

AVERAGE VALUE OF A FUNCTION

Definition (Average value of a function) : Let f be a continuous function

on $[a, b]$ then the average value of f on $[a, b]$ is $f_{av} = \frac{\int_a^b f(x) \, dx}{b - a}$.

Example : Find f_{av} of the following functions :

1. $f(x) = x^2 - 2x$ on the interval $[1, 4]$

$$\begin{aligned} \int_1^4 (x^2 - 2x) \, dx &= \left[\frac{x^3}{3} - x^2 \right]_1^4 \\ &= \left(\frac{64}{3} - 16 \right) - \left(\frac{1}{3} - 1 \right) = \frac{63}{3} - 15 = \frac{63 - 45}{3} = \frac{18}{3} = 6 \end{aligned}$$

$$\text{Hence } f_{av} = \frac{\int_1^4 (x^2 - 2x) \, dx}{4 - 1} = \frac{6}{3} = 2 .$$

2. $f(x) = \sin^2 x \cos x$ on the interval $\left[0, \frac{\pi}{2}\right]$

$$\begin{aligned} \int_0^{\frac{\pi}{2}} \sin^2 x \cos x \, dx &= \int_0^{\frac{\pi}{2}} (\sin x)^2 \cos x \, dx = \left[\frac{(\sin x)^3}{3} \right]_0^{\frac{\pi}{2}} \\ &= \frac{(\sin \frac{\pi}{2})^3}{3} - \frac{(\sin 0)^3}{3} = \frac{1}{3} - 0 = \frac{1}{3} \end{aligned}$$

$$\text{Hence } f_{av} = \frac{\int_0^{\frac{\pi}{2}} \sin^2 x \cos x \, dx}{\frac{\pi}{2} - 0} = \frac{\frac{1}{3}}{\frac{\pi}{2}} = \frac{2}{3\pi} .$$

Exercise : Find f_{av} of the function $f(x) = (2x + 1)^2$ on the interval $[0, 1]$