Reparametrisation
Math 473
Introduction to Differential Geometry
Lecture 4

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September 15, 2018

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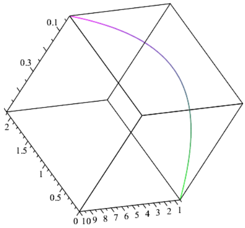
Proof

Remark

Remark The reparametrisation of the curve α in the previous Theorem (1) is called **normal Reparametrisation** of α.

Example(1) Reparametrisation the curve $\alpha(t) = \frac{1}{2}(t, \frac{1}{t}, \sqrt{2}\ln t)$, where $t \in (0, \infty)$, (Taken in Example 2(Lecture 3)) using the arc-length?(Find the normal reparametrisation of α).



Example(2) For the regular curve $\alpha(t) = (a\cos t, a\sin t, bt)$ where $a, b \in \mathbb{R}^*$.

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Example(3) Show that the curve $\alpha(t) = (\sin(e^t), \cos(e^t), \sqrt{3}e^t)$ where $t \geq 0$ is regular.

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Thanks for listening.