Faculty of Engineering
Mechanical Engineering Department

CALCULUS FOR ENGINEERS
MATH 1110

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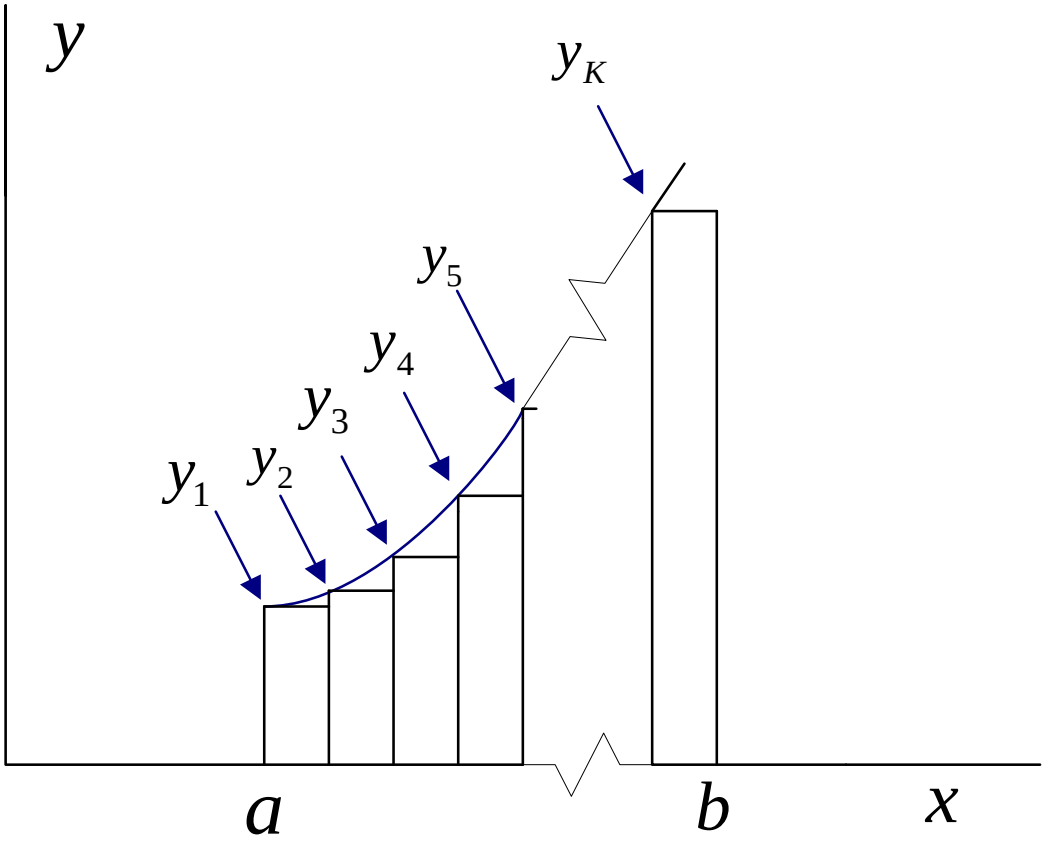
Integral Calculus

Indefinite and Definite Integrals

Indefinite $\int f(x)dx$

Definite $\int_{x_1}^{x_2} f(x)dx$

Definite Integral as Area Under the Curve



$$\text{Approximate Area} = \sum_k y_k \Delta x$$

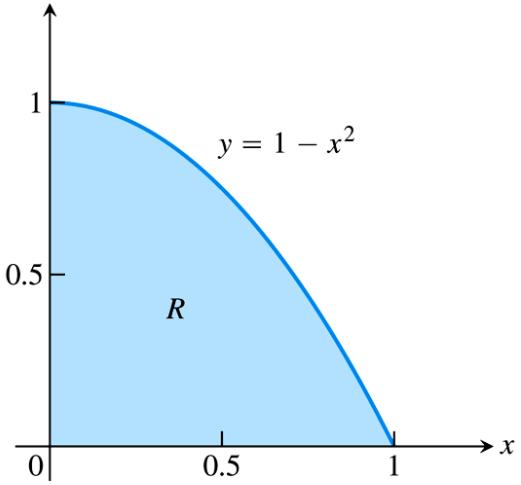
Exact Area as Definite Integral

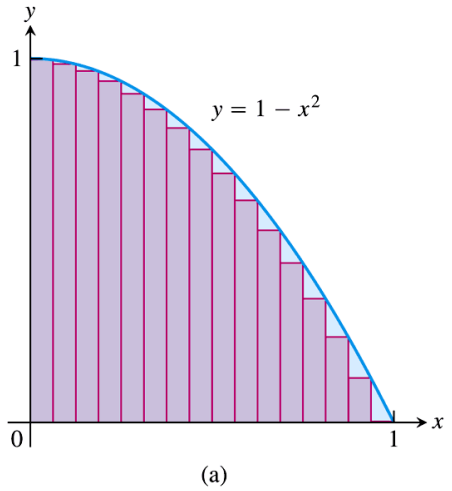
$$\int_a^b y dx = \lim_{\Delta x \to dx} \sum_k y_k \Delta x$$

Definite Integral with Variable Upper Limit

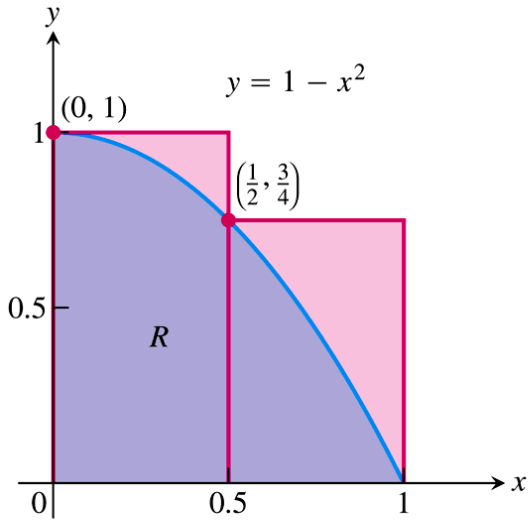
$$\int_a^x y dx$$

The Definite Integral

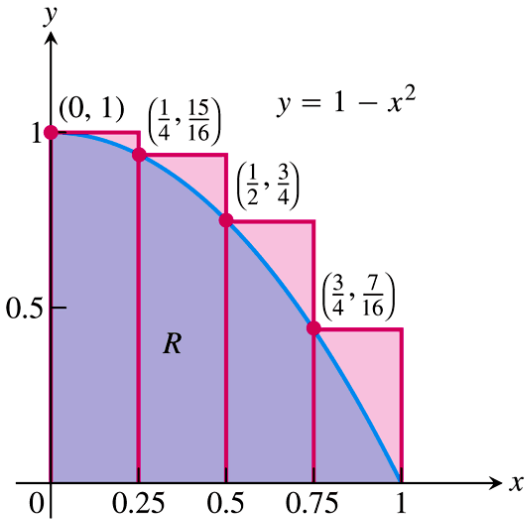




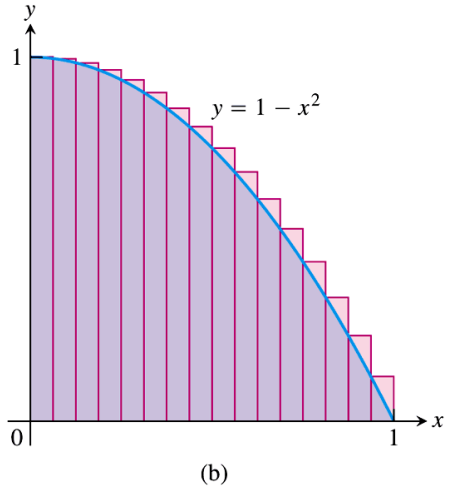
(a)



(a)



(b)



(b)

As the number of rectangles increased, the approximation of the area under the curve approaches a value.

The Definite Integral

Definition

The *definite integral* from a to b, $\int_a^b f(x)\,dx$ is the number to which all Riemann sums tend as the number of rectangles approaches infinity and as the width of all rectangles tend to zero:

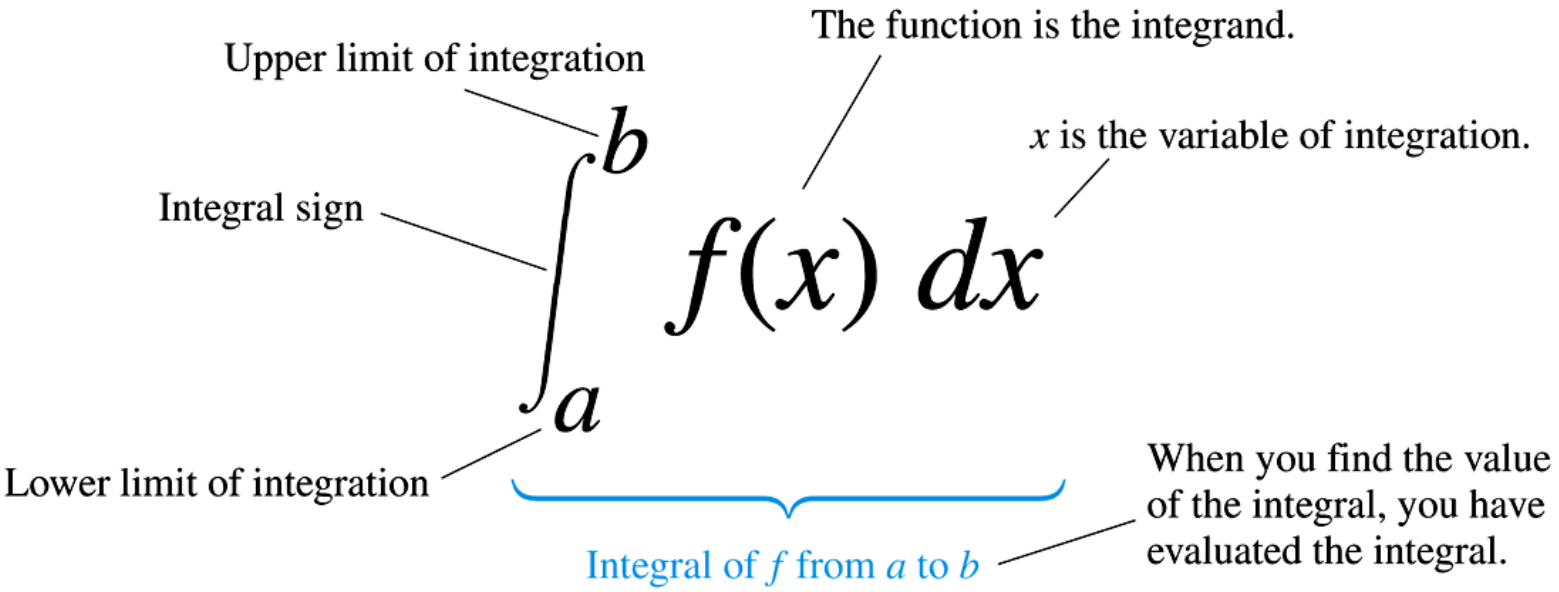
$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x_i$$

where Δx_i is the width of the ith rectangle and c_i is the x-coordinate of the point where the ith rectangle touches $f(x)$.

Note: The function f(x) must be continuous on the interval [a, b].

The Definite Integral

Parts of the Definite Integral



Properties of the Definite Integral

Rules satisfied by definite integrals

1. *Order of Integration:* $\displaystyle\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$ — A Definition

2. *Zero Width Interval:* $\displaystyle\int_a^a f(x)\,dx = 0$ — A Definition when $f(a)$ exists

3. *Constant Multiple:* $\displaystyle\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$ — Any constant k

4. *Sum and Difference:* $\displaystyle\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

5. *Additivity:* $\displaystyle\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

Using the Properties of the Definite Integral

Given: $\int_1^3 f(x)dx = 6$ $\int_3^7 f(x)dx = 9$ $\int_1^3 g(x)dx = -4$

$$\int_1^3 3f(x)dx = 3\int_1^3 f(x)dx = 3(6) = 18$$

$$\int_1^3 \big(2f(x) - 4g(x)\big)dx = 2\int_1^3 f(x)dx - 4\int_1^3 g(x)dx = 2(6) - 4(-4) = 28$$

$$\int_1^7 f(x)dx = \int_1^3 f(x)dx + \int_3^7 f(x)dx = 6 + 9 = 15$$

$$\int_3^1 f(x)dx = -\int_1^3 f(x)dx = -6$$