Applications of the Serret-Frenet Equations
Math 473
Introduction to Differential Geometry
Lecture 9

Dr. Nasser Bin Turki

King Saud University
Department of Mathematics

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Plane Curve

Definition(1): Plane Curve
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Definition(2):

Let $\alpha : I \mapsto \mathbb{R}^3$ be a regular curve. We say α is a plane curve if there is unit vector such that

$$\overrightarrow{(\alpha(t) - \alpha(0))} \bullet \overrightarrow{u} = 0$$

Theorem (1):
Let $\alpha : I \mapsto \mathbb{R}^3$ be a unit speed curve with $\kappa > 0$ and S is the arc-length. Then, α is a plane curve if and only if $\tau(t) = 0$ for all $t \in I$.

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Proof:

Theorem (2):
Let $\alpha : I \mapsto \mathbb{R}^3$ be a unit speed curve with $\kappa(t) > 0$ for all $t \in I$. If we have $\kappa(t) = \lambda$, where λ is constant, and $\tau(t) = 0$ for all $t \in I$, then the curve α is part of circle whose radius is $\frac{1}{\lambda}$.

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Thanks for listening.