Faculty of Engineering
Mechanical Engineering Department

CALCULUS FOR ENGINEERS
MATH 1110

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Integration by Substitution

Antidifferentiation of a Composite Function

Let f and g be functions such that $f \circ g$ and g' are continuous on an interval I. If F is an antiderivative of f on I, then

$$\int \underbrace{f}_{\text{Outside}}(\overbrace{g(x)}^{\text{Inside}})\underbrace{g'(x)}_{\text{Derivative of Inside}}dx = F(g(x)) + C$$

Evaluate $\int \left(x^2+1\right)^2 (2x)\,dx$

Let u = x^2 + 1

du = 2x dx

$$\frac{du}{2x} = dx$$

$$\int (u)^2 \cancel{(2x)} \frac{du}{\cancel{2x}}$$

$$= \frac{u^3}{3} + C \quad = \frac{\left(x^2+1\right)^3}{3} + C$$

Evaluate $\int 5\cos 5x\,dx$

Let u = 5x

du = 5 dx

$$\frac{du}{5} = dx$$

$$= \int \cancel{5} \cos u \frac{du}{\cancel{5}}$$

$$= \sin u + C \quad = \sin 5x + C$$

Multiplying and dividing by a constant

$$\int x\left(x^2+1\right)^2 dx$$

Let u = x² + 1

du = 2x dx

$$=\int \cancel{x}(u)^2 \frac{du}{2\cancel{x}} = \frac{u^3}{6} + C \boxed{= \frac{\left(x^2+1\right)^3}{6} + C}$$

$$\frac{du}{2x} = dx$$

$$\int \sqrt{2x-1}\,dx = \int \sqrt{u}\,\frac{du}{2} = \int u^{1/2}\frac{du}{2}$$

Let u = 2x - 1

du = 2dx

$$\frac{du}{2} = dx$$

$$= \frac{1}{\cancel{2}}\frac{\cancel{2}u^{3/2}}{3} + C \boxed{= \frac{(2x-1)^{3/2}}{3} + C}$$

Substitution and the General Power Rule
What would you let u = in the following examples?

$$\int 3(3x-1)^4 dx$$ **u = 3x - 1**

$$\int (2x+1)\left(x^2+x\right)dx$$ **u = x^2 + x**

$$\int 3x^2\sqrt{x^3-2}\, dx$$ **u = x^3 - 2**

$$\int \frac{-4x}{\left(1-2x^2\right)}dx$$ **u = 1 – $2x^2$**

$$\int \cos^2 x \sin x \, dx$$ **u = cos x**

A differential equation, a point, and slope field are given. Sketch the solution of the equation that passes through the given point. Use integration to find the particular solution of the differential equation. $\dfrac{dy}{dx} = x^2(x^3-1)^2 \quad (1,0)$

$$\int x^2\left(x^3-1\right)^2 dx \qquad u = x^3 - 1$$

$$du = 3x^2 dx$$

$$\int x^2(u)^2 \frac{du}{3x^2} \qquad \frac{du}{3x^2} = dx$$

$$\int \frac{u^2 du}{3} = \frac{u^3}{9} + C = \frac{\left(x^3-1\right)^3}{9} + C$$

$$0 = \frac{1^3-1}{9} + C$$

$$y = \frac{\left(x^3-1\right)^3}{}$$

$$C = 0$$

$$\int x^2 \sin x^3 dx$$

u = x^3

du = 3x^2 dx

$$= \int \cancel{x^2} \sin u \, \frac{du}{\cancel{3x^2}}$$

$$\frac{du}{3x^2} = dx$$

$$= \frac{1}{3} \int \sin u \; du \quad = \frac{1}{3}(-\cos u) + C$$

$$\boxed{= -\frac{1}{3} \cos x^3 + C}$$

$$\int \sin^2 3x \cos 3x \, dx$$

rewritten as

$$\int (\sin 3x)^2 \cos 3x \, dx$$

Let u = sin 3x

du = 3cos 3x dx

$$\frac{du}{3\cos 3x} = dx$$

$$= \int u^2 \cancel{\cos 3x} \frac{du}{3\cancel{\cos 3x}} \qquad = \frac{1}{3}\int u^2 du$$

$$= \frac{u^3}{9} + C \quad \boxed{= \frac{\sin^3 3x}{9} + C}$$

$$\int x\sqrt{2x-1}\,dx$$

Let u = 2x - 1

du = 2dx

$$\frac{du}{2} = dx$$

u + 1 = 2x

$$\frac{u+1}{2} = x$$

$$= \int \left(\frac{u+1}{2}\right)\sqrt{u}\,\frac{du}{2}$$

$$= \frac{1}{4}\int \left(u^{3/2} + u^{1/2}\right)du = \frac{1}{4}\left(\frac{2u^{5/2}}{5} + \frac{2u^{3/2}}{3}\right) + C$$

$$\boxed{= \frac{(2x-1)^{5/2}}{10} + \frac{(2x-1)^{3/2}}{6} + C}$$

Evaluate

$$\int_0^1 x(x^2+1)^3 dx$$

u = x^2 + 1

du = 2x dx

$$\frac{du}{2x} = dx$$

$$= \int \cancel{x}(u)^3 \frac{du}{2\cancel{x}}$$

Note that there are no upper and lower limits of integration.

$$= \frac{u^4}{8}\Bigg]_1^2$$

We must determine new upper and lower limits by substituting the old ones in for x in u = x^2 + 1

Or, we could use the old limits if we substitute x^2 + 1 back in.

$$= \frac{16}{8} - \frac{1}{8} = \frac{15}{8}$$

$$= \frac{(x^2+1)^4}{8}\Bigg]_0^1 = \frac{16}{8} - \frac{1}{8} = \frac{15}{8}$$

$$Area = \int_1^5 \frac{x}{\sqrt{2x-1}}\,dx$$

$$u = \sqrt{2x-1}$$

$$u^2 = 2x-1$$

2u du = 2 dx

u du = dx

$$x = \frac{u^2+1}{2}$$

$$= \int \frac{u^2+1}{2u}\,(udu)$$

What limits are we going to use?

$$= \frac{1}{2}\int \left(u^2+1\right)du \;= \frac{1}{2}\left(\frac{u^3}{3}+u\right)\Bigg]_1^3$$

$$= \frac{1}{2}\left[\left(\frac{27}{3}+3\right)-\left(\frac{1}{3}+1\right)\right] \;\boxed{= \frac{16}{3}}$$

See area comparisons when using different upper and lower limits. Page 302

Integration of Even and Odd Functions

If f is an <u>even function</u>, then

$$\boxed{\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx}$$

Ex. $$\int_{-2}^{2} x^3 dx = 0$$

Odd or Even?

If f is an <u>odd function</u>, then

$$\boxed{\int_{-a}^{a} f(x)dx = 0}$$