Osculating, Rectifying and Normal Planes
Math 473
Introduction to Differential Geometry
Lecture 11

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October 6, 2018

Osculating, Rectifying and Normal Planes

Let $\alpha : I \mapsto \mathbb{R}^3$ be a unit regular parametrised space curve. Let $t \in I$ be such that $\kappa(t) \neq 0$. Let $(T(t), N(t), B(t))$ be the Serret-Frenet basis of α at t.

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The **osculating plane** of α at t is the plane through the point $\alpha(t)$ spanned by $T(t)$ and $N(t)$.

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Definition (3): Normal Plane

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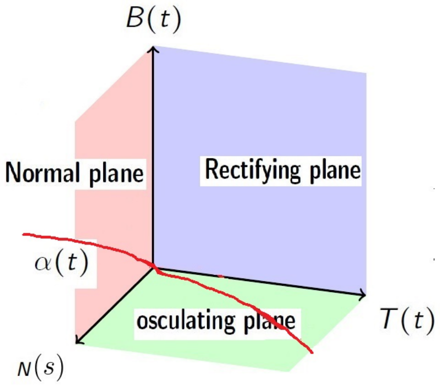
B(t)
Normal plane
Rectifying plane
α(t)
osculating plane
T(t)
N(s)

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(3) The Normal plane of α at t is the plane trough the point $\alpha(t)$ which is orthogonal to the vector $T(t)$.

Remark(2): Let $\alpha : I \mapsto \mathbb{R}^3$ be a unit regular parametrised space curve. Let $t \in I$ be such that $\kappa(t) \neq 0$. Let $(T(t), N(t), B(t))$ be the Serret-Frenet basis of α at t.

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(1) The equation of the osculating plane of α at $\alpha(t_0)$ is

$$\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \alpha(t_0) \bullet B(t_0)\right) = 0$$

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(3) The equation of the Normal plane of α at $\alpha(t_0)$ is

$$\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \alpha(t_0) \bullet T(t_0)\right) = 0$$

Example(1):

Let $\alpha : I \mapsto \mathbb{R}^3$ be given by $\alpha(t) = (\cos\frac{t}{\sqrt{2}}, \sin\frac{t}{\sqrt{2}}, \frac{t}{\sqrt{2}})$.

(i) Find the equation of the osculating plane of α at $(1, 0, 0)$.

(ii) Find the equation of the Rectifying plane of α at $(1, 0, 0)$.

(iii) Find the equation of the Normal plane of α at $(1, 0, 0)$.

Thanks for listening.