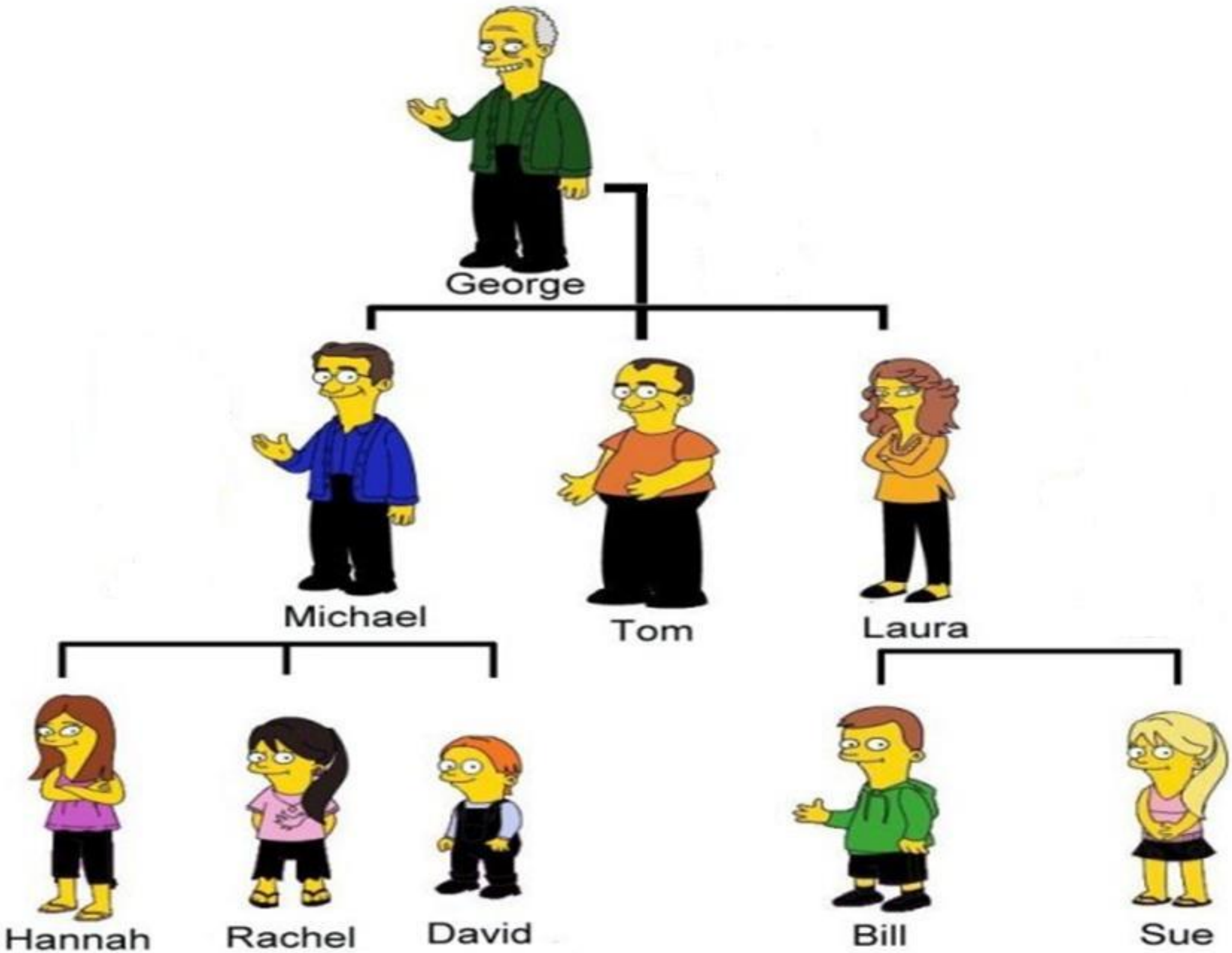
Trees

11.1 Introduction to Trees


George
Michael
Tom
Laura
Hannah
Rachel
David
Bill
Sue

Objectives:

The main purpose for this lesson is to introduce the following:

- ❑Define a tree and give examples of its.
- ❑Learn about some of tree concepts.
- ❑Define m-ary & full m-ary in rooted tree.
- ❑Properties of Trees.

Introduction to Trees

DEFINITION 1

A ***tree*** is a connected undirected graph with no simple circuits.

Because **a tree cannot have a simple circuit**, **a tree cannot contain multiple edges or loops**. Therefore **any tree must be a simple graph**.

EXAMPLE 1

Which of the graphs shown in Figure 2 are trees?

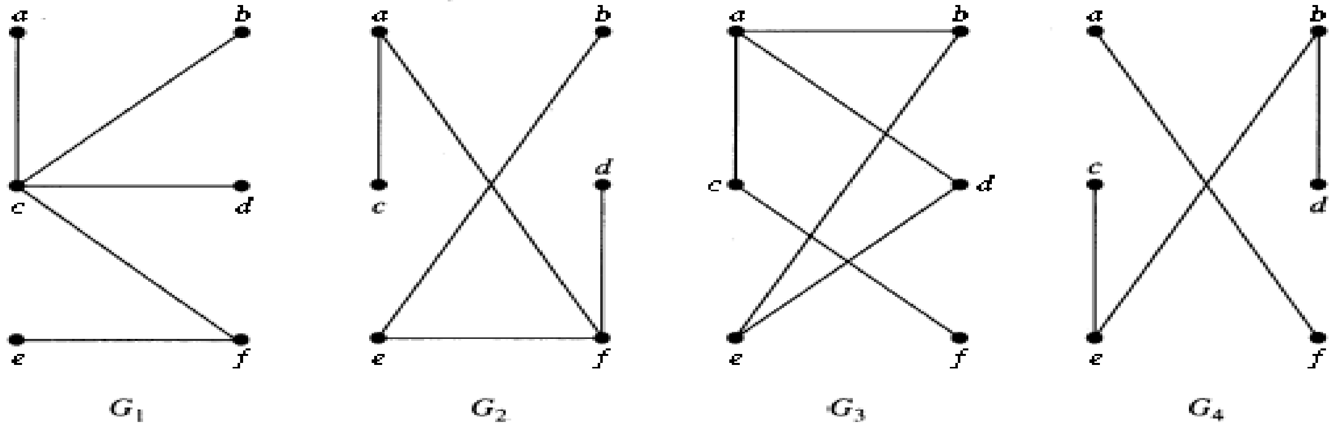


FIGURE 2 Examples of Trees and Graphs That Are Not Trees.

Solution: G_1 and G_2 are trees, because both are connected graphs with no simple circuits.

G_3 is not a tree because e, b, a, d, e is a simple circuit in this graph.

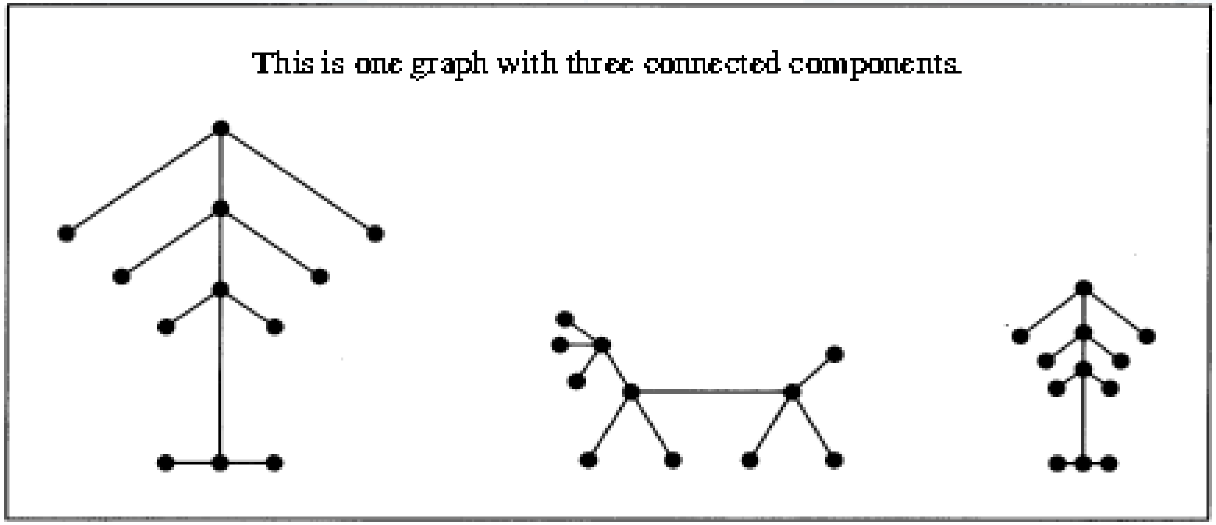
Finally, G_4 is not a tree because it is not connected.

Forests

Any connected graph that contains no simple circuits is a tree.

What about graphs containing no simple circuits that are not necessarily connected?

These graphs are called ***forests*** and have the property that ***each of their connected components is a tree.***



THEOREM 1

An undirected graph is a tree if and only if there is a unique simple path between any two of its vertices.

The Root & Rooted Trees

In many applications of trees, a particular vertex of a tree is designated as the root.

Because ***there is a unique path from the root to each vertex of the graph*** (by Theorem1), we direct each edge away from the root.

Thus, a tree together with its root produces a directed graph called a **rooted tree.**

Rooted Tree

DEFINITION 2

A rooted tree is a tree in which one vertex has been designated as the root and every edge is directed away from the root.

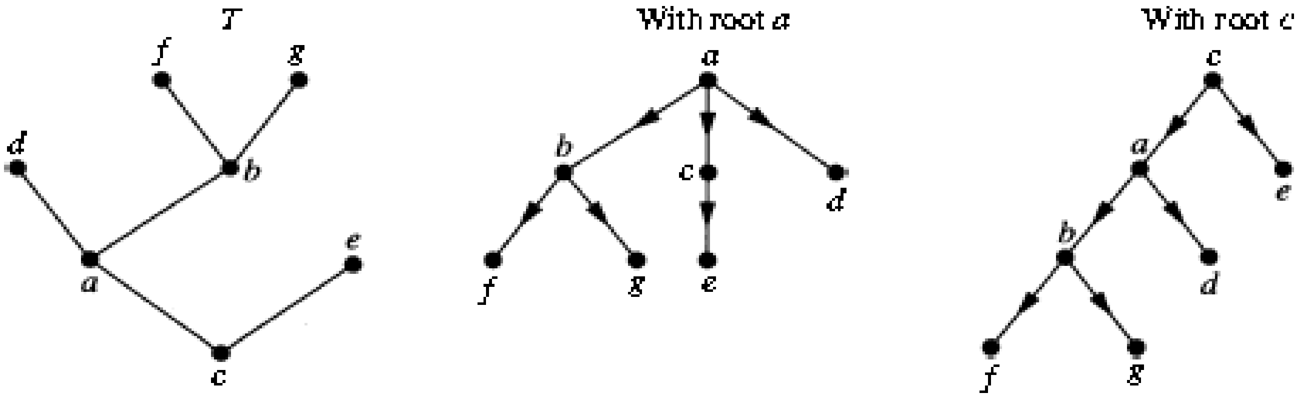
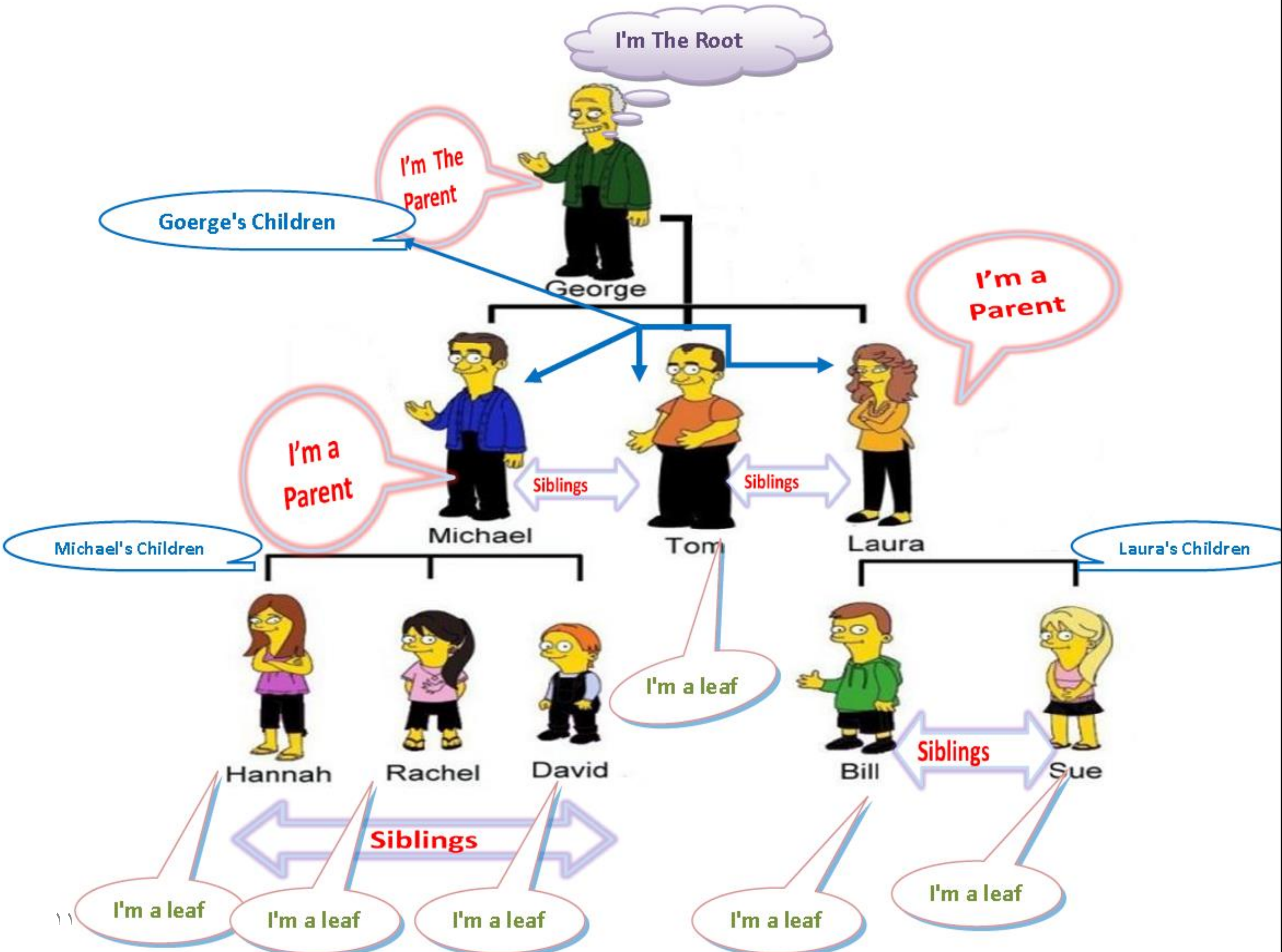


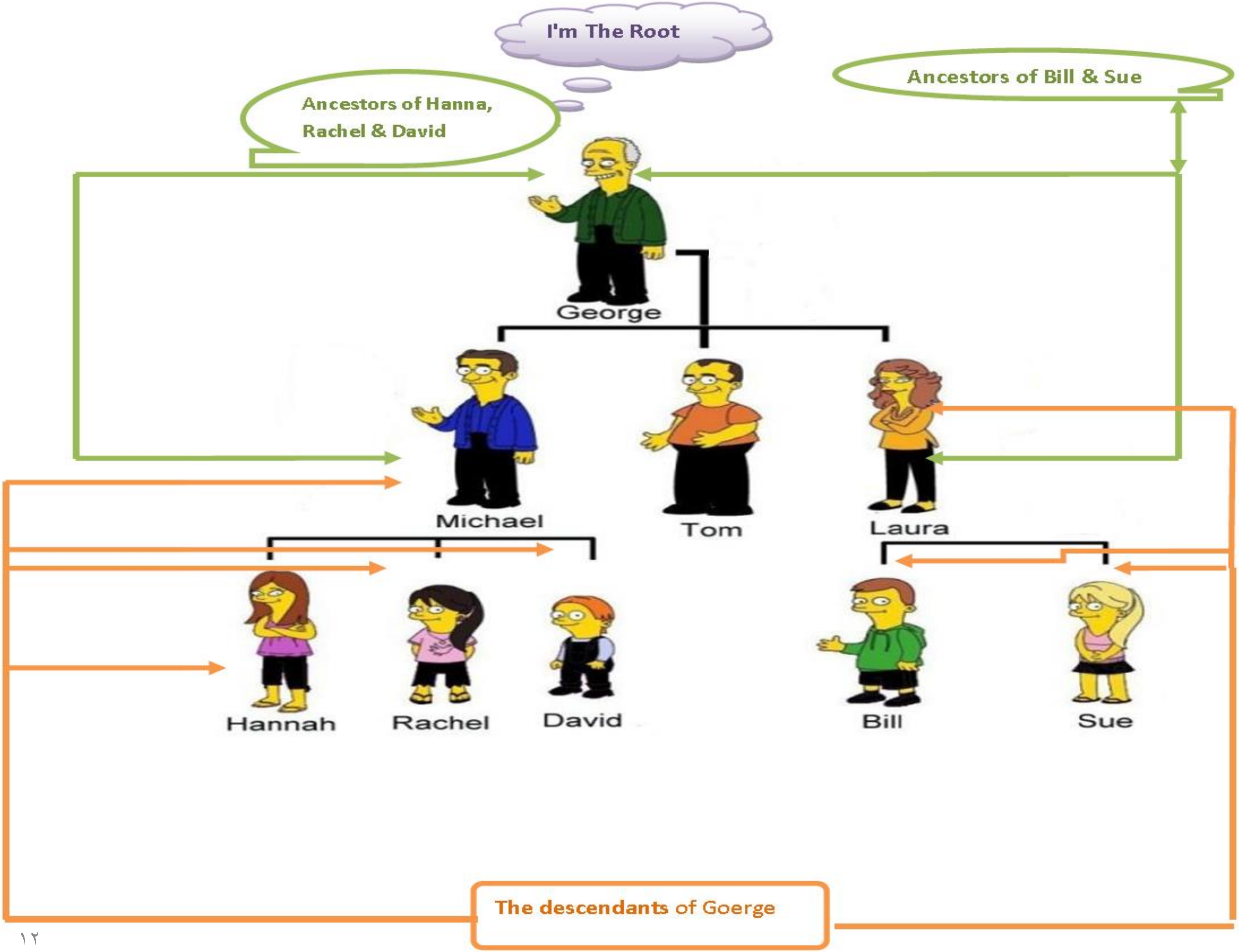
FIGURE 4 A Tree and Rooted Trees Formed by Designating Two Roots.

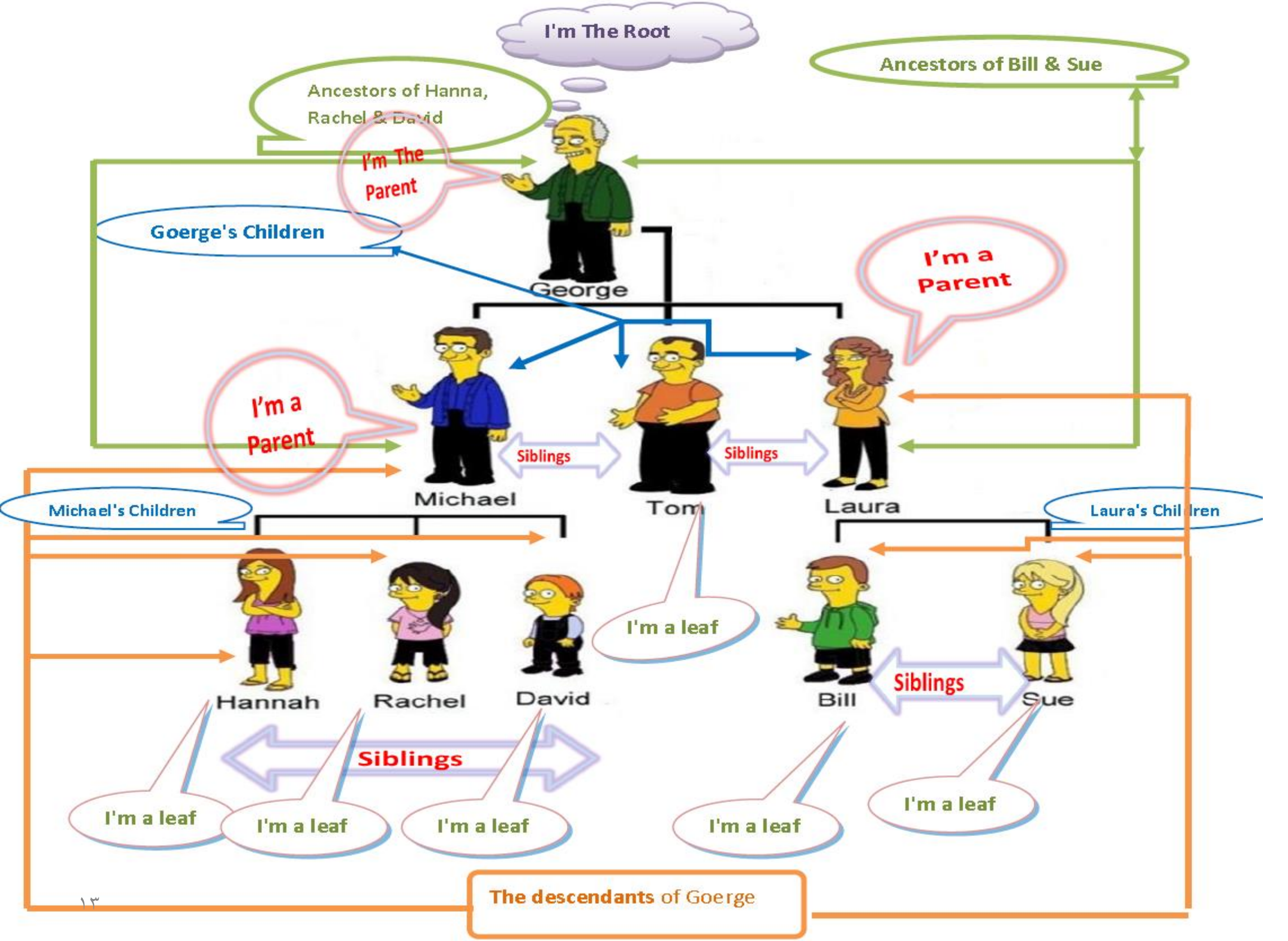
Some Concepts

Suppose that T is a rooted tree.

- If v is a vertex in T other than the root, the parent of v is the unique vertex u such that there is a directed edge from u to v.
- When u is the parent of v, v is called *a child* of u .
- Vertices with the same parent are called *siblings*.
- *The ancestors* of a vertex other than the root are the vertices in the path from the root to this vertex, excluding the vertex itself and including the root (that is, its parent, its parent's parent, and so on, until the root is reached).
- *The descendants* of a vertex v are those vertices that have v as an ancestor.
- A vertex of a tree is called *a leaf* if it has no children.
- .Vertices that have children are called *internal vertices*. The root is an internal vertex unless it is the only vertex in the graph, in which case it is a leaf.
- If a is a vertex in a tree, *the subtree* with *a* as its root is the subgraph of the tree consisting of *a* and its descendants and all edges incident to these descendants.


I'm The Root
I'm The Parent
Goerge's Children
George
I'm a Parent
Michael
Tom
Laura
I'm a Parent
Siblings
Siblings
Michael's Children
Laura's Children
Hannah
Rachel
David
Bill
Sue
Siblings
Siblings
I'm a leaf
I'm a leaf
I'm a leaf
I'm a leaf
I'm a leaf
I'm a leaf


I'm The Root
Ancestors of Hanna, Rachel & David
Ancestors of Bill & Sue
George
Michael
Tom
Laura
Hannah
Rachel
David
Bill
Sue
The descendants of Goerge


I'm The Root
Ancestors of Bill & Sue
Ancestors of Hanna, Rachel & David
I'm The Parent
Goerge's Children
George
I'm a Parent
I'm a Parent
Michael
Siblings
Tom
Siblings
Laura
Michael's Children
Laura's Children
Hannah
Rachel
David
I'm a leaf
Bill
Siblings
Sue
Siblings
I'm a leaf
I'm a leaf
I'm a leaf
I'm a leaf
I'm a leaf
The descendants of Goerge

EXAMPLE 2

In the rooted tree T (with root *a*) shown in Figure 5, find the parent of c, the children of g, the siblings of h , all ancestors of e, all descendants of b, all internal vertices, and all leaves. What is the subtree rooted at g?

Solution:

the parent of c: b

the children of g: h , i , and j

the siblings of h: i and j .

ancestors of e: c , b , and a .

all descendants of b: c , d , and e .

all internal vertices: a, b, c, g, h , and j .

all leaves: d, e, f, i , k, *l*, and m .

subtree rooted at g:

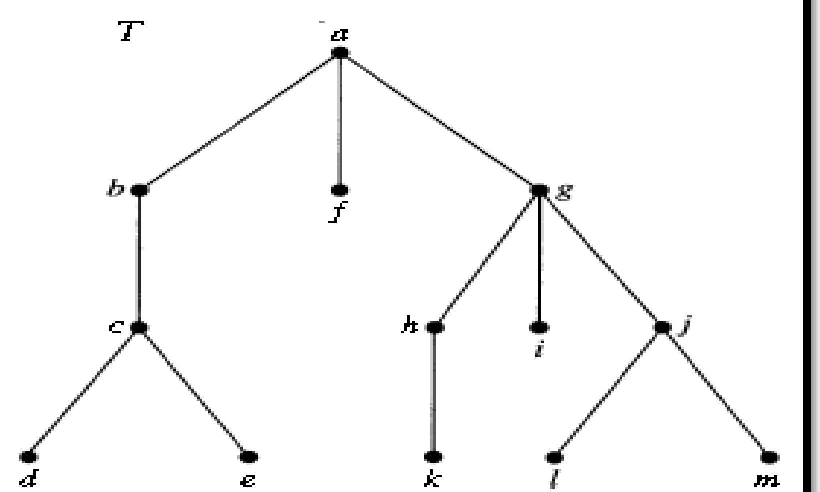


FIGURE 5 A Rooted Tree *T*.

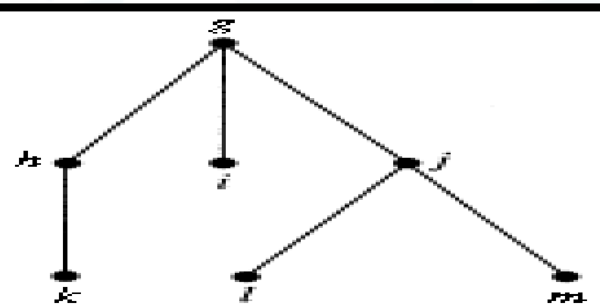


FIGURE 6 The Subtree Rooted at g.

m-ary & full m-ary

DEFINITION 3

A rooted tree is called an ***m -ary tree*** if every internal vertex has no more than m children.

The tree is called a ***full m-ary tree*** if every internal vertex has exactly m children.

An m –ary tree with m = 2 is called a ***binary tree.***

EXAMPLE 3

Are the rooted trees in Figure 7 full m -ary trees for some positive integer m

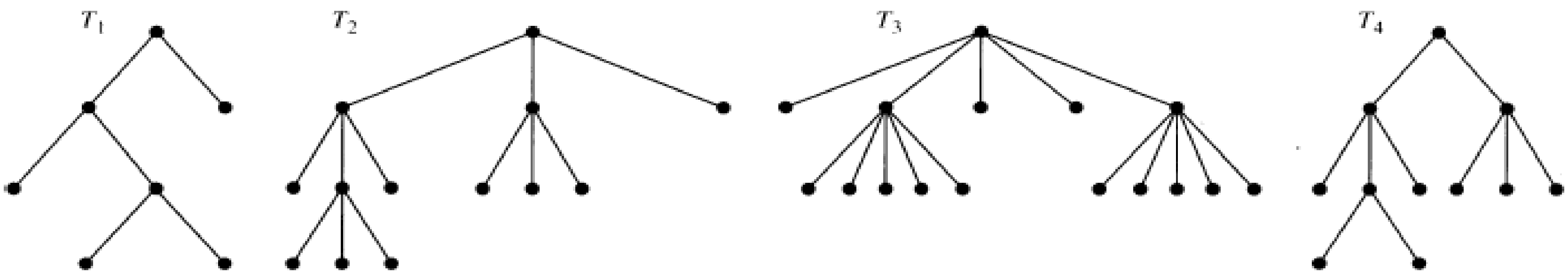


FIGURE 7 **Four Rooted Trees.**

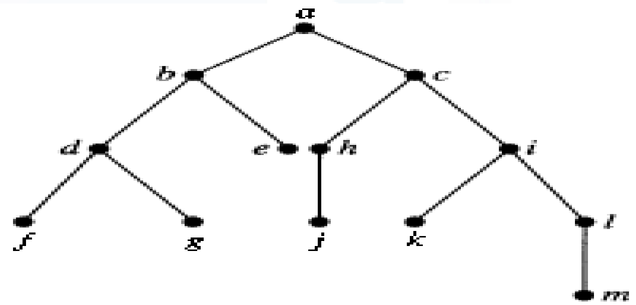
- **T_1 is a full binary tree because each of its internal vertices has two children.**
- **T_2 is a full 3-ary tree because each of its internal vertices has three children.**
- **In T_3 each internal vertex has five children, so T_3 is a full 5-ary tree.**
- **T4 is not a full m -ary tree for any m because some of its internal vertices have two children and others have three children.**

Some Concepts

- ***An ordered rooted tree*** **is a rooted tree where the children of each internal vertex are ordered.**
- **Ordered rooted trees are drawn so that the children of each internal vertex are shown in order from left to right.**
- **In an *ordered binary tree* (usually called just *a binary tree*), if an internal vertex has two children, the first child is called *the left child* and the second child is called *the right child*.**
- **The tree rooted at the left child of a vertex is called *the left subtree* of this vertex, and the tree**
- **rooted at the right child of a vertex is called *the right subtree* of the vertex.**

EXAMPLE 4

What are the left and right children of d in the binary tree T shown in Figure 8(a) (where the order is that implied by the drawing)? What are the left and right subtrees of c?

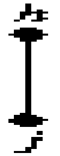


(a)

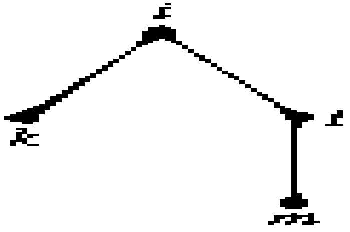
Solution:

The left child of d is f and the right child is g.

We show the left and right subtrees of c in Figures 8(b) and 8(c), respectively.



(b)



(c)

Properties of Trees

THEOREM 2

A tree with n vertices has n - 1 edges.

THEOREM 3

A full m-ary tree with i internal vertices contains

$n=m.i+1$ vertices.

Homework

Page 755/756

- 1 (a,c)
- 3 (a,b,c,d,e,f,g,h)
- 5
- 17
- 18