مثال ١ جد التكامل التالي

$$I = \int \frac{dx}{\sqrt{x}\left(\sqrt{x}+1\right)^4} = \int \left(\sqrt{x}+1\right)^{-4} \frac{1}{\sqrt{x}}\, dx$$

نفرض أن

$$u = \sqrt{x}+1 \Rightarrow du = \frac{1}{2\sqrt{x}}\, dx$$

$$\therefore\ 2\,du = \frac{1}{\sqrt{x}}\, dx$$

$$\therefore\ I = \int \left(\sqrt{x}+1\right)^{-4} \cdot \frac{1}{\sqrt{x}}\, dx = \int u^{-4} \cdot 2\,du$$

$$= 2\int u^{-4}\, du \quad , \quad \boxed{\int u^{r}\, du = \frac{u^{r+1}}{r+1},\ r \neq -1}$$

$$= 2\,\frac{u^{-3}}{-3} + C$$

$$= -\frac{2}{3}\left(\sqrt{x}+1\right)^{-3} + C$$

بشكل عام فإن:

$$\int \left[f(x)\right]^{r} \cdot f'(x)\, dx = \frac{\left[f(x)\right]^{r+1}}{r+1} + C, \quad r \neq -1.$$

مثال ١ احسب التكاملات التالية:

① $$\int (x^3+2)^7 \cdot x^2\,dx = \frac{1}{3}\int (x^3+2)^7 \cdot 3x^2\,dx$$

$$= \frac{1}{3}\frac{(x^3+2)^8}{8} + C$$

② $$\int \frac{\sqrt{1+\frac{1}{x}}}{x^2}\,dx = -\int \left(1+\frac{1}{x}\right)^{\frac{1}{2}} \cdot \frac{-1}{x^2}\,dx$$

$$= -\frac{\left(1+\frac{1}{x}\right)^{\frac{3}{2}}}{\frac{3}{2}} + C$$

③ $$\int \frac{\sec^2 x}{(5+3\tan x)^4}\,dx = \frac{1}{3}\int (5+3\tan x)^{-4} \cdot 3\sec^2 x\,dx$$

$$= \frac{1}{3}\frac{(5+3\tan x)^{-3}}{-3} + C$$

② $$\int \frac{\sec\sqrt{x}\,\tan\sqrt{x}}{\sqrt{x}\left[1+5\sec\sqrt{x}\right]^8}\,dx$$

$$= 2\cdot\frac{1}{5}\int \left[1+5\sec\sqrt{x}\right]^{-8} \cdot 5\sec\sqrt{x}\,\tan\sqrt{x} \cdot \frac{1}{2\sqrt{x}}\,dx$$

$$= \frac{2}{5}\frac{\left[1+5\sec\sqrt{x}\right]^{-7}}{-7} + C.$$

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مثال ١ احسب التكاملات التالية:

① $\int \frac{t}{\sqrt{1+t}}\,dt = \int (1+t)^{-\frac{1}{2}} \cdot t\,dt$

نفرض أن:

$u = 1+t \Rightarrow du = dt$

$\quad \rightarrow t = u-1$

$\therefore \int (1+t)^{-\frac{1}{2}}\, t\,dt = \int u^{-\frac{1}{2}} \cdot (u-1) \cdot du$

?

$= \int (u^{\frac{1}{2}} - u^{-\frac{1}{2}})\,du$

$= \frac{2}{3} u^{\frac{3}{2}} - \frac{2}{1} u^{\frac{1}{2}} + C$

$= \frac{2}{3}(1+t)^{\frac{3}{2}} - 2(1+t)^{\frac{1}{2}} + c$

② $\int_1^2 \frac{dx}{\sqrt{x}\sqrt{4-x}} = \int_1^2 \frac{1}{\sqrt{4-x}} \cdot \frac{1}{\sqrt{x}}\,dx \quad , (\sqrt{x})^2 = x$

$= \int_1^2 \frac{1}{\sqrt{4-(\sqrt{x})^2}} \cdot \frac{1}{\sqrt{x}}\,dx$

نفرض أن: $u = \sqrt{x} \Rightarrow du = \frac{1}{2\sqrt{x}}\,dx$

$\therefore 2\,du = \frac{1}{\sqrt{x}}\,dx$

$\therefore \int \frac{1}{\sqrt{4-(\sqrt{x})^2}} \frac{1}{\sqrt{x}}\,dx = \int \frac{1}{\sqrt{4-u^2}} \cdot 2\,du$

$= 2\int \frac{1}{\sqrt{4-u^2}}\,du$

$= 2\sin^{-1}\left(\frac{u}{2}\right)$

$= 2\sin^{-1}\left(\frac{\sqrt{x}}{2}\right)$

$\therefore \int_1^2 \frac{dx}{\sqrt{x}\sqrt{4-x}} = \left[2\sin^{-1}\left(\frac{\sqrt{x}}{2}\right)\right]_1^2$

$= 2\left[\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) - \sin^{-1}\left(\frac{1}{2}\right)\right]$

$= 2\left[\frac{\pi}{4} - \frac{\pi}{6}\right]$

③ $\int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}}\,dx$

نفرض أن:

$u = \tan x \Rightarrow du = \sec^2 x\,dx$

$= \int \frac{1}{\sqrt{1-u^2}}\,du$

$= \sin^{-1}(u) + C = \sin^{-1}(\tan x) + C$

④ $\int \frac{1}{x\sqrt{x^6-25}}\,dx = \int \frac{1}{x\sqrt{(x^3)^2-25}}\,dx$

نفرض أن:

$u = x^3 \Rightarrow du = 3x^2\,dx$

$= \int \frac{1}{3x^2 \cdot x\sqrt{(x^3)^2-25}}\, 3x^2\,dx$

$= \int \frac{1}{3x^3\sqrt{u^2-25}}\,du$

$= \frac{1}{3}\int \frac{1}{\ \ }\,du = \frac{1}{3} \cdot \frac{1}{5}\sec^{-1}\left(\frac{u}{5}\right) + c$

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واجب:

اوجد التكاملات التالية:

① $\int \frac{\csc^2 x}{4+\cot^2 x}\,dx$

② $\int \frac{(2-\sqrt{x})^5}{\sqrt{x}}\,dx$

③ $\int \frac{\sin(2x)}{\sqrt{9+\cos^2 x}}\,dx$

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