Engineering Mechanics
AGE 2330

Lect 4: Equilibrium

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Equivalent Systems: Resultants

Equilibrium

Equilibrium of a body is a condition in which the resultants of all forces acting on the body is zero.

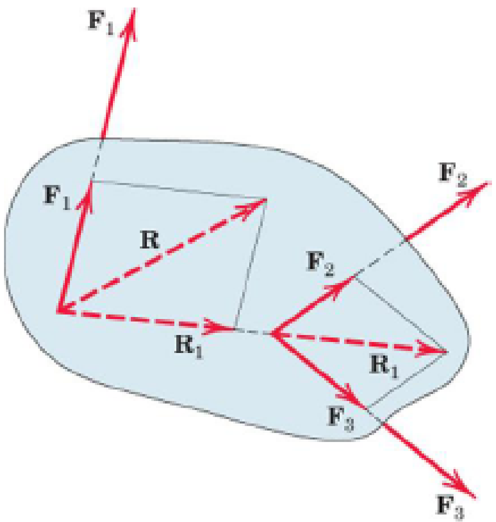
→Condition studied in Statics

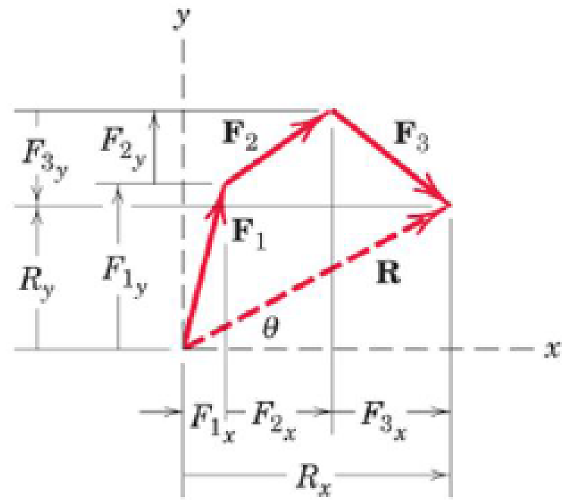
When the resultant of all forces on a body is not zero, acceleration of the body is obtained by equating the force resultant to the product of the mass and acceleration of the body.

→ Condition studied in Dynamics

Equivalent Systems: Resultants

Vector Approach: Principle of Transmissibility can be used





Magnitude and direction of the resultant force R is obtained by forming the force polygon where the forces are added head to tail in any sequence

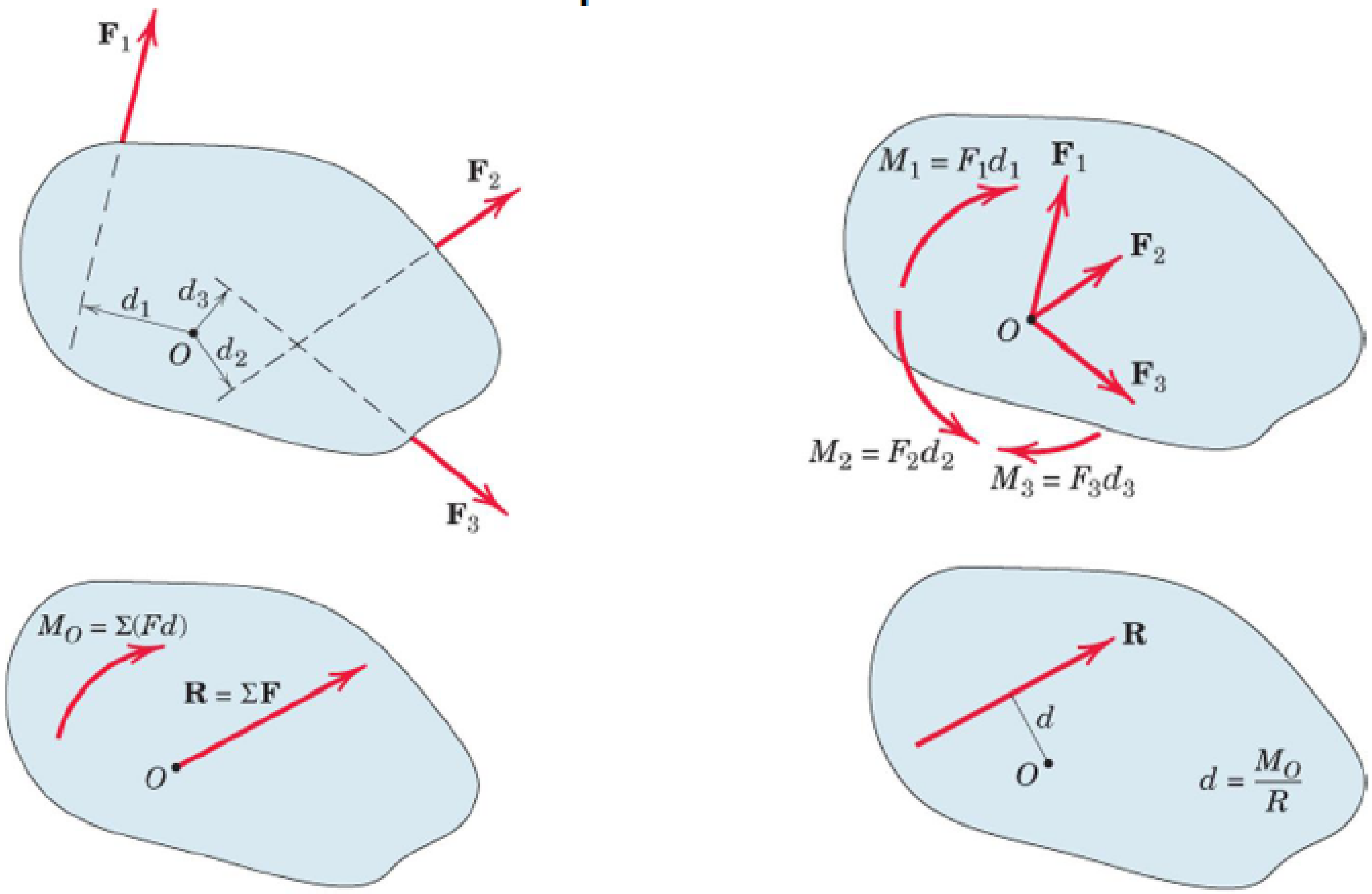
$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma\mathbf{F}$$

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{\Sigma F_y}{\Sigma F_x}$$

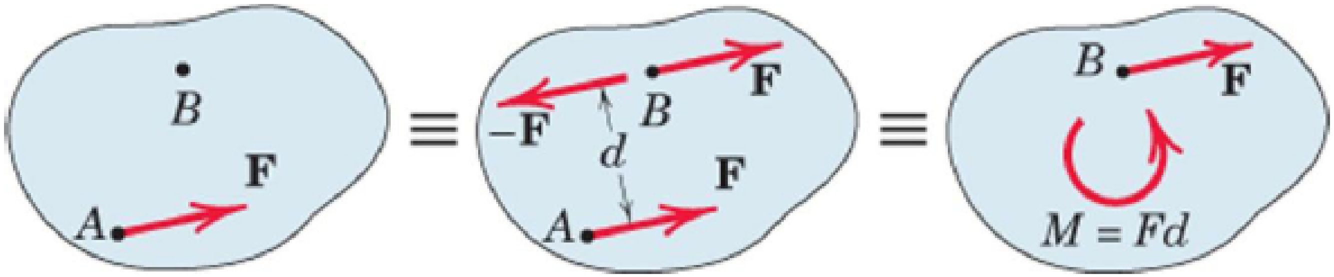
Resultant of Concurrent Forces

- Two dimensional plane

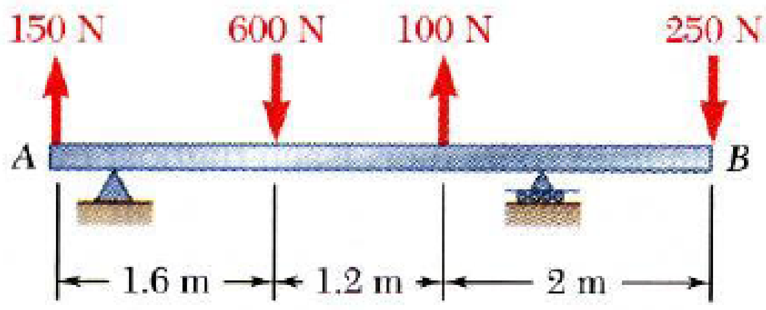


Equivalent Force and Couple

- Two dimensional plane



Example on Equivalent System



For the beam, reduce the system of forces shown to

(a) an **equivalent force-couple** system at ***A***,

(b) an **equivalent force couple** system at ***B***, and

(c) a **single force** or **resultant**

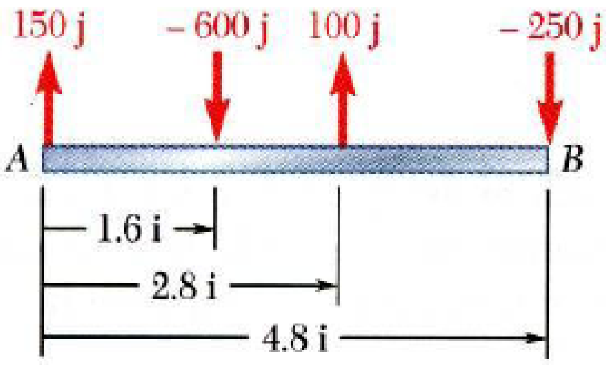
Solution:

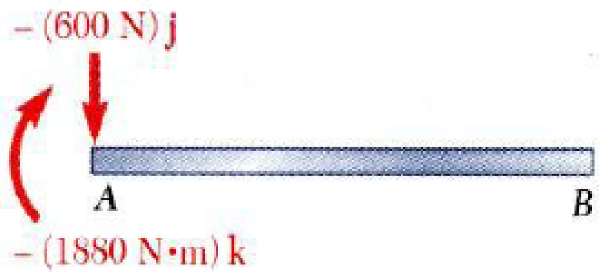
a) Compute the resultant force for the forces shown and the resultant couple for the moments of the forces about *A*.

b) Find an equivalent force-couple system at *B* based on the force-couple system at *A*.

c) Determine the point of application for the resultant force such that its moment about *A* is equal to the resultant couple at *A*.

Example on Equivalent System





SOLUTION:

a) Compute the resultant force and the resultant couple at *A*.

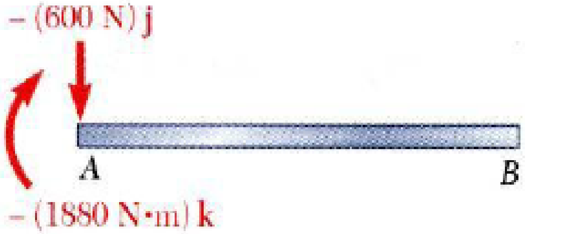
$$\vec{R} = \sum \vec{F}$$
$$= (150\ \text{N})\vec{j} - (600\ \text{N})\vec{j} + (100\ \text{N})\vec{j} - (250\ \text{N})\vec{j}$$

$$\boxed{\vec{R} = -(600\ \text{N})\vec{j}}$$

$$\vec{M}_A^R = \sum\left(\vec{r} \times \vec{F}\right)$$
$$= \left(1.6\vec{i}\right)\times\left(-600\vec{j}\right) + \left(2.8\vec{i}\right)\times\left(100\vec{j}\right) + \left(4.8\vec{i}\right)\times\left(-250\vec{j}\right)$$

$$\boxed{\vec{M}_A^R = -(1880\ \text{N}\cdot\text{m})\vec{k}}$$

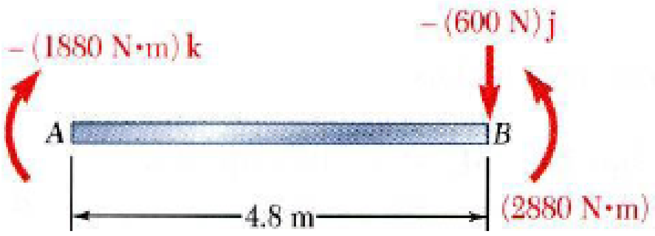
Example on Equivalent System



b) Find an equivalent force-couple system at *B* based on the force-couple system at *A*.

The force is unchanged by the movement of the force-couple system from *A* to *B*.

$$\vec{R} = -(600\ \mathrm{N})\vec{j}$$

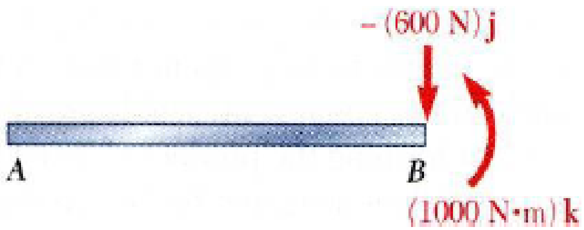


The couple at *B* is equal to the moment about *B* of the force-couple system found at *A*.

$$\vec{M}_B^R = \vec{M}_A^R + \vec{r}_{B/A} \times \vec{R}$$

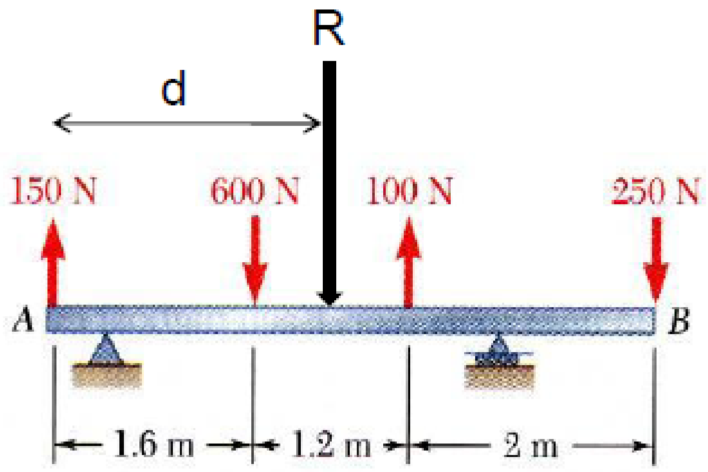
$$= -(1880\ \mathrm{N \cdot m})\vec{k} + (-4.8\ \mathrm{m})\vec{i} \times (-600\ \mathrm{N})\vec{j}$$

$$= -(1880\ \mathrm{N \cdot m})\vec{k} + (2880\ \mathrm{N \cdot m})\vec{k}$$



$$\vec{M}_B^R = +(1000\ \mathrm{N \cdot m})\vec{k}$$

Example on Equivalent System



c)

$F_R = F_1 + F_2 + F_3 + F_4$

$R = 150 - 600 + 100 - 250 = -650\ N$

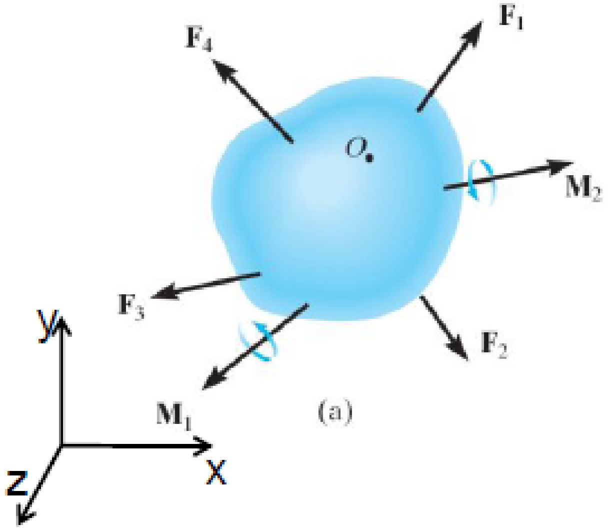
$F_R d = F_1 d_1 + F_2 d_2 + F_3 d_3 + F_4 d_4$

$d = 3.13\ m$

Rigid Body Equilibrium

A rigid body will remain in equilibrium provided

- sum of all the external forces acting on the body is equal to zero, and
- Sum of the moments of the external forces about a point is equal to zero



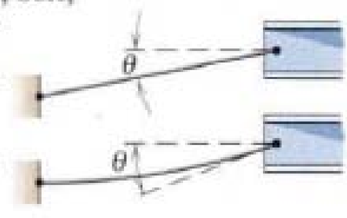
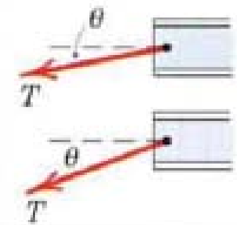
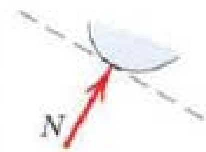
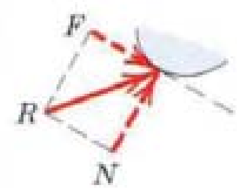
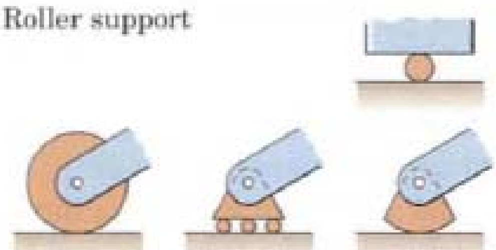
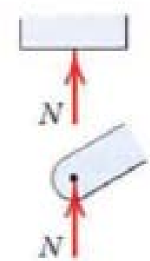
$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma F_z = 0$$

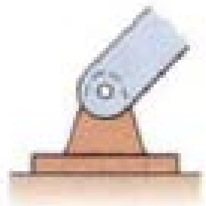
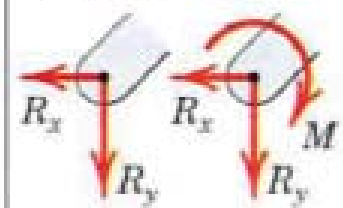
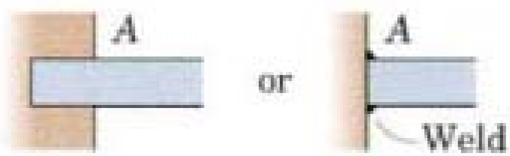
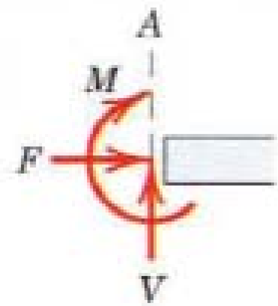
$$\Sigma M_x = 0$$
$$\Sigma M_y = 0$$
$$\Sigma M_z = 0$$

System Isolation & Free Body Diagram (FBD)

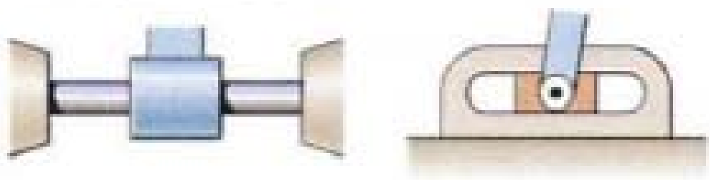
the free-body diagram is the most important single step in the solution of problems in mechanics.

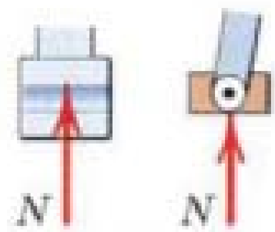
Modeling the Action of Forces

MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	Action on Body to Be Isolated
1. Flexible cable, belt, chain, or rope Weight of cable negligible Weight of cable not negligible 	 Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.
2. Smooth surfaces 	 Contact force is compressive and is normal to the surface.
3. Rough surfaces 	 Rough surfaces are capable of supporting a tangential component *F* (frictional force) as well as a normal component *N* of the resultant contact force *R*.
4. Roller support 	 Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.

MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS *(cont.)*		
Type of Contact and Force Origin	**Action on Body to Be Isolated**	
6. Pin connection 		A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the axis; usually shown as two components R_x and R_y. A pin not free to turn may also support a couple M.
7. Built-in or fixed support 		A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.
8. Gravitational attraction 		The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center mass G.

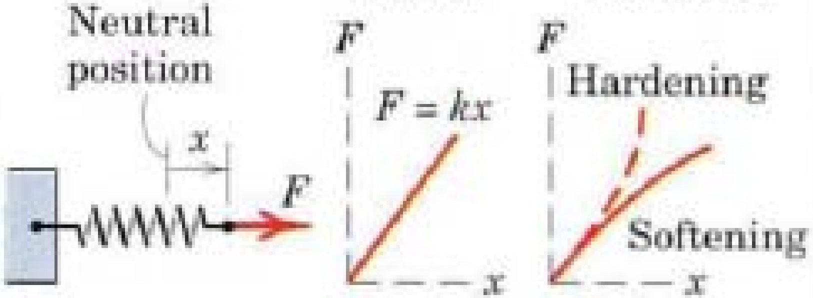
5. Freely sliding guide

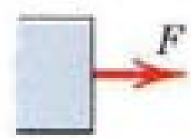




Collar or slider free to move along smooth guides; can support force normal to guide only.

9. Spring action





Spring force is tensile if spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.

Constructing FBD

- Decide which system to isolate. The system chosen should usually involve one or more of the desired unknowns.
- Next isolate the chosen system by drawing a diagram which represents its complete external boundary.
- Identify all forces which acts on the isolated system as applied by the removed contacting and attracting bodies, and represent them in their proper positions on the diagram of the isolated system.
- Show the choice of coordinate axes directly on the diagram.

Construction of Free-Body Diagrams

The full procedure for drawing a free-body diagram which isolates a body or system consists of the following steps.

Step 1. Decide which system to isolate. The system chosen should usually involve one or more of the desired unknown quantities.

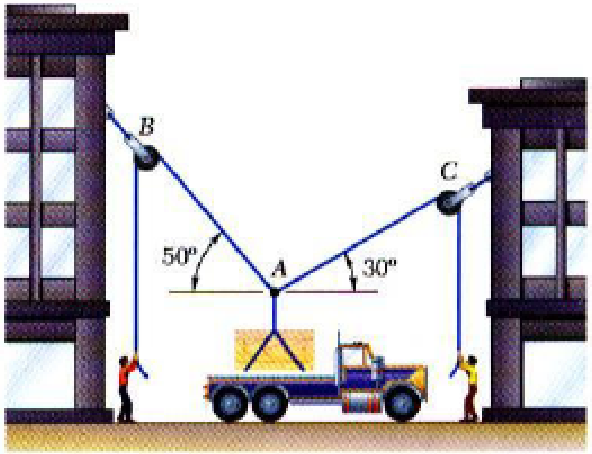
Step 2. Next isolate the chosen system by drawing a diagram which represents its *complete external boundary*. This boundary defines the isolation of the system from *all* other attracting or contacting bodies, which are considered removed. This step is often the most crucial of all. Make certain that you have *completely isolated* the system before proceeding with the next step.

Step 3. Identify all forces which act *on* the isolated system as applied *by* the removed contacting and attracting bodies, and represent them in their proper positions on the diagram of the isolated system. Make a systematic traverse of the entire boundary to identify all contact forces. Include body forces such as weights, where appreciable. Represent all known forces by vector arrows, each with its proper magnitude, direction, and sense indicated. Each unknown force should be represented by a vector arrow with the unknown magnitude or direction indicated by symbol. If the sense of the vector is also unknown, you must arbitrarily assign a sense. The subsequent calculations with the equilibrium equations will yield a positive quantity if the correct sense was assumed and a negative quantity if the incorrect sense was assumed. It is necessary to be *consistent* with the assigned characteristics of unknown forces throughout all of the calculations. If you are consistent, the solution of the equilibrium equations will reveal the correct senses.

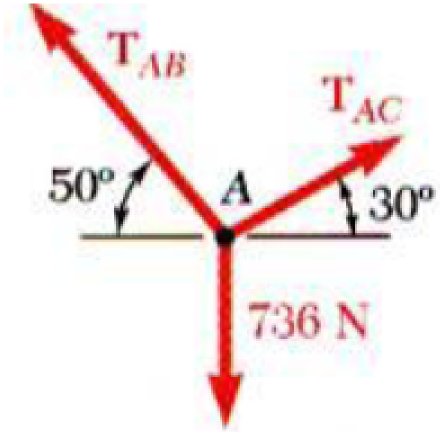
Step 4. Show the choice of coordinate axes directly on the diagram. Pertinent dimensions may also be represented for convenience. Note, however, that the free-body diagram serves the purpose of focusing attention on the action of the external forces, and therefore the diagram should not be cluttered with excessive extraneous information. Clearly distinguish force arrows from arrows representing quantities other than forces. For this purpose a colored pencil may be used.

Rigid Body Equilibrium

Free-Body Diagrams



Space Diagram: A sketch showing the physical conditions of the problem.



Free-Body Diagram: A sketch showing only the forces on the selected particle.

Rigid Body Equilibrium

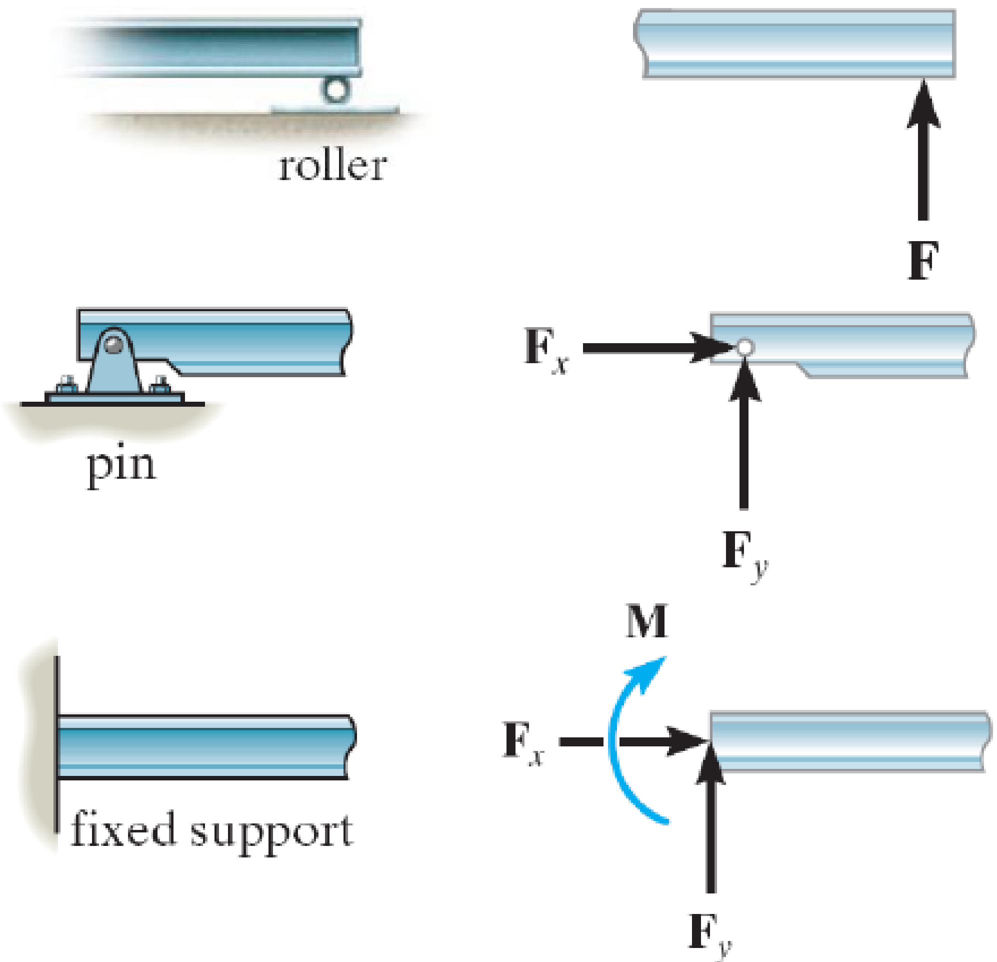
Support Reactions

Prevention of

Translation or

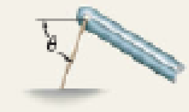
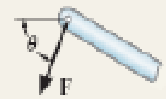
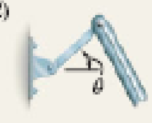
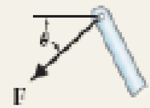
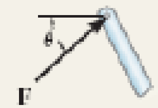
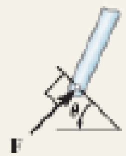
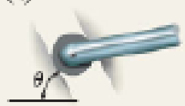
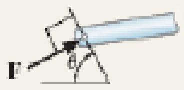
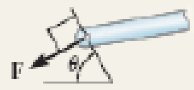
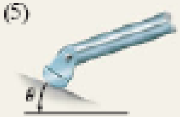
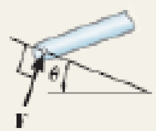
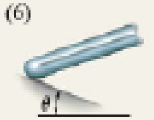
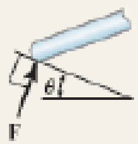
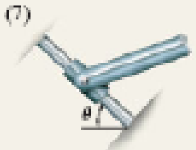
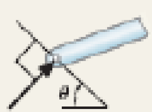
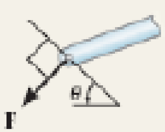
Rotation of a body

Restraints

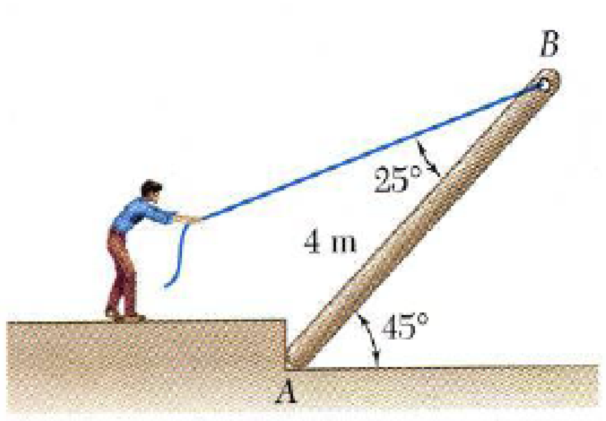


Rigid Body Equilibrium

Various Supports
2-D Force Systems

Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
(2)  weightless link	 or 	One unknown. The reaction is a force which acts along the axis of the link.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4)  roller or pin in confined smooth slot	 or 	One unknown. The reaction is a force which acts perpendicular to the slot.
(5)  rocker		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(6)  smooth contacting surface		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(7) 	 or 	One unknown. The reaction is a force which acts perpendicular to the rod.

Rigid Body Equilibrium: Example



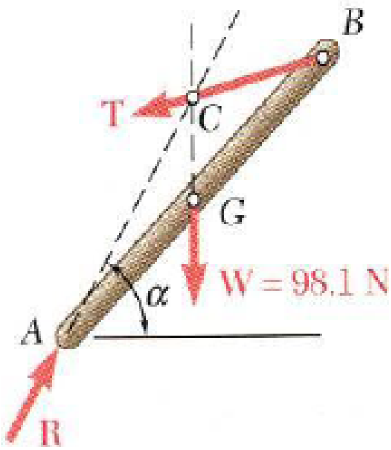
A man raises a 10 kg joist, of length 4 m, by pulling on a rope.

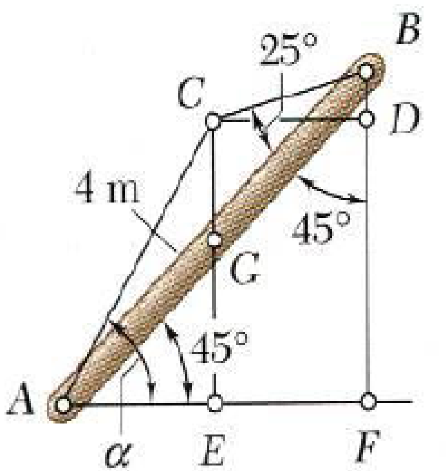
Find the **tension in the rope** and the **reaction at A**.

Solution:

- Create a **free-body diagram** of the joist.
  - The joist is a **3 force body** acted upon by the **rope**, its **weight**, and the **reaction at A**.
- The **three forces** must be **concurrent** for **static equilibrium**.
  - Reaction **R** must pass through the intersection of the lines of action of the weight and rope forces.
  - Determine the direction of the reaction force **R**.
- Utilize a **force triangle** to determine the magnitude of the reaction force **R**.

Rigid Body Equilibrium: Example





- Create a **free-body diagram** of the joist
- Determine the direction of the reaction force **R**

$$AF = AB\cos 45 = (4\,\mathrm{m})\cos 45 = 2.828\,\mathrm{m}$$

$$CD = AE = \tfrac{1}{2}AF = 1.414\,\mathrm{m}$$

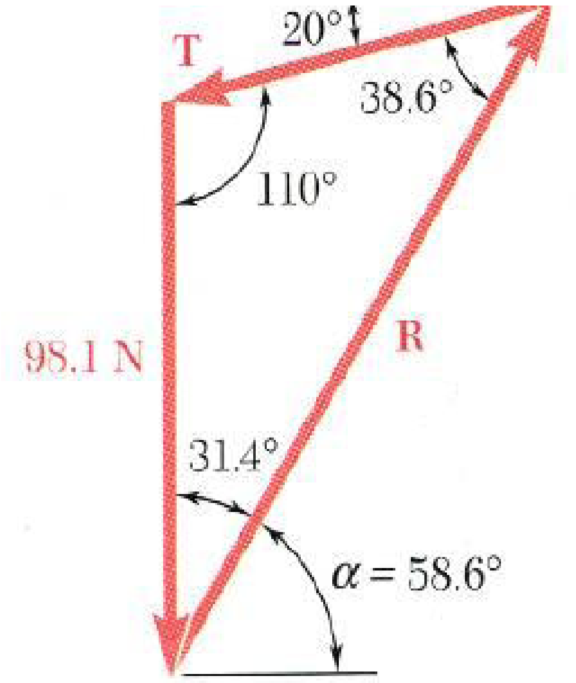
$$BD = CD\cot(45+20) = (1.414\,\mathrm{m})\tan 20 = 0.515\,\mathrm{m}$$

$$CE = BF - BD = (2.828 - 0.515)\,\mathrm{m} = 2.313\,\mathrm{m}$$

$$\tan\alpha = \frac{CE}{AE} = \frac{2.313}{1.414} = 1.636$$

$$\boxed{\alpha = 58.6^\circ}$$

Rigid Body Equilibrium: Example



- Determine the magnitude of the reaction force **R**.

$$\frac{T}{\sin 31.4^\circ} = \frac{R}{\sin 110^\circ} = \frac{98.1\,\mathrm{N}}{\sin 38.6^\circ}$$

$$T = 81.9\,\mathrm{N}$$

$$R = 147.8\,\mathrm{N}$$