10.4 Connectivity

Objectives:

The main purpose for this lesson is to introduce the following:

- ✓ Define a path and give examples of its.
- ✓ Learn about some of path concepts.
- ✓ Define connected for undirected graph.

Paths

Informally, *a path* is a sequence of edges that begins at a vertex of a graph and travels from vertex to vertex along edges of the graph.

Definition 1

- Let n be a nonnegative integer and G an undirected graph. *A path of length n* from u to v in G is a sequence of n edges $e_1, \ldots, e_n$ of G such that e_1 is associated with $\{x_0, x_1\}$, e_2 is associated with $\{X_1, X_2\}$, and so on, with e_n associated with $\{x_{n-1}, x_n\}$, where $X_0 = u$ and $X_n = v$.
- When the graph is simple, we denote this path by its vertex sequence $X_0, X_1, \ldots,$ Xn (because listing these vertices uniquely determines the path).

The path is a *circuit* if it begins and ends at the same vertex, that is, if u = v, and has length greater than zero.

- The path or circuit is said to *pass through the vertices* $X_1, X_2, \ldots, X_{n-l}$ or *traverse the edges* $e_1, e_2, \ldots, e_n$.
- *A path or circuit is simple* if it does not contain the same edge more than once.

Remark:

- in some books, the term ***walk*** is used instead of ***path***, where a ***walk*** is defined to be an alternating sequence of vertices and edges of a graph, V_0, e_1,V_1, e_2,… , V_{n-1}, e_n, V_n , where $V_{i-1}$and V_i are the endpoints of e_i for i=1,2,…,n .
- When this terminology is used, ***closed walk*** is used instead of ***circuit*** to indicate a walk that begins and ends at the same vertex,
- and ***trail*** is used to denote a walk that has no repeated

edge (replacing the term ***simple path***).

When this terminology is used, the terminology path

is often used for a trail with no repeated vertices.

EXAMPLE 1

- In the simple graph shown in Figure 1 , **a,d,c,f,e** is a **simple path of length 4**, because {a,d}, {d,c } , {c,f}, and {f,e} are all edges.
- However, **d,e,c,a** is **not a path**, because {e,c} is not an edge.
- Note that **b, c, f, e, b** is a **circuit of length 4** because {b, c}, {c,f}, {f, e}, and {e, b} are edges, and this path begins and ends at b.
- The path **a, b, e, d, a, b**, which **is of length 5**, is **not simple** because it contains the edge {a,b} twice.

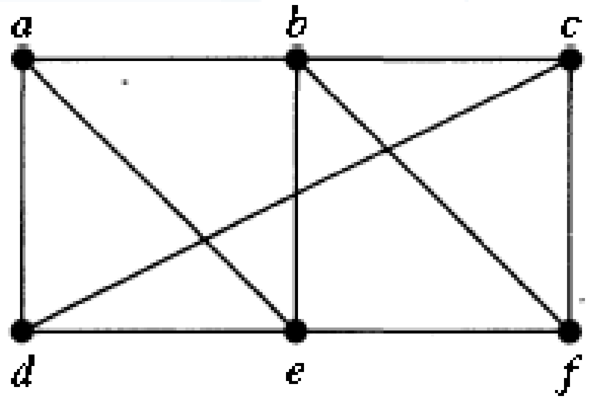

FIGURE 1 A Simple Graph.

DEFINITION 2

Let n be a nonnegative integer and G a **directed graph**.

- ***A path of length n from u to v*** in G is a sequence of edges $e_1, e_2, .., e_n$ of G such that e_1 is associated with (x_0, x_1), e_2 is associated with (x_1, x_2), and so on, with e_n associated with (x_{n-1}, x_n), where $x_0 = u$ and $x_n = v$.
- When there are no multiple edges in the directed graph, this path is denoted by its vertex sequence $x_0, x_1, x_2, \ldots, x_n$.
- A path of length greater than zero that begins and ends at the same vertex is called ***a circuit or cycle***.
- A path or circuit is called ***simple*** if it does not contain the same edge more than once.

Connectedness In Undirected Graphs

DEFINITION 3

An **undirected graph** is called ***connected*** if there is a path between every pair of distinct vertices of the graph.

EXAMPLE 5

- The graph G_1 in Figure 2 is **connected**, because for every pair of distinct vertices there is a path between them.
- However, the graph G_2 in Figure 2 is **not connected**.
- For instance, there is no path in G_2 between vertices *a* and d.

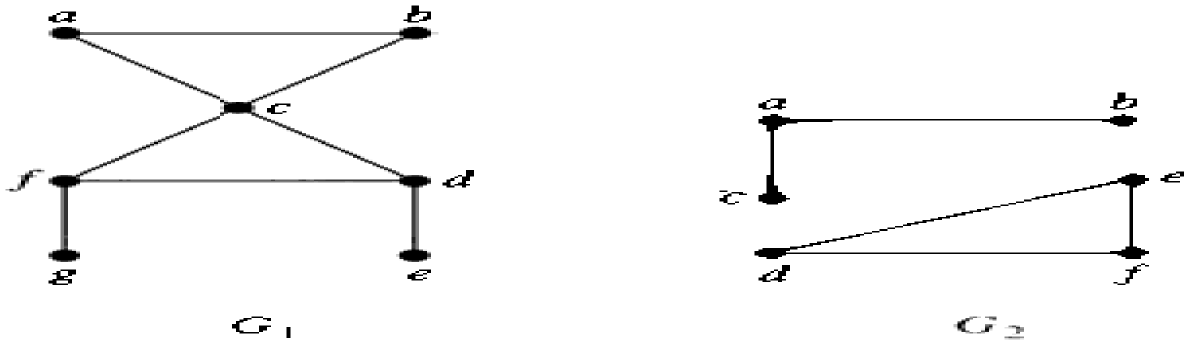

FIGURE 2 The Graphs G_1 and G_2.

Homework

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- 1(a,b,c,d)
- 3