Faculty of Engineering
Mechanical Engineering Department

CALCULUS FOR ENGINEERS
MATH 1110

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Integral Calculus

Using the Properties of the Definite Integral

Given: $\int_1^3 f(x)dx = 6$ $\int_3^7 f(x)dx = 9$ $\int_1^3 g(x)dx = -4$

$$\int_1^3 3f(x)dx = 3\int_1^3 f(x)dx = 3(6) = 18$$

$$\int_1^3 (2f(x) - 4g(x))dx = 2\int_1^3 f(x)dx - 4\int_1^3 g(x)dx = 2(6) - 4(-4) = 28$$

$$\int_1^7 f(x)dx = \int_1^3 f(x)dx + \int_3^7 f(x)dx = 6 + 9 = 15$$

$$\int_3^1 f(x)dx = -\int_1^3 f(x)dx = -6$$

Rules of the Definite Integral

$$\int_a^b c\,dx = c(b-a) \qquad \int_a^b x\,dx = \frac{b^2}{2} - \frac{a^2}{2} \qquad \int_a^b x^2\,dx = \frac{b^3}{3} - \frac{a^3}{3}$$

Examples

$$\int_2^6 4\,dx = \quad 4(6-2) = \quad 16$$

$$\int_4^8 x\,dx = \quad \frac{8^2}{2} - \frac{4^2}{2} = \quad 32 - 8 = \quad 24$$

$$\int_3^5 x^2\,dx = \frac{5^3}{3} - \frac{3^3}{3} = \quad \frac{125}{3} - \frac{27}{3} = \quad \frac{98}{3} = 32.67$$

$$\int_3^4 x^2 + 3x - 2\,dx = \int_3^4 x^2\,dx + 3\int_3^4 x\,dx - \int_3^4 2\,dx = \frac{4^3}{3} - \frac{3^3}{3} + 3\left(\frac{4^2}{2} - \frac{3^2}{2}\right) - \ 2(4-3) =$$

$$\frac{64}{3} - \frac{27}{3} + 3\left(\frac{16}{2} - \frac{9}{2}\right) - \ 2(1) = \quad 20.83$$

The Fundamental Theorem of Calculus

If f is continuous at every point in [a, b] and F is any antiderivative of f on [a, b], then

$$\int_a^b f(x)dx = F(b) - F(a).$$

Examples

$$\int_1^5 5x\,dx = \frac{5x^2}{2}\Big|_1^5 = \frac{5(5)^2}{2} - \frac{5(1)^2}{2} = \frac{125}{2} - \frac{5}{2} = \frac{120}{2} = 60$$

$$\int_{\pi/6}^{2\pi/3} sinx\,dx = -cosx\Big|_a^b = -cos\left(\frac{2\pi}{3}\right) - \left(-cos\left(\frac{\pi}{6}\right)\right) = -\left(-\frac{1}{2}\right) - \left(-\frac{\sqrt{3}}{2}\right) = 0.866$$

The Fundamental Theorem of Calculus

$$\int_a^b f(x)dx = F(b) - F(a).$$

Examples

$$\int_3^4 x^2 + 3x - 2\,dx = \left.\frac{x^3}{3} + \frac{3x^2}{2} - 2x\right|_3^4 = \frac{4^3}{3} + \frac{3(4)^2}{2} - 2(4) - \left(\frac{3^3}{3} + \frac{3(3)^2}{2} - 2(3)\right)$$

$$37.33 - 16.5 = 20.83$$

$$\int_1^{32} \frac{1}{x^{6/5}}\,dx = \int_1^{32} x^{-6/5}\,dx = \left.\frac{x^{-1/5}}{-\frac{1}{5}}\right|_1^{32} = \left.\frac{-5}{x^{1/5}}\right|_1^{32} = -\frac{5}{2} - \left(-\frac{5}{1}\right) = 2.5$$

1. Evaluate each of the following indefinite integrals.

(a) $\int 6x^5 - 18x^2 + 7\,dx$

(b) $\int 6x^5\,dx - 18x^2 + 7$

(a) $\int 6x^5 - 18x^2 + 7\,dx$

$$\int 6x^5 - 18x^2 + 7\,dx = \boxed{x^6 - 6x^3 + 7x + c}$$

(b) $\int 6x^5\,dx - 18x^2 + 7$

$$\int 6x^5\,dx - 18x^2 + 7 = \boxed{x^6 + c - 18x^2 + 7}$$

2. Evaluate each of the following indefinite integrals.

(a) $\int 40x^3 + 12x^2 - 9x + 14\,dx$

(b) $\int 40x^3 + 12x^2 - 9x\,dx + 14$

(c) $\int 40x^3 + 12x^2\,dx - 9x + 14$

(a) $\int 40x^3 + 12x^2 - 9x + 14\,dx$

$$\int 40x^3 + 12x^2 - 9x + 14\,dx = \boxed{10x^4 + 4x^3 - \tfrac{9}{2}x^2 + 14x + c}$$

(b) $\int 40x^3+12x^2-9x\,dx+14$

$$\int 40x^3+12x^2-9x\,dx+14=\boxed{10x^4+4x^3-\tfrac{9}{2}x^2+c+14}$$

(c) $\int 40x^3+12x^2\,dx-9x+14$

$$\int 40x^3+12x^2\,dx-9x+14=\boxed{10x^4+4x^3+c-9x+14}$$