Improper Integrals

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1. INTEGRALS WITH INFINITE LIMITS OF INTEGRATION

2. INTEGRALS WITH DISCONTINUOUS INTEGRANDS

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$$\int_a^{+\infty} f(x)dx, \quad \int_{-\infty}^{a} f(x)dx, \quad \int_{-\infty}^{+\infty} f(x)dx.$$

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Note that: An improper integral is called *converge* if the the limits exists as a finite number.

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$$\int_{a}^{b} f(x)dx = \lim_{t \to a^+} \int_{t}^{b} f(x)dx.$$

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3. $$\int_{0}^{\frac{\pi}{2}} \frac{1}{1-\cos(x)} dx$$
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Definition (4)

If f is continuous on the interval $[a, b]$, except for some c in the open interval (a, b) at which f has an infinite discontinuousty, then

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1. $$\int_{-3}^{1} \frac{1}{x^2}dx.$$

2. $$\int_{\frac{1}{e}}^{e} \frac{1}{x(\ln(x))^2}dx.$$