REVIEW OF SOME PARAMETRIC TESTES

OBJECTIVE

The objective of this lecture is to review some of the parametric tests you study in the previous courses such as:

- T-test (I-sample, 2-sample and paired)
- ANOVA (one and two way)
- Repeated Measures

T-TEST ANALYSIS

First, we review the Measurement Scales. We can measure and convey variables in several ways.

Nominal data:

also called categorical data, are represented by counting the number of times a particular event or condition occurs. For example, you might categorize the political alignment of a group of voters.

Dichotomous variable is a special classification of nominal data; it is simply a measure of two conditions. A dichotomous variable is either discrete or continuous. A *discrete dichotomous* variable has no particular order and might include such examples as gender (male vs. female) or a coin toss (heads vs. tails). A *continuous dichotomous* variable has some type of order to the two conditions and might include measurements such as pass/fail or young/old.

<u>Ordinal Scale</u>

Describe values that occur in some order of rank. However, distance between any two ordinal values holds no particular meaning. For example, imagine lining up a group of people according to height. It would be very unlikely that the individual heights would increase evenly.

Another example of an ordinal scale is a Likert-type scale. This scale asks the respondent to make a judgment using a scale of three, five, or seven items. The range of such a scale might use a 1 to represent strongly disagree while a 5 might represent strongly agree. This type of scale can be considered an ordinal measurement since any two respondents will vary in their interpretation of scale values.

<u>Interval scale</u>

A measure in which the relative distances between any two sequential values are the same. To borrow an example from the physical sciences, we consider the Celsius scale for measuring temperature. An increase from -8 to -7° C degrees is identical to an increase from 55 to 56°C.

<u>Ratio scale</u>

Slightly different from an interval scale. Unlike an interval scale, a ratio scale has an absolute zero value. In such a case, the zero value indicates a measurement limit or a complete absence of a particular condition. To borrow another example from the physical sciences, it would be appropriate to measure light intensity with a ratio scale. Total darkness is a complete absence of light and would receive a value of zero. On a general note, we have presented a classification of measurement scales similar to those used in many introductory statistics texts.

T-TEST

- T test was introduce in 1908 by W.S. Gosset.
- He was a brewer and agricultural statistician for the famous Guinness brewing company in Dublin. It insisted that its employees keep their work secret, so he published under the pseudonym 'Student' the distribution. This was one of the first results in modern small-sample statistics.



TYPES OF T-TEST

• One sample:

Compare the mean of a sample to a predefined value and used when you have a single sample and are comparing its mean to some value that is assumed to represent a population mean.

• Dependent (related) samples:

Compare the means of two conditions in which the same (or closely matched) participants participated

• Independent (unrelated) samples:

Used when you have two samples, each representing a different and independent group of subjects. The groups differ along one independent variable with two levels, and you compare the mean of those two groups (between-subjects design).

T-TEST ASSUMPTIONS

- The first assumption made regarding t-tests concerns the scale of measurement. The assumption for a t-test is that the scale of measurement applied to the data collected follows a <u>continuous or ordinal scale</u>, such as the scores for an IQ test.
- The second assumption made is that of a <u>simple random sample</u>, that the data is collected from a representative, randomly selected portion of the total population.
- The third assumption is the data, when plotted, results in a <u>normal</u> <u>distribution</u>, bell-shaped distribution curve.
- The fourth assumption is a reasonably large sample size is used. A larger sample size means the distribution of results should approach a normal bell-shaped curve.
- The final assumption is <u>homogeneity of variance</u>. Homogeneous, or equal, variance exists when the standard deviations of samples are approximately equal.

APPLICATIONS

- To compare the mean of a sample with population mean.
- To compare the mean of one sample with the mean of another independent sample.
- To compare between the values (readings) of one sample in 2 occasions.

ONE SAMPLE T-TEST

• The One-sample t test is used to compare a sample mean to a specific value (e.g., a population parameter; chance performance, etc.).

Assumptions:

- The data must be continuous.
- The data must follow the normal probability distribution.

Check by:

- 1- graph: (a)p-p or q-q plots (b) Box-plot
- 2-test: (a) Klomogrov and Smeranov test(b) Shabero test
- The sample is a simple random sample from its population.

EXAMPLE:

• A teacher wants to see if his students are performing well in the math class. Ten students are chosen randomly and the teacher wants the class to be able to score different than 60 on the math competition. Can the teacher have 95 percent confidence that the mean score for the class would be different than 60?

Students	Scores
I	54
2	65
3	41
4	52
5	47
6	51
7	43
8	66
9	60
10	41

Mean	52
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$$s = \sqrt{\frac{\sum \left(x_i - \bar{x}\right)^2}{n-1}}$$

 $=\sqrt{\frac{782}{9}}$ = 9.321

Scores	x-x	(x-x) ²				
54	2	4				
65	13	169				
41	-11	121				
52	0	0				
47	-5	25				
51	-1	I				
43	-9	81				
66	14	196				
60	8	64				
41	-11	121				
520/10=5 2		782				

The hypothesis:

- Null hypothesis: H₀
- Alternative hypothesis: $H_a \qquad \mu \neq 60$

 $\mu = 60$

- The α -level: $\alpha = 0.05$
- The assumptions:
- Dependent variables normally distributed.

T test

$$T = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} \qquad T = \frac{52 - 60}{\frac{9.321}{\sqrt{10}}}$$

The calculated t-value of **-2.714** while the critical t value **2.262** at 9 degrees

of freedom			μ_0					-	
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	two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	
X bar= sample	df 1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	
mean	2	0.000	0.816	1.061 0.978	1.386	1.886	2.920 2.353	4.303 3.182	
$\mu_0 = population$	4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	
mean s = standard	6 7 8	0.000	0.718	0.906	1.134	1.440 1.415 1.397	1.943 1.895 1.860	2.447 2.365 2.306	
deviation	9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	
n = sample size									

• <u>The results:</u>

The calculated t-value of 2.714 is larger in magnitude than the critical value of 2.262, therefore we can reject the null hypothesis

• <u>The conclusion:</u>

The teacher now has evidence that the class mean on the test would be different than 60.

STEP BY STEP (SPSS):

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Tests of Normality

	Kolm	nogorov-Smi	rnov ^a	Shapiro-Wilk					
	Statistic	df	Sig.	Statistic	df	Sig.			
scores	.133	10	.200	.917	10	.334			

*. This is a lower bound of the true significance.

a. Lilliefors Significance Correction



• Value of the Shapiro-Wilk Test is greater than 0.05, the data follows normal.

ONE SAMPLE T-TEST ANALYSIS

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OUTPUT

NOTE: SPSS does only two-tailed tests. The t-obtained would be the same for a one or two tailed test, but if you are doing a one-tailed test, you can divide the significance by two to calculate the significance of a one-tailed test.

	One-Sample Statistics												
	Ν	Mean	Std. Deviation	Std. Error Mean									
scores	10	52.00	9.321	2.948									

One-Sample Test

		Test Value = 60									
				Mean	95% Confidenc Differ	e Interval of the ence					
	t	df	Sig. (2-tailed)	Difference	Lower	Upper					
scores	-2.714	9	.024	-8.000	-14.67	-1.33					

RESULTS AND REPORT

Since the p-value (0.024) is less than $\alpha = 0.05$, then we reject the null hypothesis, and so we conclude that the mean score for the class on the test is not equal 60.

Report:

This study was conducted to find out wither the mean score for the student in the math class of the sample is significantly different than the average score of the overall population. At level of significant 0.05, ten students were randomly selected to serve as subjects. By using one sample t-test to analyze the data, the average score was=52 with S.d.=9.32. T-test was 2.714 with df = 9 and p-value 0.024 shows that significantly different than the average score of the overall population at level of significant 0.05. As a results, the teacher now has evidence that the class mean in the course would not equal 60.

PAIRED SAMPLES T-TEST

WHEN TO USE THE PAIRED T-TEST

- A paired t-test is used to compare two population means where you have two samples in which observations in one sample can be paired with observations in the other sample. Examples of where this might occur are:
- Before-and-after observations on the same subjects (e.g. students' diagnostic test results before and after a particular module or course).
- A comparison of two different methods of measurement or two different treatments where the measurements/treatments are applied to the same subjects (e.g. blood pressure measurements using a stethoscope).

Hypothesis testing:

- Null hypothesis H₀: The mean of this sample of differences is zero.
- Alternative hypothesis H_A: The mean is not equal zero.

- Examples
- Before-and-after pairs of measurements after giving a drug

Assumption:

- The dependent variable should be measured at the interval or ratio level (i.e., they are continuous)
- Independent variable should consist of two categorical, "related groups" or "matched pairs".
- There should be no significant outliers in the differences between the two related groups.
- The distribution of the differences in the dependent variable between the two related groups should be approximately normally distributed.

EXAMPLE

Lets suppose we had a group of 8 students whose accumulative grades were recorded pre and post an improved performance program. Is there a statistically significant difference between the two scores (pre and post) within 5% significance (α=0.05)?

Students	Pre	Post
I	10	12
2	50	52
3	20	25
4	8	10
5	115	120
6	75	80
7	45	50
8	170	175

• Hypothesis:

Null hypothesis: average pre score = average post score

Alternative hypothesis: average pre score = average post score

STEP 1: CALCULATE THE DIFFERENCES:

Students	Pre	Post	Diff.	Diff (d) ²
I	10	12	2	4
2	50	52	2	4
3	20	25	5	25
4	8	10	2	4
5	115	120	5	25
6	75	80	5	25
7	45	50	5	25
8	170	175	5	25
			31	137

STEP 2:

• Calculate T statistic:

, $\sum d$ = Sum of the differences.

$${}^{T=} \frac{\sum D}{\sqrt{\frac{n \sum D^2 - (\sum D)^2}{n-1}}} = 7.059$$
$$= \frac{\sqrt{\frac{8 \times 137 - 31 \times 31}{8-1}}}{\sqrt{\frac{8 \times 137 - 31 \times 31}{8-1}}}$$

STEP 3: T TABLE

df	.25	.20	.15	.10	.05	.025
1 2	1.000 0.816	1.376 1.061	1.963 1.386	3.078 1.886	6.314 2.920	12.71 4.303
3	0.765	0.978	1.250	1.638	2.353	3.182
4	0.741	0.941	1.190	1.533	2.132	2.776
5	0.727	0.920	1.156	1.476	2.015	2.571
6	0.718	0.906	1.134 1.119	1.440	1.943	2.447
8	0.706	0.889	1.108	1.397	1.860	2.306
9	0.703	0.883	1.100	1.383	1.833	2.262
10	0.700	0.879	1.093	1.372	1.812	2.228

STEP 4: COMPARE T CALCULATED WITH T CRITICAL

- Calculated t = 7.06
- Critical t = 2.365

as,

• t calculated > t critical

So,

We reject H_0 and Accept H_a at alpha= 0.05

LETS USE SPSS (DATA ENTERING)

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Paired-Samp	les T Test									1	BM SPSS Stat	istics Process	vhcer is ready		

OUTPUT

Paired Samples Statistics

	Mean	N	Std. Deviation	Std. Error Mean
Pair1 post	65.50	8	57.540	20.343
pre	61.63	8	56.642	20.026

Paired Samples Correlations

		Ν	Correlation	Sig.
Pair 1	post & pre	8	1.000	.000

Paired Samples Test

				Std. Error	95% Confidenc Differ	e Interval of the ence			\frown
		Mean	Std. Deviation	Mean	Lower	Upper	t	df	Sig. (2-tailed)
Pair 1	post - pre	3.875	1.553	.549	2.577	5.173	7.059	7	.000

REPORT

A paired-samples t-test was conducted to compare scores pre and post an improved performance program. There was a significant difference in the scores post performance program (M=65.5, SD=57.5) and pre performance program (M=61.6, SD=56.6); t(7)=7.06, p = 0.000. These results suggest that student's score increased an improved performance program.

INDEPENDENT Samples T-Test

- The independent samples t-test is probably the single most widely used test in statistics.
- It is used to compare differences between separate groups.
- The independent-samples t-test (independent t-test) compares the means between two unrelated groups on the same continuous, dependent variable. For example, you could use an independent t-test to understand whether there is a difference in test anxiety based on educational level (i.e., your dependent variable would be "test anxiety" and your independent variable would be "educational level", which has two groups: "undergraduates" and "postgraduates").
- Hypothesis testing procedure that uses separate samples for each treatment condition (between subjects design)

NULL AND ALTERNATIVE HYPOTHESES FOR THE INDEPENDENT T-TEST

- The null hypothesis for the independent t-test is that the population means from the two unrelated groups are equal: $H_0: \mu_1 = \mu_2$
- In most cases, we are looking to see if we can show that we can reject the null hypothesis and accept the alternative hypothesis, which is that the population means are not equal: $H_A: \mu_1 \neq \mu_2$

ASSUMPTIONS FOR THE INDEPENDENT T-TEST

- <u>Independence</u>: Observations within each sample must be independent (they don't influence each other)
- **Normal Distribution:** The scores in each population must be normally distributed
- <u>Homogeneity of Variance:</u> The two populations must have equal variances (the degree to which the distributions are spread out is approximately equal) (Levene's Test for Equality of Variances)

EXAMPLE

A researcher performed a study which aimed to define the difference in the averages for statistics among scientific department students and literature department students. The sample size was seven students from the scientific department and eight from the literature department; their score shown in the table. Is there a statically significant difference between the two samples within 0.05% significance?

Scientific Department	Literary Department
51	65
50	58
42	76
40	85
55	90
40	60
62	70
60	

65	51
58	50
76	42
85	40
90	55
60	40
70	62
	60
SUM=504	SUM= 400
Mean $\bar{x}_1 = 504/7 = 72$	Mean $\bar{x}_2 = 400/8=50$

Score for sci. dep.	x-x	(x-x) ²	Score for lit. dep.	x-x	(x-x) ²
58	-14	196	50	0	0
76	4	16	42	-8	64
85	13	169	40	-10	100
90	18	324	55	5	25
60	-12	144	40	-10	100
70	-2	4	62	12	144
			60	10	100
SUM=504	SUM=0	SUM=902	SUM= 400	SUM= 0	SUM= 534
x ₁ = 504/7=72	N= 7	$s_1^2 = \frac{902}{6}$ =150.333 s. d_1=12.261	x ₂ = 400/8=50	N=8	$s_2^2 = \frac{534}{7}$ =76.285 s. d_2=8.734

$$t = \frac{\overline{x}_1 - \overline{x}_2}{\sqrt{\frac{s_1^2(n_1 - 1) + s_2^2(n_2 - 1)}{n_1 + n_2 - 2} \times (\frac{1}{n_1} + \frac{1}{n_2})}}$$

 $\begin{array}{l} X_1 \, \text{bar} = \text{mean of } 1^{\text{st}} \text{ sample} \\ X_2 \, \text{bar} = \text{mean of } 2^{\text{nd}} \text{ sample} \\ n_1 = \text{sci. sample} \\ n_2 = \text{lit. sample} \\ s_1 = \text{Standard deviation of } 1^{\text{st}} \text{ sample} \\ s_2 = \text{Standard deviation of } 2^{\text{nd}} \text{ sample} \end{array}$

$$t = \frac{72 - 50}{\sqrt{\frac{150.333(7 - 1) + 76.285(8 - 1)}{7 + 8 - 2}}} \times \left(\frac{1}{7} + \frac{1}{8}\right)} = \frac{22}{\sqrt{\frac{901.998 + 533.995}{13}} \times \left(\frac{15}{56}\right)}}$$
$$= \frac{22}{\sqrt{29.58}} = \frac{22}{5.439} = 4.0445$$

درجات الحرية df	α = 0.05	α = <mark>0.01</mark>	$\alpha = 0.05$ $\frac{\alpha}{2} = 0.025$	$\alpha = 0.01$ $\frac{\alpha}{2} = 0.005$
1	6.31	31.8	12.70	63.65
2	2.92	6.96	4.30	9.92
3	2.35	4.54	3.18	5.84
4	2.13	3.74	2.77	4.60
5	2.01	3.36	2.57	4.03
6	1.94	3.14	2.44	3.70
7	1.89	2.99	2.36	3.49
8	1.86	2.89	2.30	3.35
9	1.83	2.82	2.26	3.25
10	1.81	2.76	2.22	3.16
11	1.7 9	2.71	2.20	3.10
12	1 78	2.68	2.17	3.05
13	1.77	2.65	2.16	3.01
14	1.76	2.62	2.14	2.97
15	1.75	2.60	2.13	2.94

At 13 degree of freedom, t value of 2.16 (critical) < 4.0445 (t calculated)

We reject H_0 and Accept H_A at alpha = 0.05

• <u>The results:</u>

- -The calculated t-value of 4.045 is larger in magnitude than the calculated t value of 2.16, therefore we reject the null hypothesis
- <u>The conclusion:</u>
 - -Scientific department ($\mu = 72$) show significantly more score in than literary department ($\mu = 50$), t(13) = 4.47, p < .05 (two-tailed).

LETS USE SPSS (DATA ENTERING)

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Group Statis	stics
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	departement	Ν	Mean	Std. Deviation	Std. Error Mean
score	scientific	7	72.0000	12.26105	4.63424
	literary	8	50.0000	8.73417	3.08800

				Indeper	ndent Samp	les Test					
		Levene's Test Varia	for Equality of nces	t-test for Equality of Means							
							Mean	Std. Error	95% Confidenc Differ	e Interval of the ence	
		F	Sig.	t	df	Sig. (2-tailed)	Difference	Difference	Lower	Upper	
score	Equal variances assumed	1.270	.280	4.045	13	.001	22.00000	5.43948	10.24872	33.75128	
	Equal variances not assumed			3.951	10.702	.002	22.00000	5.56883	9.70138	34.29862	

REPORT

• This study was conducted to find out whether scientific and literary department differ in their scores in statistic course. At level of significant 0.05. By using independent t-test to analyze the data, the average compliance score for scientific was 72 with S.d.=12.26 and for literary the average was 50 with sd = 8.7 . T-test was 4.045 with df= 13 and p-value 0.001 shows that there was a significant difference at level of significant 0.05 between the scientific and literary department.

ONE-WAY ANOVA

ONE-WAY ANOVA

 The one-way <u>AN</u>alysis <u>O</u>f <u>VA</u>riance is used to test the claim that three or more population means are equal.

• This is an extension of the two independent samples t-test.

- The *response variable* is the variable you're comparing.
- The *factor variable* is the categorical variable being used to define the groups.
 - We will assume *k* samples (groups)
- The *one-way* is because each value is classified in exactly one way.
 - Examples include comparisons by gender, race, political party, color, etc.

Conditions or Assumptions:

-The data are randomly sampled.

-The variances of each sample are assumed equal.

-The residuals are normally distributed.

• The null hypothesis is that the means are all equal:

$$H_{\scriptscriptstyle 0}: \mu_{\scriptscriptstyle 1} = \mu_{\scriptscriptstyle 2} = \mu_{\scriptscriptstyle 3} = \dots = \mu_{\scriptscriptstyle k}$$

• The alternative hypothesis is that at least one of the means is different

EXAMPLE:

• The statistics classroom is divided into three rows: front, middle, and back.

 The instructor noticed that the further the students were from him, the more likely they were to miss class or use an instant messenger during class.

He wanted to see at level of sign. 0.05:
Are the students further away did worse on the exams?

The ANOVA doesn't test that one mean is less than another, only whether they're all equal or at least one is different.



• A random sample of the students in each row was taken.

- The score for those students on the second exam was recorded:
 - Front: 82, 83, 97, 93, 55, 67, 53
 - Middle: 83, 78, 68, 61, 77, 54, 69, 51, 63
 - Back: 38, 59, 55, 66, 45, 52, 52, 61

The summary statistics for the grades of each row are shown in the table below:

Row	FRONT	MIDDLE	BACK
SAMPLE SIZE	7	9	8
MEAN	75.71	67.11	53.50
ST. DEV	17.63	10.95	8.96
VARIANCE	310.90	119.86	80.29

• VARIATION:

-Variation is the <u>Sum of the Squares</u> of the <u>deviations</u> between a <u>value</u> and the <u>mean</u> <u>of the value</u>.

-Sum of Squares is abbreviated by SS and often followed by a variable in parentheses such as SS(B) or SS(W) so we know which sum of squares we're talking about.

