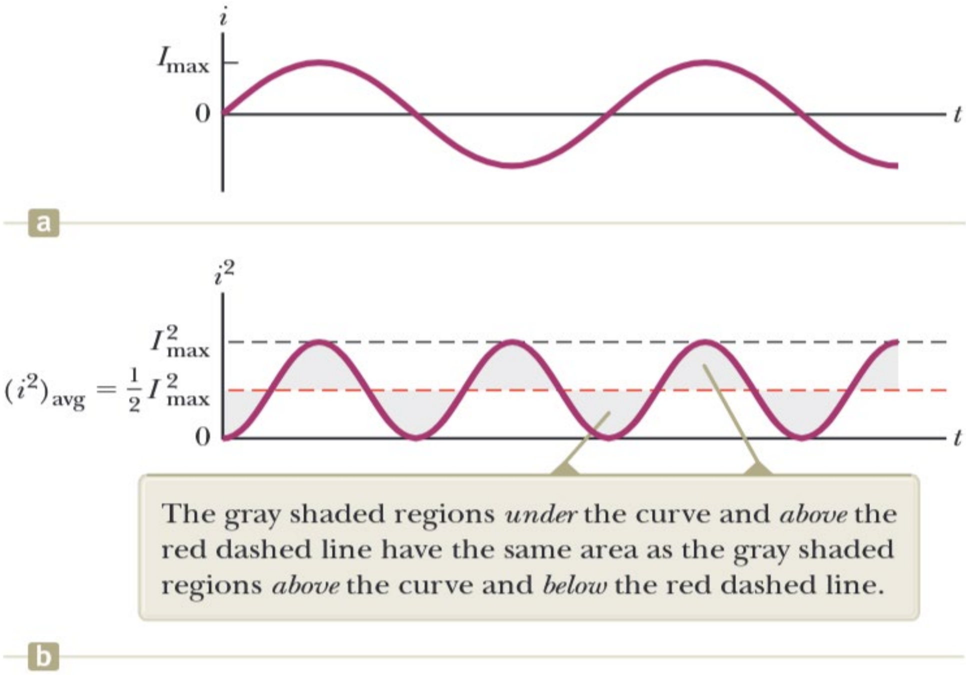
  $\mathbf{I_{max} > V_{max}}$ **if** $\mathbf{R < 1}$

  $\mathbf{I_{max} < V_{max}}$ **if** $\mathbf{R > 1}$

<u>Phasor diagram for the resistive circuit showing that the current is *in phase* with the voltage.</u>

RMS current:

In the following figure, the current squared in a resistor as a function of time.



Notice that the gray shaded regions *under* the curve and *above* the dashed line for $\frac{I^2_{max}}{2}$ have the same area as the gray shaded regions *above* the curve and *below* the dashed line for $\frac{I^2_{max}}{2}$. Thus, the average value of i^2 is $\frac{I^2_{max}}{2}$.

What is of importance in an AC circuit is the average value of current, referred to as the RMS **(root mean square) current**. $I_{rms} = \sqrt{(i^2)_{ave}}$

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = 0.707\, I_{max} \qquad 32.5$$

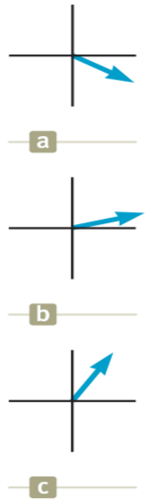
The average power delivered to a resistor that carries an alternating current is

$$P_{ave} = I^2_{rms}R$$

Similarly,

$$\Delta V_{rms} = \frac{V_{max}}{\sqrt{2}} = 0.707\, V_{max}$$

Think about it ::::



Consider the voltage phasor in the Figure at three instants in time.

(i) Choose the part of the figure, (a), (b), or (c), that represents the instant of time at which the instantaneous value of the voltage has the largest magnitude.

(ii) Choose the part of the figure that represents the instant of time at which the instantaneous value of the voltage has the smallest magnitude.

Solution:

In short:

magnitude=$|v(t)|=|V_{max}\ \sin_{\theta}|$

- **(a)** → 4th quadrant → angle just above 270° and closer to the x-axis → **negative instantaneous value**
- **(b)** → near x-axis → angle ≈ 0° → **zero instantaneous value (smallest)**
- **(c)** → near 90° → **positive maximum value** (largest)

Example-1:

The voltage output of an AC source is given by the expression $\Delta v = 200 \sin \omega t$, where Δv is in volts. Find the rms current in the circuit when this source is connected to a 47.0-Ω resistor.

Example-2

An AC source of $v(t)=120 \sin(100\pi t)$ V is connected across a 20 Ω resistor.
Find:

1. **The current as a function of time**
2. **The peak and RMS current**
3. **The average power dissipated**

Solution:

1. Voltage: $v(t) = 120\sin(100\pi t)$

   Ohm's law:

$$i(t) = \frac{v(t)}{R} = \frac{120\sin(100\pi t)}{20} = 6\sin(100\pi t)\ \mathrm{A}$$

2. Peak current: $I_{\max} = 6\ \mathrm{A}$

   RMS current:

$$I_{\mathrm{rms}} = \frac{I_{\max}}{\sqrt{2}} = \frac{6}{\sqrt{2}} \approx 4.24\ \mathrm{A}$$

3. Average power:

$$P_{\mathrm{avg}} = I_{\mathrm{rms}}^2 R = (4.24)^2 \times 20 \approx 359.3\ \mathrm{W}$$