Inverse Hyperbolic Trigonometric Functions

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1. Derivatives of Inverse Hyperbolic Trigonometric Functions

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where $u' = \frac{du}{dx}$.

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5. $f(x) = x\tanh^{-1}(x) + \ln(\sqrt{1-x^2})$.
6. $f(x) = \frac{1+\cosh(x)}{1-\cosh(x)}$.

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