Tangent Surface, Involutes and Evolutes Math 473 Introduction to Differential Geometry Lecture 17

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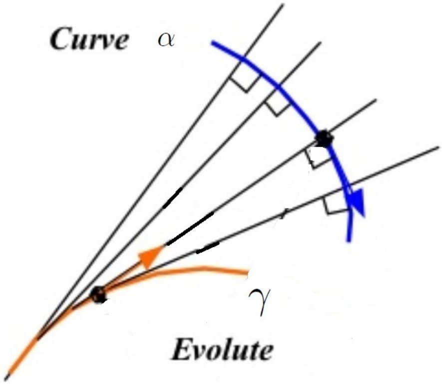
Definition (1):
Let α and γ be two regular curves defined on an interval I. The curve γ is an **evolute** of α if α is an involute of γ.



Lemma (1):

The formula of the curve γ which is evolute of α is

$$\gamma(t) = \alpha(t) + \frac{1}{\kappa_\alpha(t)} N_\alpha(t) + \frac{1}{\kappa_\alpha(t)} \cot\left(\int \tau_\alpha(t)dt + c\right) B_\alpha(t),$$

(This is the general formula).

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(This is the general formula).

Special case: If $\tau = 0$, then the formula of the curve γ which is evolute of α is

$$\gamma(t) = \alpha(t) + \frac{1}{\kappa_\alpha(t)} N_\alpha(t) + \frac{1}{\kappa_\alpha(t)} \cot c B_\alpha(t),$$

where $B_\alpha(t)$ is constant.

Proof of the general formula:

Evolute curve in the plane

Let $\alpha : I \mapsto \mathbb{R}^2$ be a regular curve with $\kappa > 0$.

$\ast$ If α is a unite speed, then the formula of the curve γ which is evolute of α is

$$\gamma(t) = \alpha(t) + \frac{1}{\kappa_\alpha(t)} N_\alpha(t).$$

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$\ast$ If α is a unite speed, then the formula of the curve γ which is evolute of α is

$$\gamma(t) = \alpha(t) + \frac{1}{\kappa_\alpha(t)} N_\alpha(t).$$

$\ast$ if α is not unit speed, then the formula of the curve γ which is evolute of α is

$$\gamma(t) = \alpha(t) + \frac{|\alpha'|^2 \omega(\alpha')}{(\alpha'' \bullet \omega(\alpha'))},$$

where $\omega : \mathbb{R}^2 \mapsto \mathbb{R}^2, \;\; \omega((x, y)) \mapsto (-y, x)$.

Examples

Example(1) Let $\alpha(t) = (\cos t, \sin t)$. Find the evolute curve by α.

Example(2) Let $\alpha(t) = (t, t^2)$. Find the evolute curve by α.

Thanks for listening.