* $\frac{d}{dx}(\ln x) = \frac{1}{x}$

* $\frac{d}{dx}\ln(f(x)) = \frac{1}{f(x)} \cdot f'(x) = \frac{f'(x)}{f(x)}$

<u>مثال</u> أوجد مشتقة الدوال التالية:

① $y = x^2 \ln x$

$\Rightarrow y' = 2x\ln x + x^2 \cdot \frac{1}{x} = 2x\ln x + x$

② $y = \sin(\ln x)$

$\Rightarrow y' = \cos(\ln x) \cdot \frac{1}{x}$

③ $y = (x^2 + \ln x)^5$

$\Rightarrow y' = 5(x^2 + \ln x)^4 \cdot \left(2x + \frac{1}{x}\right)$

④ $y = \ln(x^4 + 5x - 1)$

$\Rightarrow y' = \frac{4x^3 + 5}{x^4 + 5x - 1}$

⑤ $y = \ln(\tan^{-1} x)$

$\therefore y' = \frac{1}{\tan^{-1} x} \cdot \frac{1}{1 + x^2}$

⑥ $x\ln y + y^2 \ln x = 1$ (دالة ضمنية)

$\Rightarrow 1 \cdot \ln y + x \cdot \frac{y'}{y} + 2y y' \ln x + y^2 \cdot \frac{1}{x} = 0$

$\Rightarrow y'\left[\frac{x}{y} + 2y\ln x\right] = -\ln y - \frac{y^2}{x}$

$\therefore y' = \frac{-\ln y - \frac{y^2}{x}}{\frac{x}{y} + 2y\ln x}$

<u>مثال</u> أوجد مشتقة الدوال التالية:

① $y = \ln\left[\frac{x^3\sqrt{x^5+2}}{(2x+1)^7 \sin^2(5x)}\right]$.

يمكن تبسيط الدالة قبل الاشتقاق باستخدام خواص اللوغاريتم الطبيعي:

$\ln(ab) = \ln a + \ln b$

$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

$\ln a^r = r\ln a$

$= \ln\left[x^3 (x^5+2)^{\frac{1}{2}}\right] - \ln\left[(2x+1)^7 \sin^2(5x)\right]$

$= \ln x^3 + \ln(x^5+2)^{\frac{1}{2}} - \left[\ln(2x+1)^7 + \ln(\sin^2(5x))\right]$

$= 3\ln x + \frac{1}{2}\ln(x^5+2) - 7\ln(2x+1) - 2\ln(\sin 5x)$

$\therefore y' = 3 \cdot \frac{1}{x} + \frac{1}{2} \cdot \frac{5x^4}{x^5+2} - 7 \cdot \frac{2}{2x+1} - 2 \cdot \frac{\cos 5x \cdot 5}{\sin 5x}$

$$\boxed{\frac{d}{dx}[k f(x)] = k f'(x)}$$

② $y = \frac{x^3 (2x+5)^4}{\sqrt[3]{x^2+1}\,(x+3)^8}$

يمكن تبسيط هذه الدالة بأخذ ln للطرفين قبل الاشتقاق:

$\ln|y| = \ln\left|\frac{x^3 \cdot (2x+5)^4}{(x^2+1)^{\frac{1}{3}} \cdot (x+3)^8}\right|$

$= \ln|x^3| + \ln|2x+5|^4 - \ln(x^2+1)^{\frac{1}{3}} - \ln|x+3|^8$

$= 3\ln|x| + 4\ln|2x+5| - \frac{1}{3}\ln(x^2+1) - 8\ln|x+3|$

الآن نشتق الطرفين اشتقاقاً ضمنياً:

$\therefore \frac{y'}{y} = 3 \cdot \frac{1}{x} + 4 \cdot \frac{2}{2x+5} - \frac{1}{3}\frac{2x}{x^2+1} - 8\frac{1}{x+3}$

$\Rightarrow y' = \left[\frac{3}{x} + \frac{8}{2x+5} - \frac{2x}{3(x^2+1)} - \frac{8}{x+3}\right] \cdot y$

$= \left[\frac{3}{x} + \frac{8}{2x+5} - \frac{2x}{3(x^2+1)} - \frac{8}{x+3}\right] \cdot \frac{x^3 (2x+5)^4}{(x^2+1)^{\frac{1}{3}} (x+3)^8}$

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$$\left[\int x^r dx = \frac{x^{r+1}}{r+1},\quad r \neq -1\right]$$

بما أن: $\frac{d}{dx}(\ln|x|) = \frac{1}{x}$

$$\therefore \int \frac{1}{x}\,dx = \ln|x| + C$$

وبشكل عام فإن:

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$$

مثال: احسب التكاملات التالية:

① $$\int \frac{x^2}{2x^3+5}\,dx = \frac{1}{6}\int \frac{6x^2}{2x^3+5}\,dx$$
$$= \frac{1}{6}\ln|2x^3+5| + C$$

② $$\int \frac{\cos x}{7+\sin x}\,dx = \ln|7+\sin x| + C$$

③ $$\int \frac{1}{3x-5}\,dx = \frac{1}{3}\int \frac{3}{3x-5}\,dx$$
$$= \frac{1}{3}\ln|3x-5| + C$$

④ $$\int \frac{\sqrt{1+\ln x}}{x}\,dx = \int (\underbrace{1+\ln x}_{u})^{\frac{1}{2}} \cdot \underbrace{\frac{1}{x}\,dx}_{du}$$

نفرض أن: $u = 1+\ln x \Rightarrow du = \frac{1}{x}\,dx$

$$\therefore \int = \int u^{\frac{1}{2}}\,du = \frac{2}{3}u^{\frac{3}{2}} + C$$
$$= \frac{2}{3}(1+\ln x)^{\frac{3}{2}} + C$$

④ $$\int \frac{\sec^2(\ln x)}{x}\,dx = \int \sec^2(\ln x) \cdot \frac{1}{x}\,dx$$

نفرض أن: $u = \ln x \Rightarrow du = \frac{1}{x}\,dx$

$$\therefore \int = \int \sec^2 u\,du$$
$$= \tan u + C = \tan(\ln x) + C$$

⑤ $$\int \frac{1}{\sqrt{x}+x\sqrt{x}}\,dx = \int \frac{1}{\sqrt{x}\,(\underbrace{1+\sqrt{x}}_{u})}\,dx$$

نفرض أن: $u = 1+\sqrt{x} \Rightarrow du = \frac{1}{2\sqrt{x}}\,dx$

$$\therefore \frac{1}{\sqrt{x}}\,dx = 2\,du$$

$$\therefore \int = \int \frac{1}{u}\,2\,du$$
$$= 2\int \frac{1}{u}\,du = 2\ln|u| + C = 2\ln|1+\sqrt{x}| + C$$

⑥ $$\int \frac{x}{x+1}\,dx = \int \frac{(x+1)-1}{x+1}\,dx$$
$$= \int \left(1 - \frac{1}{x+1}\right)dx$$
$$= x - \ln|x+1| + C$$

يمكن أيضاً تقسيم مطولة ثم نكامل

⑦ $$\int \frac{x^2}{x+1}\,dx = \int \frac{(x^2-1)+1}{x+1}\,dx$$
$$= \int \frac{(x+1)(x-1)+1}{x+1}\,dx$$
$$= \int \left(x-1+\frac{1}{x+1}\right)dx$$
$$= \frac{x^2}{2} - x + \ln|x+1| + C$$

⑧ $$\int \frac{x^3}{x^2+1}\,dx = \int \frac{(x^3+x)-x}{x^2+1}\,dx$$
$$= \int \frac{x(x^2+1)-x}{x^2+1}\,dx$$
$$= \int \left(x - \frac{1}{2}\cdot\frac{2x}{x^2+1}\right)dx$$
$$= \frac{x^2}{2} - \frac{1}{2}\ln|x^2+1| + C$$

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* $\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx = -\int \frac{-\sin x}{\cos x} dx$

$= -\ln|\cos x| + c = \ln|(\cos x)^{-1}| + c$

$= \ln|\sec x| + c.$

* $\int \cot x \, dx = \int \frac{\cos x}{\sin x} dx = \ln|\sin x| + c.$

* $\int \sec x \, dx = \ln|\sec x + \tan x| + c$

* $\int \csc x \, dx = \ln|\csc x - \cot x| + c.$