

# General Exponential and Logarithmic Functions

## Math 106

### Lecture 9

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Def: General Exponential Function:

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when  $a = e$ , we will obtain that

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Then the function is increasing since  $x$  increased.
- $a < 1 \rightarrow \ln a < 0$   
Then the function is decreasing.
- $a = 1 \rightarrow f(x) = 1$



$$f(x) = a^x$$

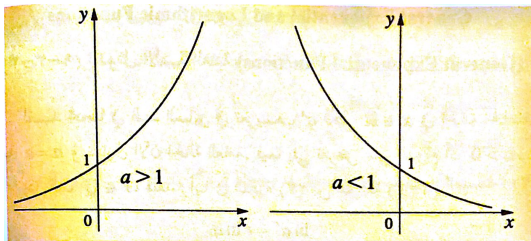


Figure:  $a^x$ .

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- $a^x b^x = (ab)^x$ .

## Derivative of General Exponential Function :

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Thm:

$$\frac{d}{dx} (x^p) = px^{p-1}, \quad \forall x \in \mathbb{R}^+, \forall p \in \mathbb{R}.$$

EX: Find the derivative:

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$$(2) \int \frac{9^{\sqrt{x}}}{\sqrt{x}} dx.$$

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- $\forall x, y \in \mathbb{R}^+, r \in \mathbb{R}$ :

$$\log_a(xy) = \log_a(x) + \log_a(y),$$

$$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y),$$

$$\log_a(x^r) = r \log_a x.$$

$$\log_a(x)$$

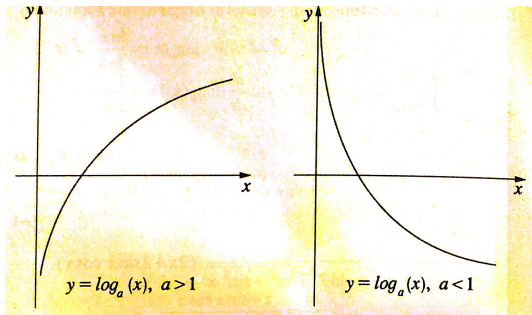


Figure:  $\log_a(x)$ .

## Derivative of General Logarithmic Function :

Thm:

$$\frac{d}{dx} \log_a(x) = \frac{1}{\ln a} \left( \frac{1}{x} \right),$$

$$\frac{d}{dx} \log_a f(x) = \frac{1}{\ln a} \left( \frac{f'(x)}{f(x)} \right).$$

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$$(2)y = \log(\sin x).$$

Integration of General Logarithmic Function:

$$\int \frac{1}{x \ln x} dx = \log_a(x) + c.$$

EX: find the integral:

$$(1) \int \frac{dx}{x \log x}$$

*Thanks for listening.*