Serret-Frenet Equations for Space Curve
Math 473
Introduction to Differential Geometry
Lecture 8

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Theorem (1):
Let $\alpha : I \mapsto \mathbb{R}^3$ be a unit speed curve with $\kappa = 0$ on the interval I. Then the curve segment α is a straight line.

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Proof:

Theorem (2):
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$$T' = \kappa \cdot s \cdot N,$$
$$N' = -\kappa \cdot s \cdot T + \tau \cdot s \cdot B,$$
$$B' = -\tau \cdot s \cdot N.$$

Serret-Frenet Equations

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For a unit speed curve, we have $s = |\alpha'| = 1$ and the Serret-Frenet equations have the following form:

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T' &= \kappa \cdot N, \\
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Remark(1): The Serret-Frenet equations can be written in the following matrix form

$$\begin{pmatrix} T' & N' & B' \end{pmatrix} = |\alpha'| \cdot \begin{pmatrix} T & N & B \end{pmatrix} \cdot \begin{pmatrix} 0 & -\kappa & 0 \\ \kappa & 0 & -\tau \\ 0 & \tau & 0 \end{pmatrix}.$$

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Remark(2): For a unit speed curve we have $s = |\alpha'| = 1$ and the Serret-Frenet equations can be written as

$$\begin{pmatrix} T' & N' & B' \end{pmatrix} = \begin{pmatrix} T & N & B \end{pmatrix} \cdot \begin{pmatrix} 0 & -\kappa & 0 \\ \kappa & 0 & -\tau \\ 0 & \tau & 0 \end{pmatrix}.$$

Proof of Theorem (2) (for α is a unit speed):

Thanks for listening.