Applications of the Serret-Frenet Equations Math 473 Introduction to Differential Geometry Lecture 10

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We can use Serret-Frenet equations to prove the useful formulas for curvature and torsion.

Theorem (1):
For a regular parametrised space curve $\alpha : I \mapsto \mathbb{R}^3$ the curvature κ and the torsion τ can be computed as

$$\kappa = \frac{|\alpha' \times \alpha''|}{|\alpha'|^3}, \quad \tau = \frac{[\alpha', \alpha'', \alpha''']}{|\alpha' \times \alpha''|^2},$$

where $[\alpha'\alpha'', \alpha''']$ is the triple scalar product given by
$[\alpha', \alpha'', \alpha'''] = (\alpha' \times \alpha'') \bullet \alpha'''$.

Proof:

Theorem (2):
The Serret-Frenet basis can also be computed as

$$T = \frac{\alpha'}{|\alpha'|}, \quad B = \frac{\alpha' \times \alpha''}{|\alpha' \times \alpha''|}, \quad N = B \times T.$$

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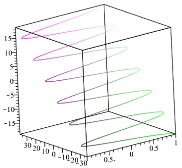
Example(1):

Let $\alpha : \mathbb{R} \mapsto \mathbb{R}^3$ be given by $\alpha(t) = (2t + \sin t, \cos t, t)$. Compute the velocity and the speed of α. Show that α is a regular space curve. Compute the unit tangent T, the binormal B, the principal normal N, the curvature κ and the torsion τ of α. You need not attempt to simplify the expressions obtained.



Thanks for listening.