-No, so there is some variation in the data

## This is called the total variation

-Denoted SS(Total) for the total Sum of Squares (variation)

- Sum of Squares is another name for variation
- ARE ALL OF THE SAMPLE MEANS IDENTICAL?
- No, so there is some variation between the groups.


## This is called variation Between Group

- Sometimes called the variation due to the factor.
- Denoted SS(B) for Sum of Squares (variation) between the groups


## - Are each of the values within each group identical?

- No, there is some variation within the groups.


## This is called variation within group

- Sometimes called the error variation.
- Denoted SS(W) for Sum of Squares (variation) within the groups


## - THERE ARE TWO SOURCES OF VARIATION:

- The variation between the groups, $\mathbf{S S}(\mathrm{B})$, or the variation due to the factor.
- The variation within the groups, SS(W), or the variation that can't be explained by the factor so it's called the error variation.
- Here is the basic one-way ANOVA table

| Source | SS | df | MS | F | P |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Between |  |  |  |  |  |
| Within |  |  |  |  |  |
| Total |  |  |  |  |  |

## - Grand Mean

- The grand mean is the average of all the values when the factor is ignored.
- It is a weighted average of the individual sample means.

$$
\overline{\bar{x}}=\frac{\sum_{i=1}^{k} n_{i} \bar{x}_{i}}{\sum_{i=1}^{k} n_{i}}
$$

$$
\overline{\bar{x}}=\frac{n_{1} \bar{x}_{1}+n_{2} \bar{x}_{2}+\mathrm{L}+n_{k} \bar{x}_{k}}{n_{1}+n_{2}+\mathrm{L}+n_{k}}
$$

- Grand Mean for our example is 65.08

$$
\begin{aligned}
& \overline{\bar{x}}=\frac{7(75.71)+9(67.11)+8(53.50)}{7+9+8} \\
& \overline{\bar{x}}=\frac{1562}{24} \\
& \overline{\bar{x}}=65.08
\end{aligned}
$$

## - Between Group Variation, SS(B)

- The between group variation is the variation between each sample mean and the grand mean
- Each individual variation is weighted by the sample size

$$
S S(B)=\sum_{i=1}^{k} n_{i}\left(\bar{x}_{i}-\overline{\bar{x}}\right)^{2}
$$

$$
S S(B)=n_{1}\left(\bar{x}_{1}-\overline{\bar{x}}\right)^{2}+n_{2}\left(\bar{x}_{2}-\overline{\bar{x}}\right)^{2}+\mathrm{L}+n_{k}\left(\bar{x}_{k}-\overline{\bar{x}}\right)^{2}
$$

The Between Group Variation for our example is SS(B)=1902

$$
S S(B)=7(75.71-65.08)^{2}+9(67.11-65.08)^{2}+8(53.50-65.08)^{2}
$$

$$
\operatorname{SS}(B)=1900.8376 \approx 1902
$$

## - Within Group Variation, SS(W)

-The Within Group Variation is the weighted total of the individual variations.
-The weighting is done with the degrees of freedom.
-The df for each sample is one less than the sample size for that sample.

## Within Group Variation

$$
S S(W)=\sum_{i=1}^{K} d f S_{i}^{2}
$$

$$
S S(W)=d f_{1} s_{1}^{2}+d f_{2} s_{2}^{2}+\mathrm{L}+d f_{1} s_{k}^{2}
$$

- The within group variation for our example is 3386

$$
S S(W)=6(310.90)+8(119.86)+7(80.29)
$$

$$
S S(W)=3386.31 \approx 3386
$$

- After filling in the sum of squares, we have ...

| Source | SS | df | MS | F | $p$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Between | 1902 |  |  |  |  |
| Within | 3386 |  |  |  |  |
| Total | 5288 |  |  |  |  |

## - Degrees of Freedom, df

- A degree of freedom occurs for each value that can vary before the rest of the values are predetermined.
- For example, if you had six numbers that had an average of 40, you would know that the total had to be 240 . Five of the six numbers could be anything, but once the first five are known, the last one is fixed so the sum is 240 . The df would be $6-1=5$
- The df is often one less than the number of values
- The Between Group df is one less than the number of groups
- We have three groups, so

$$
\operatorname{df}(B)=2
$$

- The Within Group df is the sum of the individual df's of each group
- The sample sizes are 7, 9, and 8

$$
\operatorname{df}(W)=6+8+7=21
$$

- The total df is one less than the sample size

$$
\operatorname{df}(\text { Total })=24-1=23
$$

- Filling in the degrees of freedom gives this ...

| Source | SS | df | MS | F | p |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Between | 1902 | 2 |  |  |  |
| Within | 3386 | 21 |  |  |  |
| Total | 5288 | 23 |  |  |  |

## - VARIANCES

- The variances are also called the Mean of the Squares and abbreviated by MS, often with an accompanying variable MS(B) or MS(W)
- They are an average squared deviation from the mean and are found by dividing the variation by the degrees of freedom
MS = SS / df
Vawianon_ Variation
- MS(B) = $1902 / 2=951.0$
- MS(W) = 3386/21 = 161.2
-MS(T) = 5288/23 = 229.9
- Notice that the MS(Total) is NOT the sum of MS(Between) and MS(Within).
- This works for the sum of squares $\operatorname{SS}$ (Total), but not the mean square MS(Total)
- The MS(Total) isn't usually shown
- Completing the MS gives ...

| Source | SS | df | MS | F | $p$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Between | 1902 | 2 | 951.0 |  |  |
| Within | 3386 | 21 | 161.2 |  |  |
| Total | 5288 | 23 |  |  |  |

## Special Variances

- The MS(Within) is also known as the pooled estimate of the variance since it is a weighted average of the individual variances
- The MS(Total) is the variance of the response variable, not technically part of ANOVA table.


## F test statistic

- An F test statistic is the ratio of two sample variances.
- The MS(B) and MS(W) are two sample variances and that's what we divide to find $F$.

$$
F=M S(B) / M S(W)
$$

- For our data, $F=951.0 / 161.2=5.9$
- Adding F to the table ...

| Source | SS | df | MS | F | P |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Between | 1902 | 2 | 951.0 | 5.9 |  |
| Within | 3386 | 21 | 161.2 |  |  |
| Total | 5288 | 23 | 229.9 |  |  |

- The F test is a right tail test
- The F test statistic has an F distribution with $d f(B)$ numerator $d f$ and $d f(W)$ denominator df
- The $p$-value is the area to the right of the test statistic:

$$
P(F 2,21>5.9)=0.009
$$

- Completing the table with the p-value

| Source | SS | df | MS | F | $p$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Between | 1902 | 2 | 951.0 | 5.9 | 0.009 |
| Within | 3386 | 21 | 161.2 |  |  |
| Total | 5288 | 23 |  |  |  |

- The p -value is 0.009 , which is less than the significance level of 0.05 , so we reject the null hypothesis.
- The null hypothesis is that the means of the three rows in class were the same, but we reject that, so at least one row has a different mean.
- There is enough evidence to support the claim that there is a difference in the mean scores of the front, middle, and back rows in class.
- The ANOVA doesn't tell which row is different, you would need to look at confidence intervals or run post hoc tests to determine that.


## MULTIPLE-COMPARISON PROCEDURE [POST HOC TEST]

- Tells which population means are significantly different
- e.g.: $\mu_{1}=\mu_{2} \neq \mu_{3}$
- Done after rejection of equal means in ANOVA
- Allows pair-wise comparisons
- Compare absolute mean differences with critical range



## TUKEY-KRAMER CRITICAL RANGE

Critical Range $=q_{\alpha} \sqrt{\frac{\mathrm{MSW}}{2}\left(\frac{1}{\mathrm{n}_{\mathrm{i}}}+\frac{1}{\mathrm{n}_{\mathrm{j}}}\right)}$
where:
$\mathrm{q}_{\alpha}=$ Value from standardized range table with $k$ and $N-k$ degrees of freedom for the desired level of $\alpha$.

MSW = Mean Square Within
$n_{i}$ and $n_{j}=$ Sample sizes from populations (levels) $i$ and $j$

## SPSS-statistical Example

## EXAMPLE OF ONE WAY ANOVA

- A manager wants to raise the productivity at his company by increasing the speed at which his employees can use a particular spreadsheet program.



## Example of one way anova

He employs an external agency which provides training in this spreadsheet program. They offer 3 courses: a beginner, intermediate and advanced course.

He is unsure which course is needed for the type of work they do at his company.


| beginner | intermediate | advanced |
| :---: | :---: | :---: |
| 5 employees | 5 employees | 5 employees |

- When they all return from the training,
- problem to solve using the spreadsheet program, $\rightarrow$ time to complete the problem.
- (beginner, intermediate, advanced) $\rightarrow$ differences in the average time it took to complete the problem.

Open data file called "ANOVA_I" and then follow the following steps:


## One-Way ANOVA





+ One-Way ANOVA: Post Hoc Multiple Comparisons

〔Equal Variances Assumed

-Equal Variances Not Assumed
$\square$ Tamhane's T2 $\square$ Dunnett's T3 $\square$ Games-Howell $\square$ Dunnett's C

Significance level: 0.05

> Continuel Cancel Help

Dependent List:


## 米 time [time]

Post Hoc...
Options...
Bootstrap...

## OK

Paste
Reset
Cancel
Help

Statistics

## - Descriptive

Fixed and random effects
$\square$ Homogeneity of variance test
$\square$ Brown-Forsythe
$\square$ Welch


Means plot
「Missing Values
(o) Exclude cases analysis by analysis
© Exclude cases listwise

Cancel
Help


## SPSS STATISTICS OUTPUT OF the ONe-way anova

## DESCRIPTIVE TABLE

## Descriptives

time

|  | N | Mean | Std. Deviation | Std. Error | 95\% Confidence Interval for Mean |  | Minimum | Maximum |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | Lower Bound | Upper Bound |  |  |
| beginners | 5 | 25.2000 | 2.38747 | 1.06771 | 22.2356 | 28.1644 | 22.00 | 28.00 |
| intermediate | 5 | 29.2000 | 2.28035 | 1.01980 | 26.3686 | 32.0314 | 27.00 | 33.00 |
| Advanced | 5 | 21.6000 | 3.04959 | 1.36382 | 17.8134 | 25.3866 | 18.00 | 26.00 |
| Total | 15 | 25.3333 | 4.01189 | 1.03586 | 23.1116 | 27.5550 | 18.00 | 33.00 |

## ANOVA TABLE

This is the table that shows the output of the ANOVA analysis and whether we have a statistically significant difference between our group means.

## ANOVA

time

|  Sum of <br> Squares df Mean Square F Sig. <br> Between Groups 144.533 2 72.267 10.733 .002 |
| :--- |
| there is a statistically significant difference in the mean <br> length of time to complete the spreadsheet problem <br> between the different courses taken |

## MULTIPLE COMPARAISONS TABLE

## Multiple Comparisons

Dependent Variable: time
Tukey HSD

| (1) course | (J) course | Mean Difference (lJ) | Std. Error | Sig. | 95\% Confidence Interval |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | Lower Bound | Upper Bound |
| beginners | intermediate | -4.00000- | 1.64114 | 075 | -8.3783- | . 3783 |
|  | Advanced | 3.60000 | 1.64114 | 113 | -.7783- | 7.9783 |
| intermediate | beginners | 4.00000 | 1.64114 | . 075 | -.3783- | 8.3783 |
|  | Advanced | $7.60000^{\prime \prime}$ | 1.64114 | 002 | 3.2217 | 11.9783 |
| Advanced | beginners | -3.60000- | 1.64114 | 113 | -7.9783- | . 7783 |
|  | intermediate | -7.60000-* | 1.64114 | . 002 | -11.9783- | -3.2217- |

*. The mean difference is significant at the 0.05 level.

## REPORTING THE OUTPUT OF THE ONEWAY ANOVA

- There was a statistically significant difference between groups as determined by one-way ANOVA $(F(2, I 2)=10.733, p=$ .002).
- A Tukey post-hoc test revealed that the time to complete the problem was statistically significantly lower after taking the Advanced ( $21.6 \pm 3.0 \mathrm{~min}$ ) compared to the Intermediate $(29.2 \pm 2.2 \mathrm{~min})$ course and the beginners course $(25.2 \pm 2.3$ min ). There were no statistically significant differences between the beginners and intermediate groups ( $p=.075$ ) or between the beginners and Advanced group ( $p=.| | 3$ ) while it was significance between the intermediate and Advanced ( $\mathrm{p}=.002$ ).

A TWO-WAY ANOVA is useful when we desire to compare the effect of multiple levels of two factors and we have multiple observations at each level.

- "Two-Way" means groups are defined by 2 independent variables.
- These IVs are typically called factors.
- An experiment in which any combination of values for the 2 factors can occur is called a completely crossed factorial design.
- If all cells have the same $n$, the design is said to be balanced.
- Still have only 1 dependent variable
- What kind of variables?

Continuous (scale / interval / ratio) and 2 independent categorical variables (factors)

- Common Applications:

Comparing means of a single variable at different levels of two conditions (factors) in scientific experiments.

- THEVARIABLES INTHETWO-WAY ANOVA
- There are two kinds of variables Two-Way ANOVA

ONE DEPENDENTVARIABLE

TWO INDEPENDENTVARIABLE

## ASSUMPTIONS

-Independent random samples are drawn

- Populations are normally distributed
- Populations have equal variances
- A two-way ANOVA always involves two independent variables.
- Each independent variable, is made up of, or defined by, two or more elements called levels.

- Let us suppose that we desire to know if patient phobia/ Anxiety in the clinic varies according to age and gender.
- The variable of interest is therefore patient phobia .
- There are two factors being studied - age and gender.

Further suppose that the patients have been classified into three groups or levels:

I/ age less than 30, 2 / 30 to $40 \quad 3$ / above 40
In addition patients have been labeled into gender classification
(levels):

- male
- Female

- Testing for Interaction

There are two versions of the Two-Way ANOVA:
I. The basic version has one observation in each cell - one phobia score from one patient each of the six cells.
2. The second version has more than one observation per cell but the number of observations in each cell must be equal. The advantage of the second version is it also helps us to test if there is any interaction between the two factors.
For instance, in the example above, we may be interested to know if there is any interaction between age and gender.

## SOUREES OF VARIATION

- Suppose that the two factors of interest are: A and B, and
- $\mathrm{a}=$ number of levels of factor A
- $b=$ number of levels of factor $B$
- $\mathrm{N}=$ total number of observations in all cells


## $\mathrm{SST}=\mathrm{SS}_{\mathrm{A}}+\mathrm{SS}_{\mathrm{B}}+\mathrm{SS}_{\mathrm{AB}}+\mathrm{SSE}$

Degrees of Freedom:


## MEAN SQUARE CALCULATIONS

$M S_{A}=$ Mean squarefactor $A=\frac{S S_{A}}{a-1}$
$M S_{B}=$ Mean squarefactor $B=\frac{S S_{B}}{b-1}$
$\mathrm{MS}_{\mathrm{AB}}=$ Mean squareinteraction $=\frac{\mathrm{SS}_{\mathrm{AB}}}{(\mathrm{a}-1)(\mathrm{b}-1)}$
MSE $=$ Mean squareerror $=\frac{S S E}{N-a b}$

## TWO-WAY ANOVA: THE F TEST STATISTIC

## F Test for Factor A Main Effect

$\mathrm{H}_{0}: \mu_{\mathrm{A} 1}=\mu_{\mathrm{A} 2}=\mu_{\mathrm{A} 3}=\cdots$
$H_{A}$ : Not all $\mu_{A i}$ are equal

$$
\mathrm{F}=\frac{\mathrm{MS}_{\mathrm{A}}}{\mathrm{MSE}}
$$

Reject $\mathrm{H}_{0}$ if $F>F_{\alpha}$

F Test for Factor B Main Effect

$$
\mathrm{F}=\frac{\mathrm{MS}_{\mathrm{B}}}{\mathrm{MSE}} \quad \begin{aligned}
& \text { Reject } \mathrm{H}_{0} \\
& \text { if } \mathrm{F}>\mathrm{F}_{\alpha}
\end{aligned}
$$

$H_{0}$ : factors $A$ and $B$ do not interact to affect the mean response
$H_{A}$ : factors $A$ and $B$ do interact

F Test for Interaction Effect

$$
\mathrm{F}=\frac{\mathrm{MS}_{\mathrm{AB}}}{\mathrm{MSE}}
$$

Reject $\mathrm{H}_{0}$
if $F>F_{\alpha}$

## two-way anova summary table

| Source of Variation | Sum of Squares | Degrees of Freedom | Mean Squares | F <br> Statistic |
| :---: | :---: | :---: | :---: | :---: |
| Factor A | $\mathrm{SS}_{\mathrm{A}}$ | a-1 | $\begin{gathered} \mathbf{M S}_{\mathbf{A}} \\ =\mathrm{SS}_{\mathrm{A}} /(\mathrm{a}-1) \end{gathered}$ | $\frac{\mathrm{MS}_{\mathrm{A}}}{\mathrm{MSE}}$ |
| Factor B | $\mathrm{SS}_{\mathrm{B}}$ | b-1 | $\begin{gathered} \mathrm{MS}_{\mathbf{B}} \\ =\mathrm{SS}_{\mathrm{B}} /(\mathrm{b}-1) \end{gathered}$ | $\frac{\text { MS }_{\mathrm{B}}}{\mathrm{MSE}}$ |
| $\begin{gathered} \mathrm{AB} \\ \text { (Interaction) } \end{gathered}$ | SS ${ }_{\text {AB }}$ | $(a-1)(b-1)$ | $\begin{gathered} \mathbf{M S}_{\mathbf{A B}} \\ =\mathrm{SS}_{\mathrm{AB}} /[(\mathrm{a}-1)(\mathrm{b}-1)] \end{gathered}$ | $\begin{gathered} \mathrm{MS}_{\mathrm{AB}} \\ \hline \mathrm{MSE} \end{gathered}$ |
| Error | SSE | N-ab | $\begin{gathered} \text { MSE = } \\ \text { SSE/(N - ab) } \end{gathered}$ |  |
| Total | SST | N-1 |  |  |

- Degrees of freedom always add up
$-\mathrm{N}-\mathrm{I}=(\mathrm{N}-\mathrm{ab})+(\mathrm{a}-\mathrm{I})+(\mathrm{b}-\mathrm{I})+(\mathrm{a}-\mathrm{I})(\mathrm{b}-\mathrm{I})$
- Total $=$ error + factor $A+$ factor $B+$ interaction
- The denominator of the F Test is always the same but the numerator is different
- The sums of squares always add up
$-\mathrm{SST}=\mathrm{SSE}+\mathrm{SS}_{\mathrm{A}}+\mathrm{SS}_{\mathrm{B}}+\mathrm{SS}_{\mathrm{AB}}$
- Total $=$ error + factor $A+$ factor $B+$ interaction


## GENERALEXAMPLE

A researcher is interested in whether emergency rooms patients get more agitated when the machine monitoring their vital signs emits lots of noises. He measures their pulse as a way to index their agitation, and have some of the patients hooked up to machines that are very noisy or machines that have the volume off.

DV = Pulse (beats per minute)
IV1: Monitor Volume (volume on, volume off)
He is also interested in whether men or women tend to have higher pulse rates when in the ER, and in particular, whether each gender responds differently to the presence of noisy monitoring devices.

## IV2: Gender (male, female)

A: Monitor Volume (volume on, volume off)

B: Gender (male, female)

Ax B: Interaction of monitor volume and gender

|  | Male | Female |
| :--- | :--- | :--- |
| Volume on |  |  |
| Volume off |  |  |

## KIND OF THE INFORMATION

## MAIN EFFECT FOR IV1:

Monitor Volume (volume on, volume off)
Overall, did pulse rate differ on average as a function of monitor volume?
Did the monitor volume matter?

## MAIN EFFECT FOR IV2:

Gender (male, female)
Overall, did pulse rates differ on average as a function of gender?
Did gender matter?

## INTERACTION IV1xIV2:

Interaction of monitor volume and gender
Did the influence of the monitor's volume on pulse rates depend on
whether the person was male or female?

|  | Male | Female |
| :--- | :--- | :--- |
| Volume on |  |  |
| Volume off |  |  |

## SOME TERMINOLOGY

## $2 \times 2 \quad 2 \times 32 \times 4$ factorial design

Number of numbers = how many factors (IVs)

How many factors = how many "ways" one factor = one-way, two factors = two-way, etc.

Number itself = how many levels of that IV
Total number of conditions = product So a $2 \times 2$ design has 4 conditions; a $2 \times 3$ has 6 conditions

## MAIN EFFECTS

IV2

## Male Female

## IVI Volume on Volume off

## Cell means = means for each condition

So the males whose heart rate was measured with the volume on average had a pulse of 100

The womens whose heart rate was measured with the volume on had a pulse of 80 on average

## Male Female

| IV I Volume on | 100 | 80 | $(90)$ | $(100+80) / 2$ |
| :---: | :--- | :---: | :---: | :--- |
| Volume off | 70 | 60 | $(65)$ | $(70+60) / 2$ |
|  | $(85)$ | $(70)$ |  |  |
|  | $(100+70) / 2$ | $(80+60) / 2$ |  |  |

Marginal means = means for main effects (average of cell means in respective column or row)

## Male <br> Female

IVI Volume on Volume off
100
70
$(85)$

Main effect for IV1:
Does monitor volume matter? Does it effect pulse rate? 90 vs. 65
Main effect for IV2:
Does gender matter?
Does it effect pulse rate? 85 vs. 70

A main effect lets you know overall whether that IV influenced the DV, ignoring the other IV
Refer to marginal means when interpreting main effects. Look at $F$ value and corresponding $p$ value for each main effect to see whether it is significant. Is it probably a real effect?

Using a factorial design is like having two separate studies rolled into one:
You get to see the overall effects of each variable

As in a one-way ANOVA, if the main effect is significant and there are only 2 levels of IV, you know where the difference is.

If 3 or more levels of IV, you know that at least one condition is different from the others but do not know which differences are real. You would need to run additional tests to see which differences between 2 conditions are real (e.g., Tukey's HSD if equal n; Fisher's protected t if unequal n )

|  | Male | Female |
| :--- | :--- | :--- |
| Very noisy |  |  |
| Somewhat noisy |  |  |
| Volume off |  |  |

## Plot the cell means to see what the data are showing




Factorial designs not only yield info about main effects, but they provide a third - and often critical - piece of information

## Interaction:

Main effects do not tell full story; need to consider IV1 in relation to IV2

Do the effects of one IV on the DV depend on the level of the $2^{\text {nd }} \operatorname{IV}$ ?

Is the pattern of one IV across the levels of the other IV different depending on the level of the other IV (not parallel)?

## INTERACTIONS

## Example 1:

Crossover interaction

|  | Male |  | Female |  |
| :---: | ---: | :--- | :---: | :---: |
| Volume on | 100 | $>$ | 50 | $(75)$ |
| Volume off | 50 | $<$ | 100 | $(75)$ |
|  | $(75)$ | $=$ | $(75)$ |  |

The main effects would lead you to conclude that neither variable influenced pulse rates. But that is not true. Look at the patterns shown by the cell means. The pattern seen in the differences in average pulse rates between men and women differ depending on whether the volume is on or off

## Example 2:

## Treatment works for one level but not for other

$$
\text { Male } \quad \text { Female }
$$

| Volume on | 100 | $>$ | 50 | $(75)$ |
| :--- | :---: | :---: | :---: | :---: |
| Volume off | 80 | $=$ | 80 | $(80)$ |
|  | $(90)$ | $>$ | $(65)$ |  |

Main effects would lead you to conclude that males always have higher pulse rates on average, but that is only true when the volume is on (not when the volume is off)

Nonparallel lines suggest an interaction is present


Interactions can be seen easily when line graphs are made

Nonparallel lines = Interaction

Parallel lines = No interaction

## Parallel lines suggest there is not an interaction





## Example 3: Ordinal Interaction

Sometimes an interaction is significant but it does not change the conclusions you draw from the main effects. The patterns are the same, it is HOW MUCH the difference varies

|  | Male |  | Female |  |
| :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |
| Volume on | 60 | $<$ | 90 | (75) |
| Volume off | 50 | $<$ | 60 | $(55)$ |
|  | $(55)$ | $<$ | $(75)$ |  |

Women always have higher average pulse rates than men but the relative difference is more pronounced when the volume is on than when it is off

Volume on: 60 vs. $90=30$ point difference Volume off: 50 vs. $60=10$ point difference
The volume being on always produced higher average pulse rates than the volume being off, but women are especially affected by this Men: 60 vs. $50=10$ point difference

Women: 90 vs. $60=30$ point difference

## SOME REMARKS

## Remark 1:

## Graphing the data really drive interactions home

The lines are not parallel but are moving in the same direction (i.e., slopes have same sign but different values); Women always have higher pulses than men but especially so when the volume is off


## Remark 2:

Graphs can be made with either variable on the x axis; a good researcher thinks about whether the data tell the story better with IV1 or IV2 on the x axis



## Remark 3:

- Remind that when talking about main effects, always use marginal means
- When talking about interactions, always use cell means
- When making graphs, graph the cell means, not the marginal means


## HYPOTHESIS TESTING USING ANOVA

Testing for the Main Effect of IV1 and similarly for IV2

NULL HYPOTHESIS $\mathrm{H}_{0}$ :

- The IV1 (IV2) does not influence the DV
- Any differences in average scores between the different conditions of IV1 (IV2) are probably just due to chance (measurement error, random sampling error)
- The (population) means at all levels of the IV1 (IV2) are equal


## RESEARCH OR ALTERNATIVE HYPOTHESIS H ${\underset{A}{A}}^{\text {B }}$ :

- The IV1 does influence the DV
- The differences in average scores between the different conditions are probably not due to chance but show a real effect of the IV1 on the DV

At least one level of the IV1 has a different (population) mean

## Testing for the Interaction of IV1 and IV2

## NULL HYPOTHESIS $\mathbf{H}_{\mathbf{0}}$ :

- There is not an interaction between IV1 and IV2
- IV1 and IV2 are independent
- The effect of IV1 does not depend on the level of IV2 (and IV2 does not depend on IV1)


## RESEARCH OR ALTERNATIVE HYPOTHESIS H ${ }_{A}$ :

- There is an interaction between IV1 and IV2
- IV1 and IV2 are dependent The effect of IV1 does depend on the level of IV2 (and IV2 does depend on IV1)
So, under $H_{A}$ the (population) cell means cannot be modeled only from the main effects.


## HOW TO REPORT TWO-WAY ANOVAS

## STEP 1.

Describe the design itself. A two-way ANOVA of [IV1] (level 1, level 2) and [IV2] (level 1, level 2) on [DV] was conducted
[DV] was analyzed in a two-way [between, within, mixed] ANOVA, with [IV1] (level 1, level 2) as a [between subjects; within subjects] variable and [IV2] (level 1, level 2) as a [between subjects; within subjects] variable.

A 2 x 2 factorial [between; within; mixed] ANOVA was conducted on [DV], with [IV1] (level 1, level 2) and [IV2] (level 1, level 2) as the independent variables

## For Example:

* A two-way ANOVA of monitor volume (volume on, volume off) and patient's gender (male, female) on pulse rate as measured by beats per minute was conducted.
** The number of beats per minute was analyzed in a two-way mixed factorial ANOVA, with monitor volume (volume on, volume off) manipulated within-subjects and gender (male, female) as a between-subjects variable
** A $2 \times 2$ between-subjects ANOVA was conducted on pulse rate, with monitor volume and patient's gender as factors


## STEP 2:

Report the main effect for IV1:
A significant main effect of [IV1] on [DV] was found, F (dfbet, dferror) $=\mathrm{x} . \mathrm{xx}, \mathrm{p}=\mathrm{xxx}$.

The main effect of [IV1] on [DV] was/was not significant, F (dfbet, dferror) $=\mathrm{x} . \mathrm{xx}, \mathrm{p}=\mathrm{xxx}$.

## Step 2a:

If the main effect is significant, then describe it by reporting the marginal means
[DV] was higher / lower for [IV1, Level 1] ( $\mathrm{M}=\mathrm{x} . \mathrm{xx}$ ) than for [IV1, Level 2] ( $\mathrm{M}=\mathrm{x} . \mathrm{xx}$ ).
[DV] did not significantly differ between [IV1, Level 1] ( $\mathrm{M}=$ x.xx) and [IV1, Level 2] ( $\mathrm{M}=\mathrm{x} . \mathrm{xx}$ ).

## FOR EXAMPLE:

A significant main effect of monitor volume on pulse rate was found, $\mathrm{F}(*, * *)=++, \mathrm{p}=*<.05$.
Patients' average pulse rate was higher when the volume was on $(M=100.53)$ than when the volume was off $(M=75.13)$

The main effect of monitor volume on pulse rate was not significant, $\mathrm{F}(*, * * *)=+++, \mathrm{p}=* *>0.05$.
This means the average pulse rate did not differ significantly whether the volume was on or off.

So there is no need for an additional sentence, though some people like to say: Thus pulse rates did not differ on average when the volume was on $(\mathrm{M}=88.23)$ or off $(\mathrm{M}=84.66)$ ]

## STEP 3:

## Report the main effect for IV2

## STEP 3A:

If the main effect is significant, the describe it by reporting the marginal means

Use same sentence structures as Step 2 and 2a.

A significant main effect of gender on pulse rate was found, $\mathrm{F}(*, * *)=* *, \mathrm{p}=*<.05$. Men's average pulse rate was higher $(M=105.88)$ than women's $(M=85.31)$

The main effect of gender on pulse rate was not significant, $\mathrm{F}\left({ }^{*}\right.$, $\left.{ }^{* *}\right)={ }^{* *}, \mathrm{p}=*$.

No need for an additional sentence, though some people like to say: Thus pulse rates did not differ between men $(M=86.25)$ or women ( $\mathrm{M}=89.32$ ).

## STEP 4:

Report the interaction

The [IV1] x [IV2] interaction was/was not significant, $F($ dfiV1xIV2, dferror $)=x . x x, p=x x x$

When the interaction is NOT significant (i.e., when p > .05), that is all you need to do. This means that the two IVs influence the DV independently from each other.

## NOTE:

## A significant interaction trumps the main effects!!!

When the interaction is significant, you have to report the cell means and describe their patterns.

You also need to be careful what you say about the main effects. The emphasis should be on the interaction when it is significant.

You may examine CELL MEANS and see that the pattern for one IV across the other IV is not the same (i.e., the lines in the graph are not parallel).

When the patterns are different, the statement you make about a main effect is NOT true for all levels of the other IV. So the emphasis in your write up should be about the interaction, not about the main effects.

