PHYS-453

6 - The infinite square well

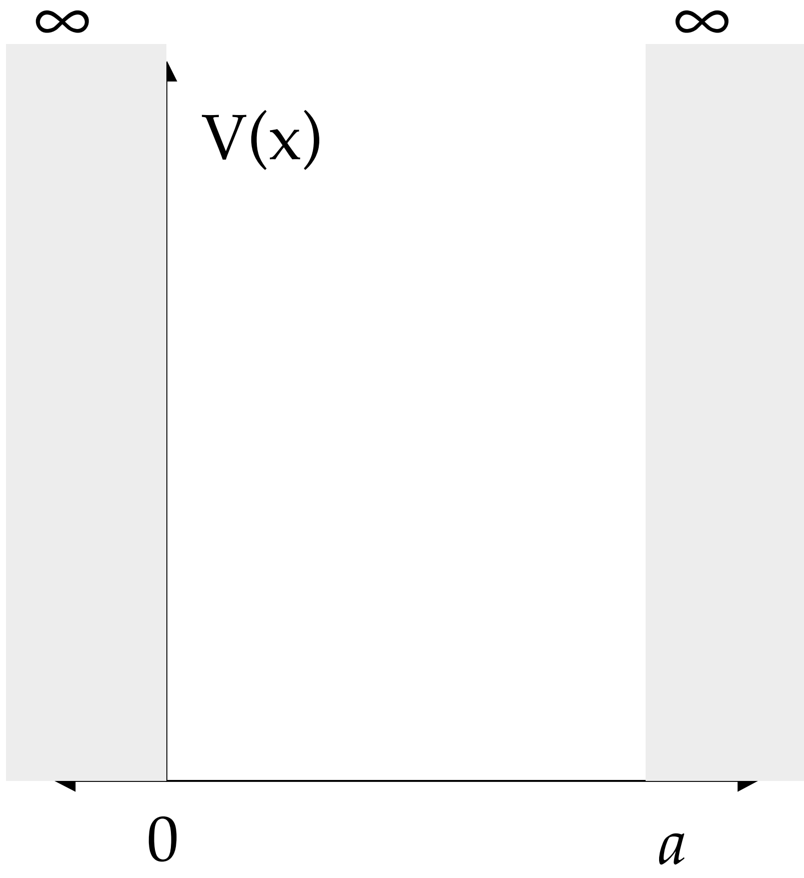
The infinite square well-a

- The infinite square well is the simplest problem that can be solved with Schroedinger equation. With these problem we can show easily some very important properties of the wave functions.

The infinite square well-b

The potential in this problem has the form:

$$V(x) = \begin{cases} 0 & 0 < x < a \\ \infty, & x < 0,\ x > a \end{cases}$$



Due to the infinity of the potential the wave function obeys the following initial conditions

$$\psi(0) = 0, \qquad \psi(a) = 0$$

The solution of Schroedinger's equation-c

The solution of the time independent Schroedinger equation has the form:

$$\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right) \qquad n = 1,\ 2,\ \ldots\ ,\infty$$

With corresponding energies

$$E_n = \frac{\hbar^2\pi^2}{2ma^2}n^2, \qquad n = 1,\ 2,\ \ldots\ ,\infty$$

Discussion of results-a

a) The energy of the particle is quantized

b) The paricle cannot be localized. The narrower the well the higher is the ground state energy!

c) The wave functions are alternately **even** and **odd** with respect to the center of the well.

d) As you go up in energy each successive state has on more node. The ground state has no nodes. The first excited has one node etc.

e) To each energy corresponds only one wave function. The energy spectrum is non-degenerate. This is a property of all one dimensional bound problems.

Discussion of results-b

f) The eigenfunctions are mutually orthogonal. This means that

$$\int \psi_m(x)^* \psi_n(x)dx = 0, \qquad \text{for} \quad m \neq n$$

$$\int \psi_m(x)^* \psi_n(x)dx = 1, \qquad \text{for} \quad m = n$$

Or by using the so-called **Kronecker delta**

$$\int \psi_m(x)^* \psi_n(x)dx = \delta_{mn} \qquad \delta_{mn} = \begin{cases} 0, & m \neq n \\ 1, & m = n \end{cases}$$

Discussion of results-b

g) They are complete, in the sense that any other function can be expressed as a linear combination of them

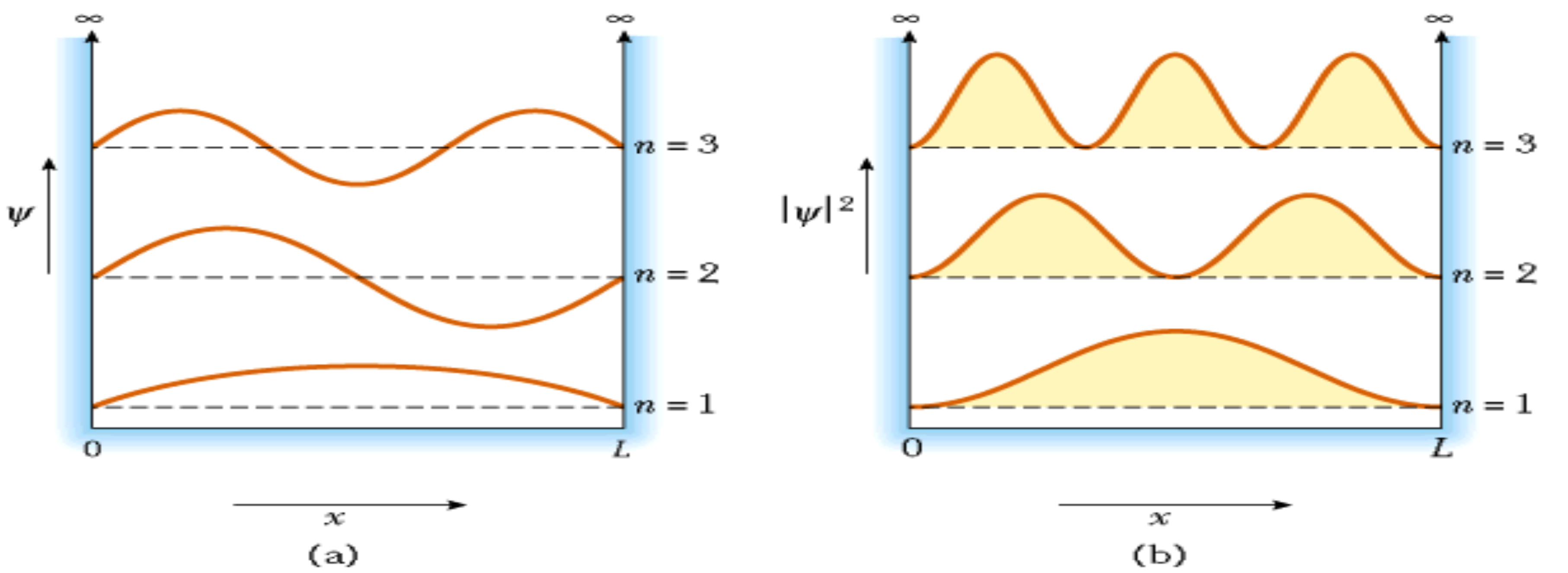
$$f(x) = \sum_{n=1}^{\infty} c_n \psi_n(x) = \sqrt{\frac{2}{a}} \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi}{a} x\right)$$

with

$$c_n = \int \psi_n(x)^* f(x) dx$$

Eigenfunctions and probability density

Serway, Physics for Scientists and Engineers, 5/e
Figure 41.11



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