

INTEGRAL MEAN VALUE THEOREM

Theorem (Integral Mean value Theorem) :

If f is a continuous function on the interval $[a, b]$ then there exists a number

$$c \in (a, b) \text{ for which } f(c) = \frac{\int_a^b f(x) \, dx}{b - a} .$$

Example : Find the value that satisfies the integral Mean value theorem for the function $f(x) = 4x^3 - 1$ on the interval $[1, 2]$

$$\text{Answer : } f(c) = \frac{\int_1^2 (4x^3 - 1) \, dx}{2 - 1}$$

$$4c^3 - 1 = [x^4 - x]_1^2$$

$$4c^3 - 1 = (16 - 2) - (1 - 1)$$

$$4c^3 - 1 = 14$$

$$c^3 = \frac{15}{4}$$

$$c = \sqrt[3]{\frac{15}{4}}$$

Note that $c = \sqrt[3]{\frac{15}{4}} \in (1, 2)$.