Velocity, Speed and Unit Tangent Vector for Regular Curve
Math 473
Introduction to Differential Geometry
Lecture 2

Dr. Nasser Bin Turki

King Saud University
Department of Mathematics

September 15, 2018

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$$|\alpha'(t)| = \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}.$$

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$$T(t) = \frac{\alpha'(t)}{|\alpha'(t)|}$$

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- **(ii)** compute the velocity,
- **(iii)** compute the speed,
- **(iv)** compute the unit tangent vector of the curve.

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Thanks for listening.