Engineering Mechanics
AGE 2330

Lect 11: Kinetics of particles

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Kinetics of Particles

Kinetics:

Study of the relations between unbalanced forces and the resulting changes in motion.

Newton's Second Law of Motion : The acceleration of a particle is proportional to the resultant force acting on it and is in the direction of this force.
→ **A particle will accelerate when it is subjected to unbalanced forces**

Three approaches to solution of Kinetics problems:

1. Force-Mass-Acceleration method (direct application of Newton's Second Law)
2. Use of Work and Energy principles
3. Impulse and Momentum methods

Limitations of this chapter:

- Motion of bodies that can be treated as particles (motion of the mass centre of the body)
- Forces are concurrent through the mass center (action of non-concurrent forces on the motion of bodies will be discussed in chapter on Kinetics of rigid bodies).

Kinetics of Particles

Force-Mass-Acceleration method

Equation of Motion

Particle of mass *m* subjected to the action of concurrent forces $\mathbf{F}_1$, $\mathbf{F}_2$,… whose vector sum is $\sum\mathbf{F}$:

→ Equation of motion: $\sum\mathbf{F} = m\mathbf{a}$

→ Force-Mass-Acceleration equation

Equation of Motion gives the instantaneous value of the acceleration corresponding to the instantaneous value of the forces.

- The equation of motion can be used in scalar component form in any coordinate system.
- For a 3 DOF problem, all three scalar components of equation of motion will be required to be integrated to obtain the space coordinates as a function of time.
- All forces, both applied or reactive, which act on the particle must be accounted for while using the equation of motion.

Free Body Diagrams:

In Statics: Resultant of all forces acting on the body = 0

In Dynamics: Resultant of all forces acting on the body = *ma* → Motion of body

Kinetics of Particles: Force-Mass-Acceleration method

Rectilinear Motion

Motion of a particle along a straight line

For motion along *x*-direction, accelerations along *y*- and *z*-direction will be zero

→ $\sum F_x = ma_x$
$\sum F_y = 0$
$\sum F_z = 0$

For a general case:

→ $\sum F_x = ma_x$
$\sum F_y = ma_y$
$\sum F_z = ma_z$

The acceleration and resultant force are given by:

$$\mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$$

$$a = \sqrt{a_x{}^2 + a_y{}^2 + a_z{}^2}$$

$$\Sigma\mathbf{F} = \Sigma F_x\mathbf{i} + \Sigma F_y\mathbf{j} + \Sigma F_z\mathbf{k}$$

$$|\Sigma\mathbf{F}| = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2}$$

Kinetics of Particles: Force-Mass-Acceleration method

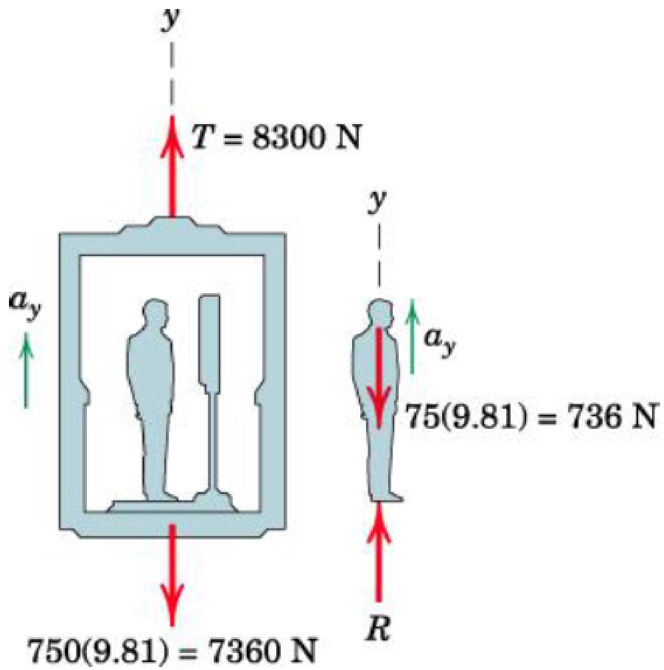
Rectilinear Motion

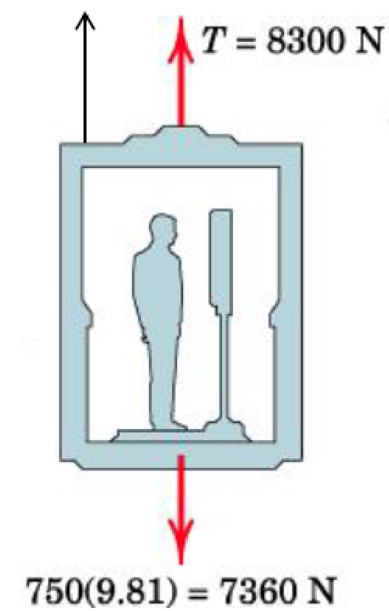
Example

A 75 kg man stands on a spring scale in an elevator. During the first 3 seconds of motion from rest, the tension *T* in the hoisting cable is 8300 N. Find the reading *R* of the scale in Newton during this interval and the upward velocity v of the elevator at the end of the 3 seconds. Total mass of elevator, man, and scale is 750 kg.

Solution

Draw the FBD of the elevator and the man alone





Kinetics of Particles: Force-Mass-Acceleration method

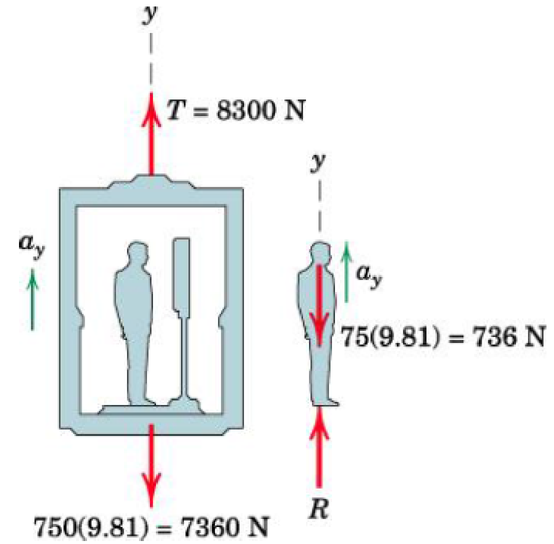
Rectilinear Motion

Example

Solution

During first 3 seconds, the forces acting on the elevator are constant. Therefore, the acceleration a_y will also remain constant during this time.

Force registered by the scale and the velocity of the elevator depend on the acceleration a_y



From FBD of the elevator, scale, and man taken together:

$\sum F_y = ma_y \rightarrow$ 8300-7360 = $750a_y \rightarrow a_y$ = 1.257 m/s^2

From FBD of the man alone:

$\sum F_y = ma_y \rightarrow R$-736 = $75a_y \rightarrow R$ = 830 N

Velocity reached at the end of the 3 sec:

$\Delta v = \int a\, dt \rightarrow \quad v - 0 = \int_0^3 1.257\, dt$

v = 3.77 m/s

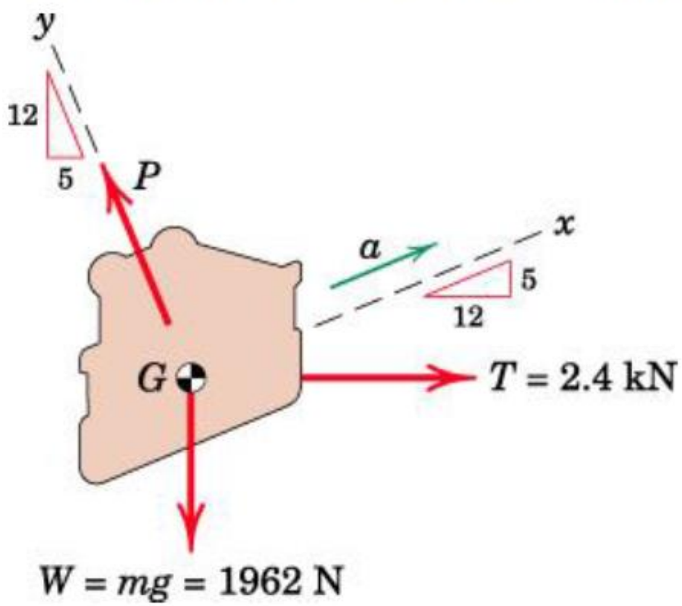
Kinetics of Particles: Force-Mass-Acceleration method

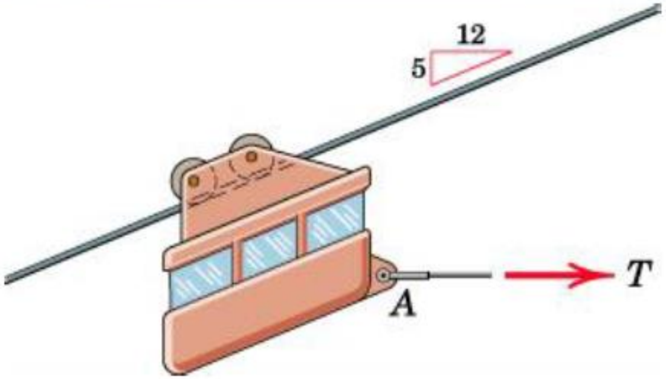
Rectilinear Motion

Example

A small inspection car with a mass of 200 kg runs along the fixed overhead cable and is controlled by the attached cable at A. Determine the acceleration of the car when the control cable is horizontal and under a tension $T = 2.4$ kN. Also find the total force P exerted by the supporting cable on the wheels.

Solution: Draw the FBD of the system





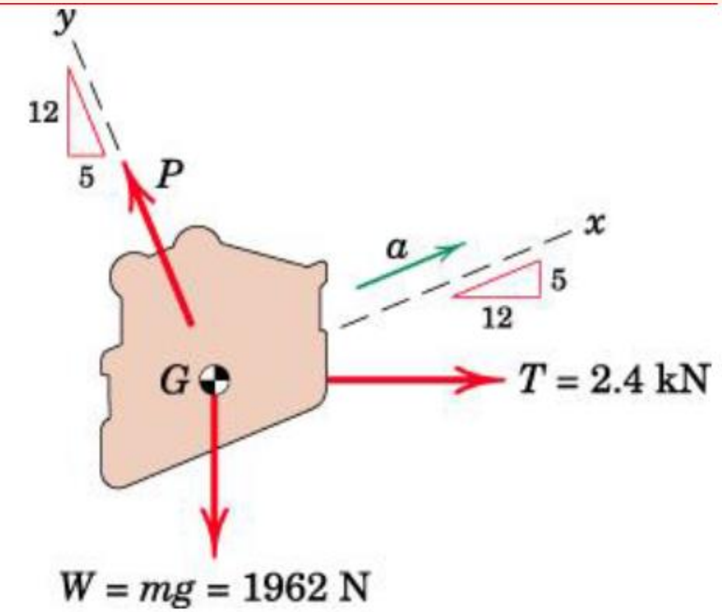
Choosing the *x-y* coordinate system such that the axes are along and normal to the motion (acceleration)
→ calculations simplified

The car is in equilibrium in the y-direction since there is no acceleration in this direction. Thus,

$$[\Sigma F_y = 0] \qquad P - 2.4(\tfrac{5}{13}) - 1.962(\tfrac{12}{13}) = 0 \qquad P = 2.73 \text{ kN} \qquad \textit{Ans.}$$

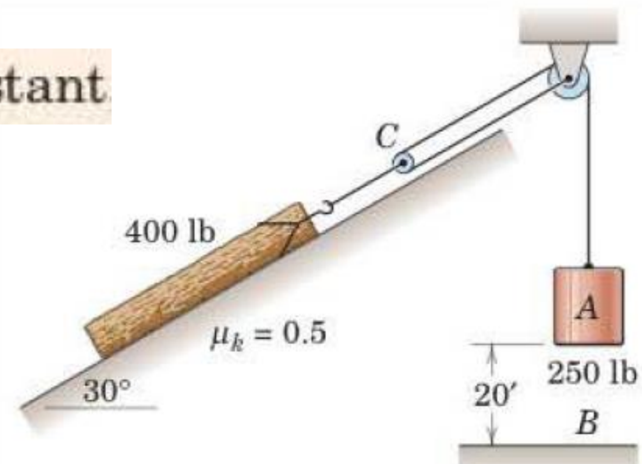
In the x-direction the equation of motion gives

$$[\Sigma F_x = ma_x] \qquad 2400(\tfrac{12}{13}) - 1962(\tfrac{5}{13}) = 200a \qquad a = 7.30 \text{ m/s}^2 \qquad \textit{Ans.}$$



Sample Problem 3/3

The 250-lb concrete block A is released from rest in the position shown and pulls the 400-lb log up the 30° ramp. If the coefficient of kinetic friction between the log and the ramp is 0.5, determine the velocity of the block as it hits the ground at B.

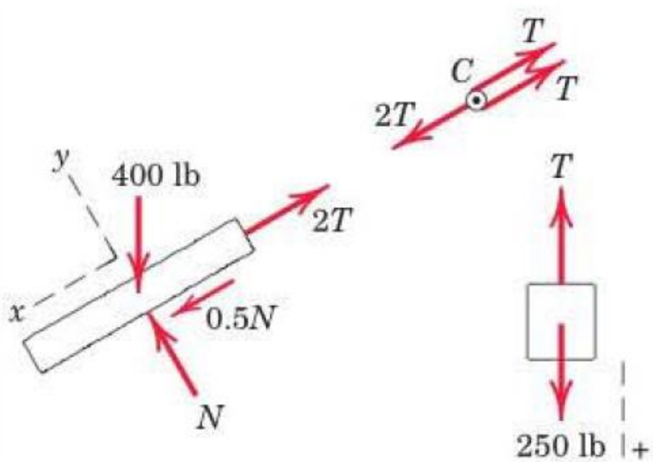


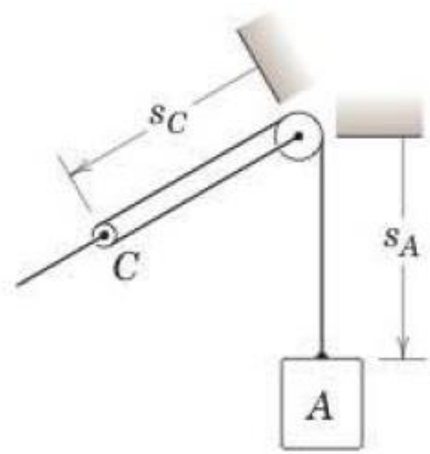
total length of the cable is $L = 2s_C + s_A + \text{constant}$

Differentiating the above equation two times gives:

$$0 = 2\ddot{s}_C + \ddot{s}_A$$

$$0 = 2a_C + a_A$$





$[\Sigma F_y = 0]$ $\quad N - 400 \cos 30° = 0 \quad N = 346 \text{ lb}$

$[\Sigma F_x = ma_x]$ $\quad 0.5(346) - 2T + 400 \sin 30° = \dfrac{400}{32.2} a_C$

For the block in the positive downward direction, we have

$[+\downarrow \Sigma F = ma]$

$$250 - T = \frac{250}{32.2} a_A$$

Solving the three equations in a_C, a_A, and T gives us

$$a_A = 5.83 \text{ ft/sec}^2 \qquad a_C = -2.92 \text{ ft/sec}^2 \qquad T = 205 \text{ lb}$$

For the 20-ft drop with constant acceleration, the block acquires a velocity

$[v^2 = 2ax]$

$$v_A = \sqrt{2(5.83)(20)} = 15.27 \text{ ft/sec}$$

Kinetics of Particles: Force-Mass-Acceleration method

Curvilinear Motion: Particles move along plane curvilinear paths.

Rectangular Coordinates

$$\sum F_x = ma_x \qquad a_x = \ddot{x}$$
$$\sum F_y = ma_y \qquad a_y = \ddot{y}$$

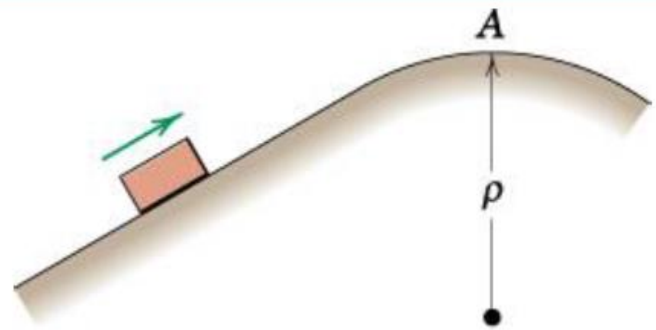
Normal and Tangential Coordinates

$$\sum F_n = ma_n$$
$$\sum F_t = ma_t$$

$$a_n = \frac{v^2}{\rho} = \rho\dot{\beta}^2 = v\dot{\beta}$$
$$a_t = \dot{v} = \ddot{s}$$
$$v = \rho\dot{\beta}$$

Example (1) on curvilinear motion

Determine the maximum speed v which the sliding block may have as it passes point A without losing contact with the surface.

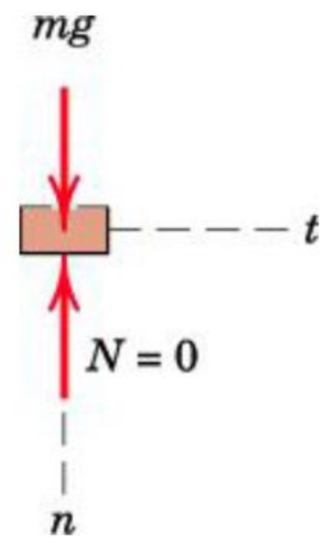


Solution:

The condition for loss of contact: Normal force N exerted by the surface on the block is equal to zero.

Draw the FBD of the block and using n-t coordinate system

Let m be the mass of the block.



Along n-direction:

$\sum F_n = ma_n$

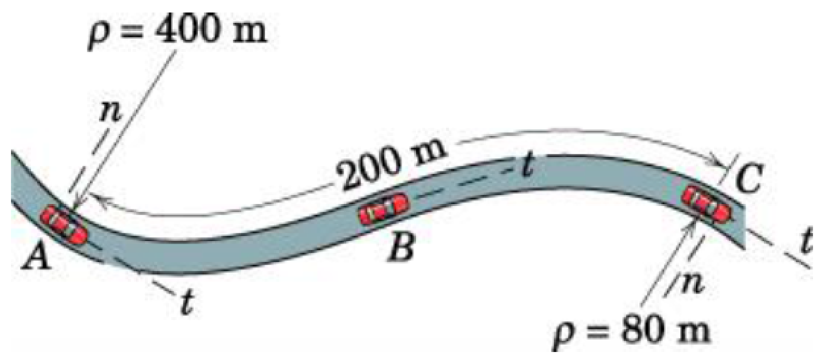
$mg - N = ma_n$

$mg = m(v^2/\rho)$

→ $$v = \sqrt{g\rho}$$

Example (2) on curvilinear motion

A 1500 kg car enters a section of curved road in the horizontal plane and slows down at a uniform rate from a speed of 100 km/h at *A* to 50 km/h at *C*. Find the total horz force exerted by the road on the tires at positions *A*, *B*, and *C*. Point *B* is the inflection point where curvature changes sign.

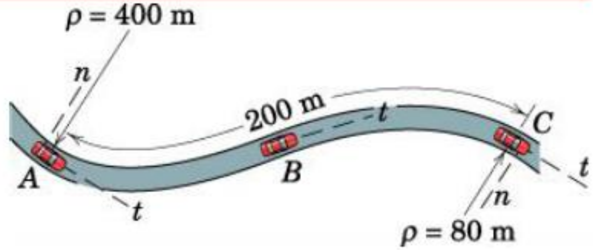


Solution:
The car will be treated as a particle → all the forces exerted by the road on tires will be treated as a single force.

Normal and tangential coordinates will be used to specify the acceleration of the car since the motion is described along the direction of the road.

Forces can be determined from the accelerations.

Example (2) on curvilinear motion

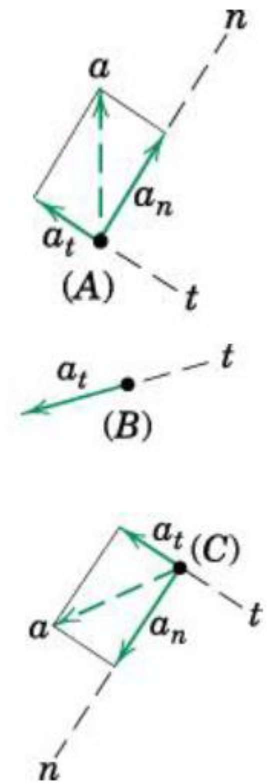


Solution:

The acceleration is constant and its direction will be along negative *t*-direction.

Magnitude of acceleration:

$$[v_C^2 = v_A^2 + 2a_t \Delta s] \qquad a_t = \left| \frac{(50/3.6)^2 - (100/3.6)^2}{2(200)} \right| = 1.447 \text{ m/s}^2$$



Normal components of the acceleration at *A*, *B*, and *C*:

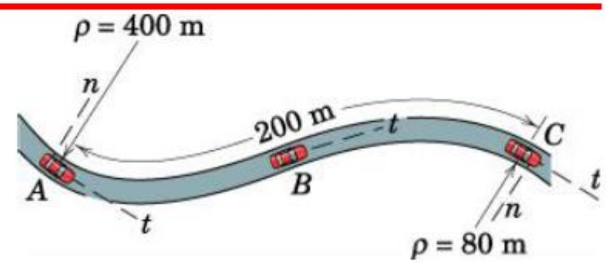
$$[a_n = v^2/\rho] \qquad \text{At } A, \quad a_n = \frac{(100/3.6)^2}{400} = 1.929 \text{ m/s}^2$$

$$\text{At } B, \quad a_n = 0$$

$$\text{At } C, \quad a_n = \frac{(50/3.6)^2}{80} = 2.41 \text{ m/s}^2$$

Forces can be found out by using equation of motion along *n*- and *t*-directions

Example (2) on curvilinear motion



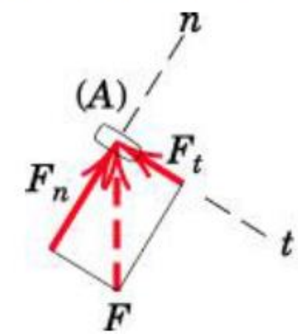
Solution:
Applying equation of motion to the FBD of the car along *n*- and *t*-directions

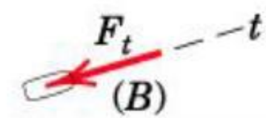
$[\Sigma F_t = ma_t]$ $\quad F_t = 1500(1.447) = 2170 \text{ N}$

$[\Sigma F_n = ma_n]$ $\quad$ At A, $\quad F_n = 1500(1.929) = 2890 \text{ N}$

At B, $\quad F_n = 0$

At C, $\quad F_n = 1500(2.41) = 3620 \text{ N}$



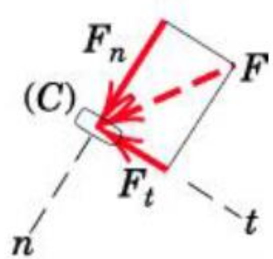


Total horz force acting on tires of car:

At A, $\quad F = \sqrt{F_n^2 + F_t^2} = \sqrt{(2890)^2 + (2170)^2} = 3620 \text{ N}$

At B, $\quad F = F_t = 2170 \text{ N}$

At C, $\quad F = \sqrt{F_n^2 + F_t^2} = \sqrt{(3620)^2 + (2170)^2} = 4220 \text{ N}$



Directions of forces will match with those of accelerations.

Kinetics of Particles

Work and Energy

•Second approach to solution of Kinetics problems

Work and Kinetic Energy

•Previous discussion: instantaneous relationship between the net force acting on a particle and the resulting acceleration of the particle.

- Change in velocity and corresponding displacement of the particle determined by integrating the computed accelerations using kinematic equations

•Cumulative effects of unbalanced forces acting on a particle

→Integration of the forces wrt displacement of the particle

→ leads to equations of work and energy

→ Integration of the forces wrt time they are applied

→ leads to equations of impulse and momentum

Kinetics of Particles: Work and Energy

Work and Kinetic Energy

Work

Work done by the force **F** during the displacement $d\mathbf{r}$

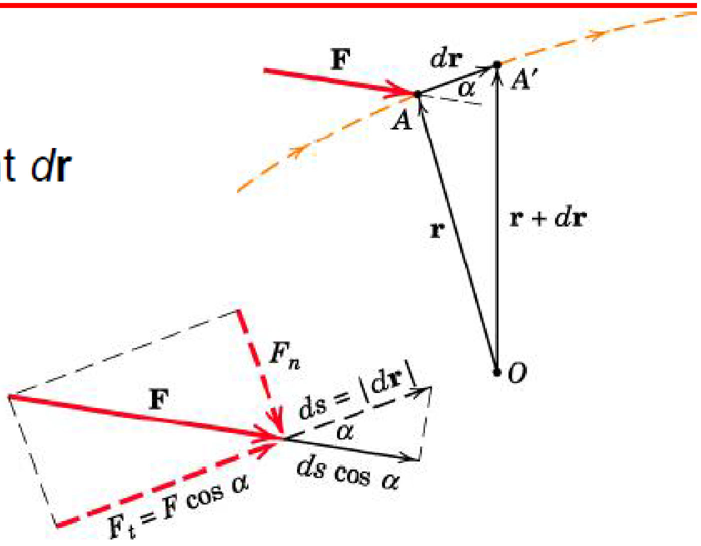
$dU = \mathbf{F} \cdot d\mathbf{r}$

$dU = F\, ds \cos\alpha$

The normal component of the force: $F_n = F \sin\alpha$ does no work.

→ $dU = F_t\, ds$

Units of Work: Joules (J) or Nm

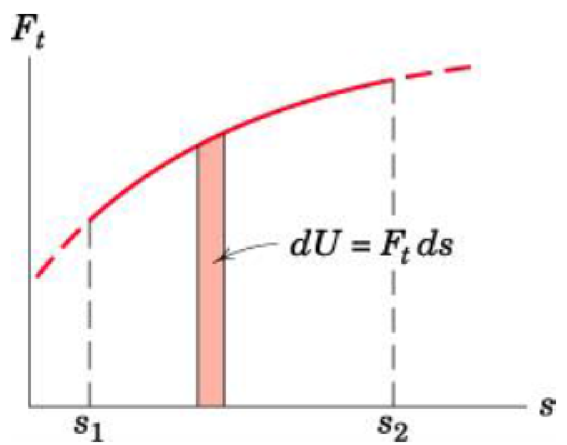


Calculation of Work:

$$U = \int_1^2 \mathbf{F} \cdot d\mathbf{r} = \int_1^2 \left(F_x dx + F_y dy + F_z dz\right)$$

or

$$U = \int_{s_1}^{s_2} F_t\, ds$$



Kinetics of Particles: Work and Energy

Work and Kinetic Energy

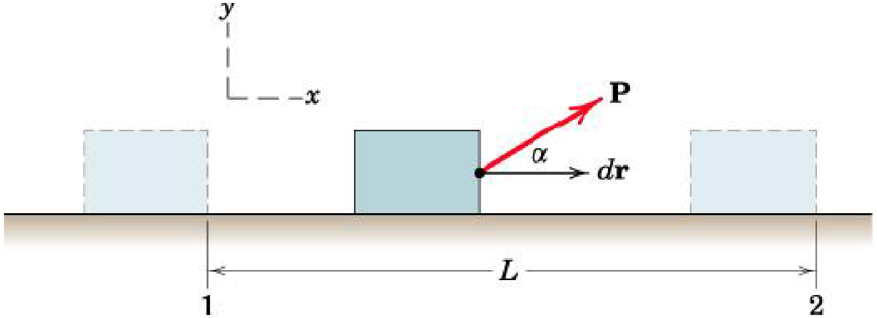
Examples of Work

Computing the work associated with three frequently associated forces:

Constant Forces, Spring Forces, and Weight

(a) Work associated with a constant external force

Work done by the constant force *P* on the body while it moves from position 1 to 2:



$$U_{1-2} = \int_1^2 \mathbf{F}\cdot d\mathbf{r} = \int_1^2 \left[(P\cos\alpha)\mathbf{i} + (P\sin\alpha)\mathbf{j}\right]\cdot dx\mathbf{i}$$

$$= \int_{x_1}^{x_2} P\cos\alpha\, dx = P\cos\alpha\,(x_2 - x_1)$$

$$= PL\cos\alpha$$

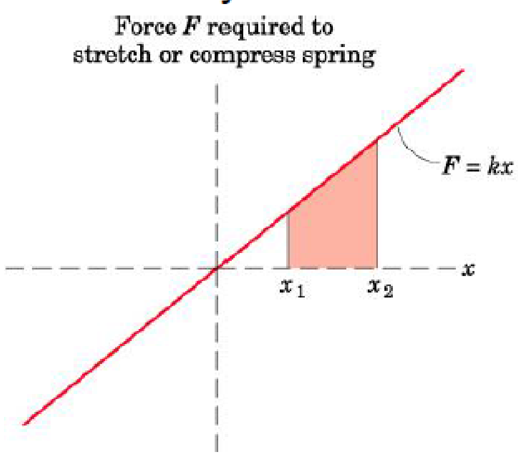
- The normal force *Psinα* does no work.
- Work done will be negative if *α* lies between 90^0 to 270^0

Kinetics of Particles: Work and Energy

Work and Kinetic Energy

Examples of Work (b) Work associated with a spring force

Force required to compress or stretch a linear spring of stiffness k is proportional to the deformation x. Work done by the spring force on the body while the body moves from initial position x_1 to final position x_2:

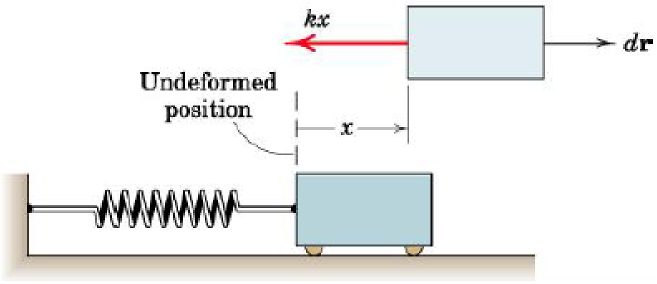


Force exerted by the spring on the body:

F = - *kx***i** (this is the force exerted on the body)

$$U_{1-2} = \int_1^2 \mathbf{F} \cdot d\mathbf{r} = \int_1^2 (-kx\mathbf{i}) \cdot dx\mathbf{i} = -\int_1^2 kx\, dx$$
$$= \frac{1}{2} k\left(x_1^2 - x_2^2\right)$$

- If the initial position x_1 is zero (zero spring deformation), work done is –ve for any final position $x_2 \neq 0$.
- If we move from an arbitrary initial posn $x_1 \neq 0$ to the undeformed final position $x_2 = 0$, work done will be positive (same dirn of force & disp)



Mass of the spring is assumed to be small compared to the masses of other accelerating parts of the system → no appreciable error in using the linear static relationship *F=kx*.

Kinetics of Particles: Work and Energy

Work and Kinetic Energy

Examples of Work

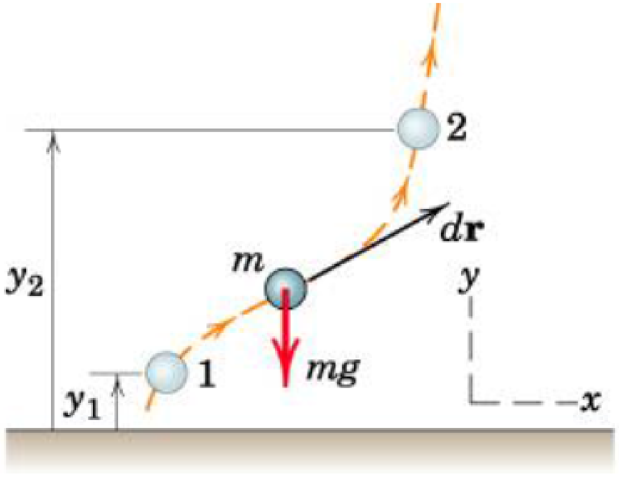
(c) Work associated with weight

Case (i) g = constant → altitude variation is sufficiently small

Work done by weight mg of the body as it is displaced from y_1 to final altitude y_2:

$$U_{1-2} = \int_1^2 \mathbf{F} \cdot d\mathbf{r} = \int_1^2 (-mg\mathbf{j}) \cdot (dx\mathbf{i} + dy\mathbf{j})$$

$$= -mg \int_{y_1}^{y_2} dy = -mg(y_2 - y_1)$$



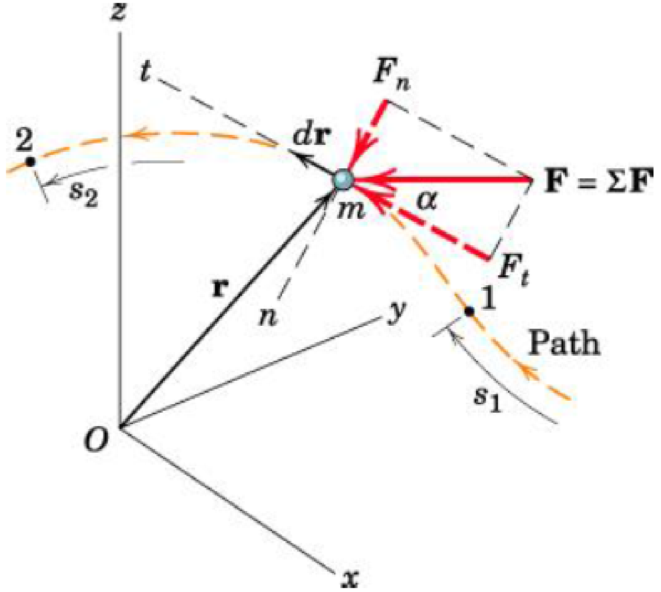
- Horz movement does not contribute to this work
- If the body rises (y_2-y_1 > 0) → Negative Work (opposite direction of force and displacement)
- If the body falls (y_2-y_1 < 0) → Positive Work (same direction of force and displacement)

Kinetics of Particles: Work and Energy

Work and Curvilinear Motion

Work done on a particle of mass *m* moving along a curved path (from 1 to 2) under the action of **F**:

- Position of m specified by position vector **r**
- Disp of *m* along its path during *dt* represented by the change *d***r** in the position vector.



$$U_{1-2} = \int_1^2 \mathbf{F} \cdot d\mathbf{r} = \int_{s_1}^{s_2} F_t \, ds$$

Substituting Newton's Second law **F** = *m***a**:

$$U_{1-2} = \int_1^2 \mathbf{F} \cdot d\mathbf{r} = \int_1^2 m\mathbf{a} \cdot d\mathbf{r}$$

$\mathbf{a} \cdot d\mathbf{r} = a_t \, ds$ a_t is the tangential component of acceleration of mass

Also, $a_t \, ds = v \, dv$

$$U_{1-2} = \int_1^2 m\mathbf{a} \cdot d\mathbf{r} = \int_{v_1}^{v_2} mv dv = \frac{1}{2} m\left(v_2^2 - v_1^2\right)$$

v_1 and v_2 are the velocities at points 1 and 2, respectively.

Kinetics of Particles: Work and Energy

Principle of Work and Kinetic Energy

The Kinetic Energy T of the particle is defined as:

$$T = \frac{1}{2}mv^2$$

Scalar quantity with units of Work (Joules or Nm)
T is always positive regardless of direction of velocity

Which is the total work required to be done on the particle to bring it from a state of rest to a velocity *v*.

Rewriting the equation for Work done: $$U_{1-2} = \frac{1}{2}m\left(v_2^2 - v_1^2\right)$$

→ $$U_{1-2} = T_2 - T_1 = \Delta T$$ May be positive, negative, or zero

→ Work Energy equation for a particle
"*Total Work Done by all forces acting on a particle as it moves from point 1 to 2 equals the corresponding change in the Kinetic Energy of the particle*"

→ Work always results in change in Kinetic Energy

Alternatively, the work-energy equation may be expressed as:

$$T_1 + U_{1-2} = T_2$$

→ Corresponds to natural sequence of events

Kinetics of Particles: Work and Energy

Work and Kinetic Energy

Advantages of Work Energy Method

- No need to compute acceleration; leads directly to velocity changes as functions of forces, which do work.
- Involves only those forces, which do work, and thus, produces change in magnitudes of velocities.
- Two or more particles connected by rigid and frictionless members can be analyzed without dismembering the system.
  - the internal forces in the connection will be equal and opposite
  - net work done by the internal forces = 0
  - the total kinetic energy of the system is the sum of the kinetic energies of both elements of the system

Method of Analysis:

- Isolate the particles of the system
- For a single particle, draw FBDs showing all externally applied forces
- For a system of particles connected without springs, draw Active Force Diagrams showing only those external forces which do work.

Kinetics of Particles: Work and Energy

Work and Kinetic Energy

Power

Capacity of a machine is measured by the time rate at which it can do work or deliver energy → Power (= time rate of doing work)

Power P developed by a force $\mathbf{F}$, which does an amount of work U:

$P = dU/dt = \mathbf{F} \cdot d\mathbf{r}/dt$

$d\mathbf{r}/dt$ is the velocity $\mathbf{v}$ at the point of application of the force

→ $P = \mathbf{F} \cdot \mathbf{v}$

- Power is a scalar quantity
- Units: Nm/s = J/s

  Special unit: Watt (W) [US customary unit: Horsepower (hp)]

  1 W = 1 J/s

  1 hp = 746 W = 0.746 kW

Kinetics of Particles: Work and Energy

Work and Kinetic Energy

Efficiency

Mechanical Efficiency of machine (e_m) = Ratio of the work done by a machine to the work done on the machine during the same interval of time

- Basic assumption: machines operates uniformly → no accumulation or depletion of energy within it.
- Efficiency is always less than unity due to loss of energy and since energy cannot be created within the machine.
- In mechanical devices, loss of energy due to negative work done by kinetic friction forces.

At any instant of time, mechanical efficiency and mechanical power are related by:

$$e_m = \frac{P_{output}}{P_{input}}$$

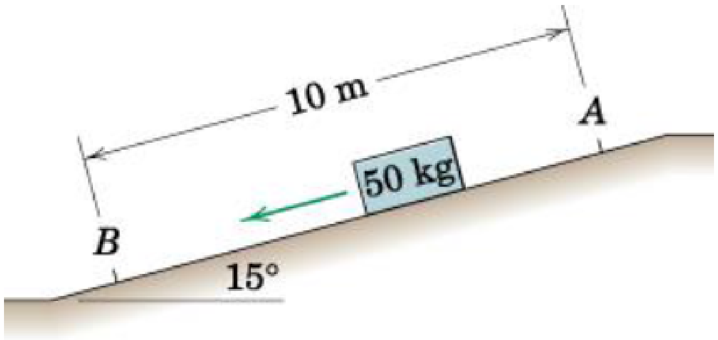
- Other energy losses are: electrical energy loss and thermal energy loss
  → electrical efficiency e_e and thermal efficiency e_t should also be considered

  Overall Efficiency: $e = e_m \, e_e \, e_t$

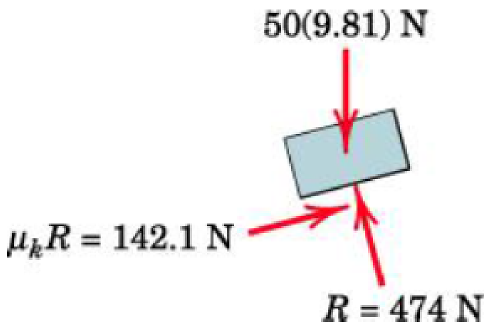
Kinetics of Particles: Work and Kinetic Energy

Example

Calculate the velocity of the 50 kg box when it reaches point B if it is given an initial velocity of 4 m/s down the slope at A. μ_k = 0.3. Use the principle of work.



Solution: Draw the FBD of the box



Normal reaction R = 50(9.81)cos15 = 474 N
Friction Force: $\mu_k R$ = 0.3x474 = 142.1 N
Work done by the weight will be positive and
Work done by the friction force will be negative.
Total work done on the box during the motion:
$U = Fs \rightarrow U_{1\text{-}2}$ = 50(9.81)(10sin15) – 142.1(10) = -151.9 J
Using work-energy equation:

$$T_1 + U_{1-2} = T_2$$

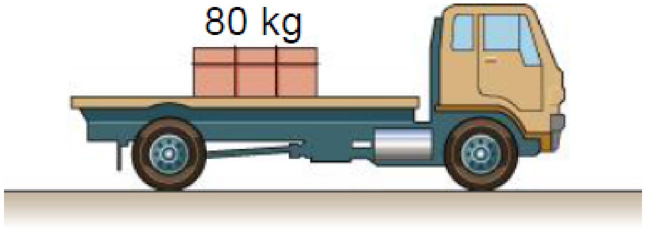
$$\frac{1}{2}mv_1^2 + U_{1-2} = \frac{1}{2}mv_2^2$$

$$\frac{1}{2}(50)(4)^2 - 151.9 = \frac{1}{2}(50)(v_2)^2 \quad \Rightarrow v_2 = 3.15\,\text{m/s}$$

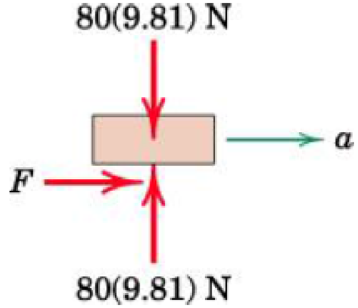
Work done is negative → velocity reduces → Kinetic Energy reduces

Kinetics of Particles: Work and Kinetic Energy

Example: A flatbed truck, which carries an 80kg crate, starts from rest and attains a speed of 72km/h in a distance of 75m on a level road with constant acceleration. Calculate the work done by the friction force acting on the crate during this interval if μ_s and μ_k between the crate and the truck bed are (a) 0.3 and 0.28, and (b) 0.25 and 0.2.



Solution: Draw the FBD of the crate

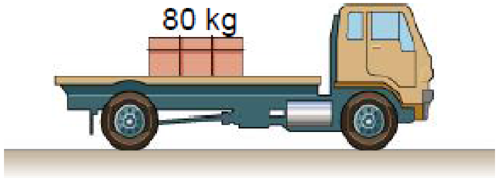


If the crate does not slip on the flatbed, accln of the crate will be equal to that of the truck:

$$[v^2 = 2as]$$

$$a = \frac{v^2}{2s} = \frac{(72/3.6)^2}{2(75)} = 2.67 \text{ m/s}^2$$

Kinetics of Particles: Work and Kinetic Energy



Example: **Solution**: Acceleration of the crate = 2.67 m/s²

Case (a): μ_s = 0.3, μ_k = 0.28

The accln of the crate requires a force (friction force) on the flatbed: $F = ma$ = (80)2.67 = 213 N

Maximum possible value of frictional force (limiting friction for impending motion):

$F_{lim} = \mu_s N$ = 0.3(80)(9.81) = 235 N which is more than F.

→ The crate does not slip and work done by the actual static friction force (213 N):

$U = Fs$ = 213(75) = 16000 J = 16 kJ

Case (b): μ_s = 0.25, μ_k = 0.20

The accln of the crate requires a force (friction force) on the flatbed:

$F = ma$ = (80)2.67 = 213 N

Maximum possible value of frictional force (limiting friction for impending motion):

$F_{lim} = \mu_s N$ = 0.25(80)(9.81) = 196.2 N which is less than F required for no slipping.

→ The crate slips, and the actual friction force is: $F = \mu_k N$ = 0.2(80)(9.81) = 157 N

→ And the actual accln of the crate becomes: a = F/m = 157/80 = 1.962 m/s²

The distances travelled by the crate and the truck are in proportion to their acclns.

→ Crate has a displacement of: (1.962/2.67)75 = 55.2 m.

→ Work done by the kinetic friction: $U = Fs$ = 157(55.2) = 8660 J = 8.66 kJ

Kinetics of Particles: Work and Energy

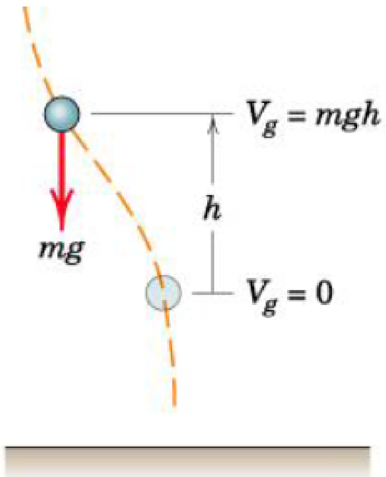
Potential Energy

- In work energy method, work done by gravity forces, spring forces, and other externally applied forces was determined by isolating particles.
- Potential Energy approach can be used to specifically treat the work done by gravity forces and spring forces → Simplify analysis of many problems.

Gravitational Potential Energy

- Motion in close proximity to earth's surface → g constant
- The gravitational potential energy of a particle V_g = work done (mgh) against the gravitational field to elevate the particle a distance h above some arbitrary reference plane, where V_g is taken as zero → $V_g = mgh$

This work is called potential energy because it may be converted into energy if the particle is allowed to do work on supporting body while it returns to its lower original datum.



In going from one level h_1 to higher level h_2, change in potential energy: $\Delta V_g = mg(h_2 - h_1) = mg\Delta h$

The corresponding work done by the gravitational force on particle is $-mg\Delta h$ → work done by the gravitational force is the negative of the change in V_g.

Kinetics of Particles: Work and Energy

Potential Energy

Elastic Potential Energy

- Work done on linear elastic spring to deform it is stored in the spring and is called its elastic potential energy V_e.
- Recoverable energy in the form of work done by the spring on the body attached to its movable end during release of the deformation of spring.

Elastic potential energy of the spring = work done on it to deform at an amount x:

$$V_e = \int_0^x kx\,dx = \frac{1}{2}kx^2$$

k is the spring stiffness

If the deformation of the spring increases from x_1 to x_2:

Change in potential energy of the spring is final value minus initial value:

$$\Delta V_e = \frac{1}{2}k({x_2}^2 - {x_1}^2)$$

Always positive as long as deformation increases

If the deformation of spring decreases during the motion interval → negative Δv_e

Force exerted on spring by moving body is equal and opposite to the force exerted by the spring on the body → work done on the spring is the negative of the work done on the spring

→ Replace work done U by the spring on the body by $-V_e$, negative of the potential energy change for the spring → the spring will be included in the system

Kinetics of Particles: Work and Energy

Potential Energy: **Work-Energy Equation**

Total work done is given by: $U_{1-2} = T_2 - T_1 = \Delta T$

Modifying this eqn to account for the potential energy terms:

$U'_{1-2} + (-\Delta V_g) + (-\Delta V_e) = \Delta T \rightarrow$ $U'_{1-2} = \Delta T + \Delta V$

U'_{1-2} is work of all external forces other than the gravitational and spring forces (Gravitational and spring forces are also known as Conservative Forces and all other external forces that do work are also known as Non-Conservative Forces)

ΔT is the change in kinetic energy of the particle

ΔV is the change in total potential energy

- The new work-energy equation is often far more convenient to use because only the end point positions of the particle and end point lengths of elastic spring are of significance.

Further, following the natural sequence of events: $T_1 + V_1 + U'_{1-2} = T_2 + V_2$

If the only forces acting are gravitational, elastic, and nonworking constraint forces $\rightarrow$ U'_{1-2} term will be zero, and the energy equation becomes:

$T_1 + V_1 = T_2 + V_2 \quad or \quad E_1 = E_2$

E = T+V is the total mechanical energy of the particle and its attached spring

$\rightarrow$ This equation expresses the "Law of Conservation of Dynamical Energy"

Kinetics of Particles: Work and Energy

Conservation of Energy $T_1 + V_1 = T_2 + V_2 \quad or \quad E_1 = E_2$

- During the motion sum of the particle's kinetic and potential energies remains constant. For this to occur, kinetic energy must be transformed into potential energy, and vice versa.

A ball of weight *W* is dropped from a height *h* above the ground (datum)

- PE of the ball is maximum before it is dropped, at which time its KE is zero. Total mechanical energy of the ball in its initial position is:

$$E = T_1 + V_1 = 0 + Wh = Wh$$

- When the ball has fallen a distance *h*/2, its speed is: $v^2 = v_0^2 + 2a_c(y - y_0)$

  $v = \sqrt{2g(h/2)} = \sqrt{gh}.$

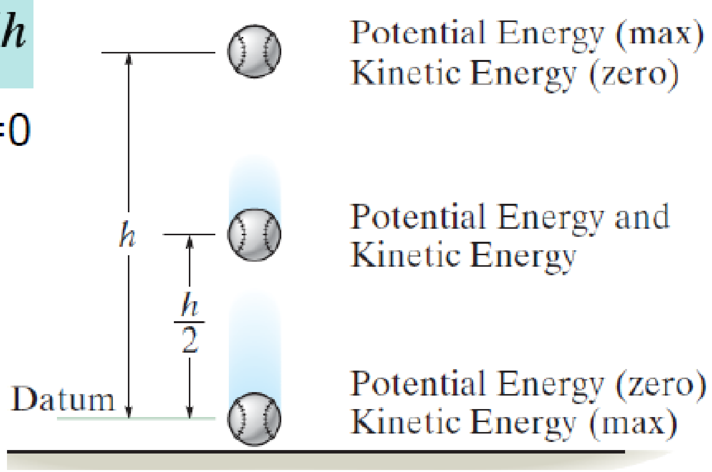
  Energy of the ball at mid-height position:

$$E = T_2 + V_2 = \frac{1}{2}\frac{W}{g}\left(\sqrt{gh}\right)^2 + W\left(\frac{h}{2}\right) = Wh$$

- Just before the ball strikes the ground, its PE=0 and its speed is: $v = \sqrt{2gh}$

  The total mechanical energy of the ball:

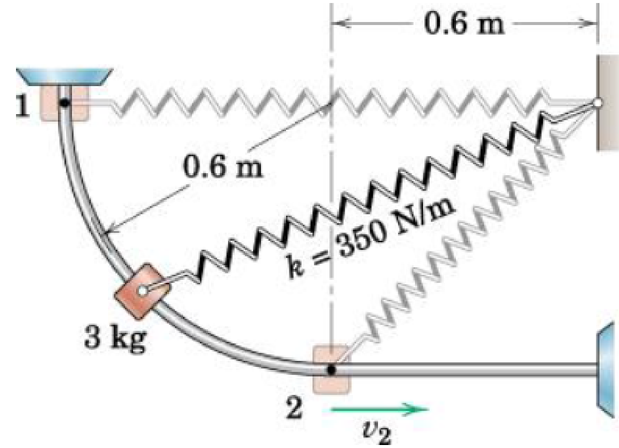
$$E = T_3 + V_3 = \frac{1}{2}\frac{W}{g}\left(\sqrt{2gh}\right)^2 + 0 = Wh$$



Kinetics of Particles: Work and Energy

Potential Energy

Example: A 3 kg slider is released from rest at position 1 and slides with negligible friction in vertical plane along the circular rod. Determine the velocity of the slider as it passes position 2. The spring has an unstretched length of 0.6 m.



Solution:

•**Reaction** of rod on slider is **normal** to the **motion** → **does no work** → $U'_{1\text{-}2} = 0$
Defining the datum to be at the level of position 1
Kinetic Energy: $T_1 = 0$ and $T_2 = \frac{1}{2}(3)(v_2)^2$
Gravitational Potential Energies:
$V_1 = 0$ and $V_2 = -mgh = -3(9.81)(0.6) = -17.66$ J
Initial and final elastic potential energies:
$V_1 = \frac{1}{2}kx_1^2 = 0.5(350)(0.6)^2 = 63$J and $V_2 = \frac{1}{2}kx_2^2 = 0.5(350)(0.6\sqrt{2} - 0.6)^2 = 10.81$J

$$T_1 + V_1 + U'_{1-2} = T_2 + V_2$$

→$0 + (0+63) + 0 = \frac{1}{2}(3)(v_2)^2 + (-17.66 + 10.81)$
→$v_2 = 6.82$ m/s