1-SCHROEDINGER EQUATION

…and its statistical interpretation

The wave particle duality principle-a

Objects have a double nature. They are simultaneously particles and waves. The relations which relate the particle-like and the wave-like behavior are the following:

$$E = hf\,, \qquad p = h/\lambda \qquad (1.1)$$

This is a universal principle. It has been proved for all particles and fields in nature except the gravitational.

The wave particle duality principle-b

- The next task was to find a way to assign particle properties to a wave and wave properties to a particle.
- The quantization of light (or quantum electrodynamics) was a difficult task and solved between 1951-1953).

The wave particle duality principle-c

- The quantization of Newtonian mechanics, or quantum mechanics was solved in 1927 with the *statistical interpretation* according to which the coexistence of wave and particle properties on the same physical object is possible only if we consider the wave as a *probability wave*. That is a wave which represent not a vibration but the probability of finding the particle in a certain region of space.

The Schrödinger equation-a

- The fundamental quantity of the quantum mechanics is the wave-function $\psi(\mathbf{r},t)$.
- The function itself has not a direct physical meaning.
- It is the probability density which has a physical meaning.

The Schrödinger equation-b

- Thus, the probability density of finding a particle at position **r** at a certain time *t*, is given by :

$$\psi^*\left(\mathbf{r},t\right)\psi\left(\mathbf{r},t\right)dV = \left|\psi\left(\mathbf{r},t\right)\right|^2 dV \qquad (1.3)$$

- From the above discussion it is obvious that the following *normalization condition* holds:

$$\int_{-\infty}^{+\infty}\left|\psi\left(\mathbf{r},t\right)\right|^2 dV = 1 \qquad (1.4)$$

The Schrödinger equation-c

- If only $|\psi(\mathbf{r},t)|^2 dV$ has physical meaning, why don't we use this probability rather than $\psi(\mathbf{r},t)$?
- The wave function itself plays an important role in the interpretation of many phenomena in quantum physics. Briefly, $\psi(\mathbf{r},t)$ describes the wave nature of a particle, and $|\psi(\mathbf{r},t)|^2 dV$ describes where we might find the particle.

The Schrödinger equation-d

- The probability of finding the particle between two points *a* and *b* at time *t* is given by

$$\int_a^b \left|\psi\left(x,t\right)\right|^2 dx \qquad (1.6)$$

- The Schrödinger equation for a particle which moves in one direction and is subjected to a potential *V(x)* is given by

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi \qquad (1.7)$$

The Schrödinger equation-e

The Schrödinger equation for a particle which moves in three dimensions and is subjected to a potential *V(x,y,z)* is given by

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right) + V(x,y,z)\psi$$

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V(x,y,z)\psi \qquad (1.8)$$

The Schrödinger equation-f

- The wave function ψ has two important properties:
- It is square integrable

$$\int_{-\infty}^{+\infty} \left|\psi\left(x\right)\right|^2 dx < \infty$$

- This implies that $\psi\left(+\infty\right) = \psi\left(-\infty\right) = 0$
- It preserves the probability

How the Schrödinger equation is derived-a

- The function which describes a classical wave is given by:

$$u(x,t) = e^{i(kx-\omega t)}$$

- But remember the following relations between wave and particle properties:

$$E = hf \Rightarrow E = \frac{h}{2\pi} \cdot 2\pi f \Rightarrow E = \hbar \cdot \omega \Rightarrow \omega = E / \hbar$$

$$p = \frac{h}{\lambda} \Rightarrow p = \frac{h / 2\pi}{\lambda / 2\pi} \Rightarrow p = \hbar k \Rightarrow k = \frac{p}{\hbar}$$

How the Schrödinger equation is derived-b

- So the function which describes matter wave is given by:

$$y(x,t) = e^{i(px-Et)/\hbar}$$

- The equation which is obeyed by such a matter wave must be:

1. Linear and homogeneous (so superposition holds)

How the Schrödinger equation is derived-c

2. With constant coefficients since for V=0 all points of space must be equivalent, similarly all time moments (homogeneity of space and time).

3. It must reproduce the correct form of energy $E=p^2/2m$.

There are now two ways to arrive at the Schrödinger equation and they will be shown in the class.

The proof is given in the class.