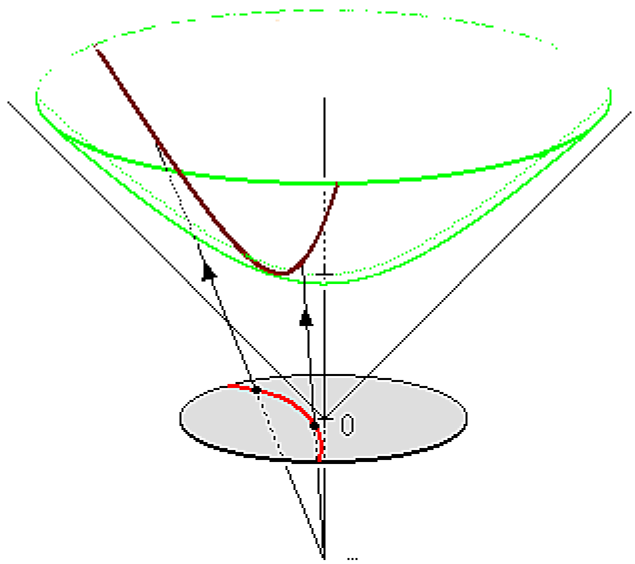


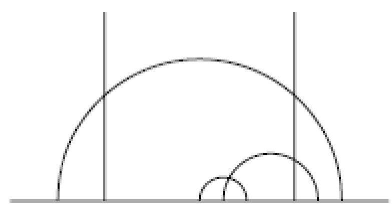


Figure 1.1: Hyperbolic lines in $\mathbb{H}$

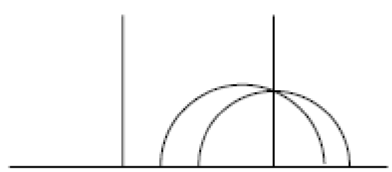


Figure 1.2: Several parallel hyperbolic lines

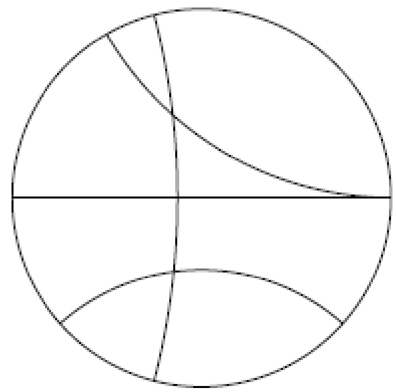
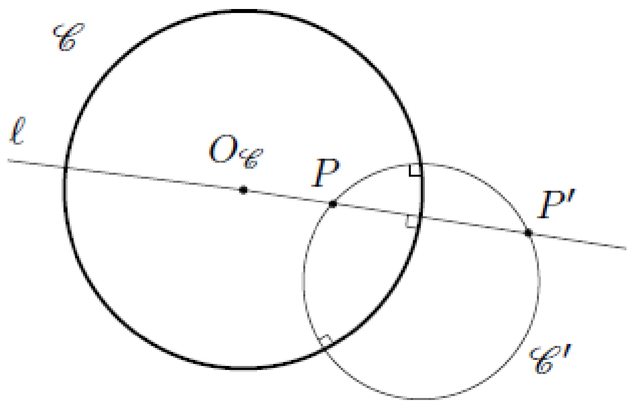
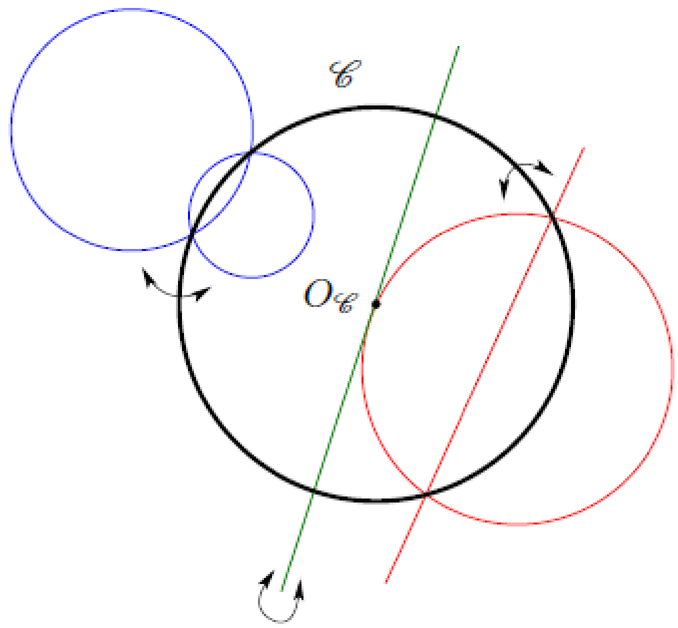


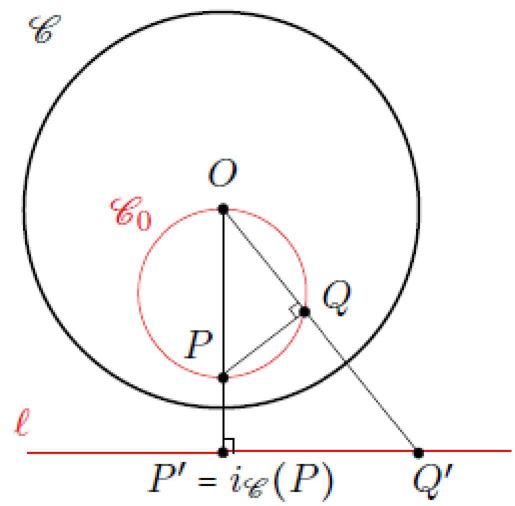
Figure 4.1: Some hyperbolic lines in $\mathbb{D}$



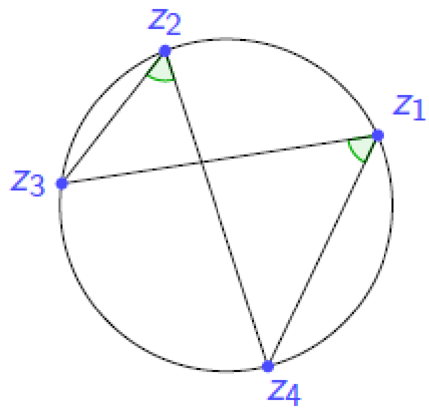
$$O_{\mathscr{C}}P \cdot O_{\mathscr{C}}P' = r_{\mathscr{C}}^2.$$



$$i_{\mathscr{C}}(z) = \frac{r^2}{\overline{z}-\overline{a}} + a.$$



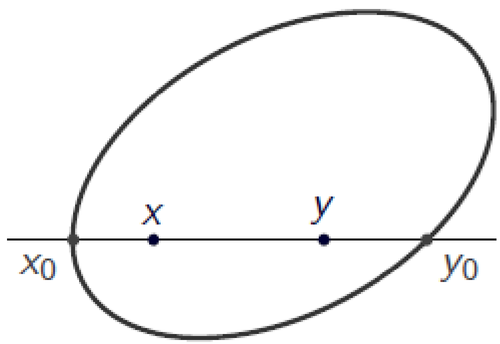
$$\varphi_A(z) = \begin{cases} \frac{az+b}{cz+d} & \text{si } z \neq \infty, z \neq -\frac{d}{c}, \\ \infty & \text{si } z = -\frac{d}{c}, \\ \frac{a}{c} & \text{si } z = \infty. \end{cases}$$



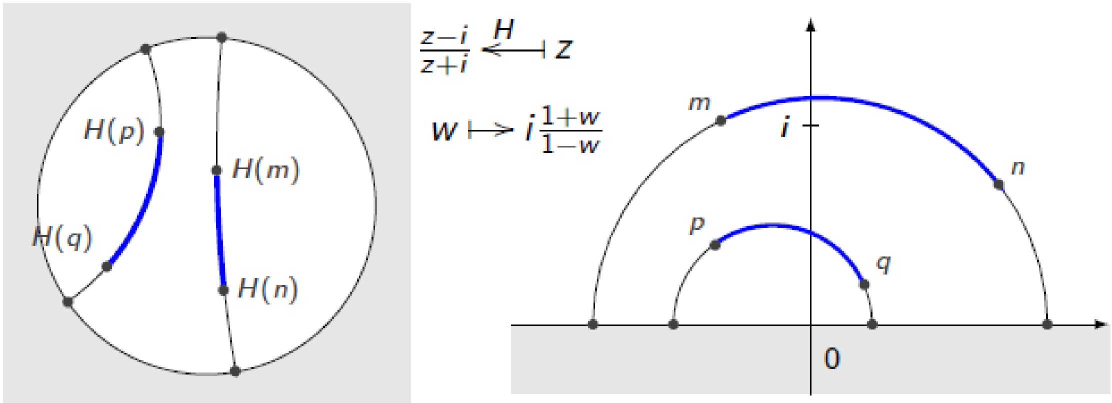
$$[z_1, z_2, z_3, z_4] \in \mathbb{R}.$$

$$(z_1, z_2, z_3, z_4) = \frac{z_1 - z_3}{z_1 - z_4} : \frac{z_2 - z_3}{z_2 - z_4}.$$

$$(z_1, z_2, z_3, \infty) = \frac{z_1 - z_3}{z_2 - z_3}.$$



$$d(x, y) = \ln[x, y, y_0, x_0] = \ln \frac{y - x_0}{x - x_0} \frac{y_0 - x}{y_0 - y} \geq 0.$$



law of sines:

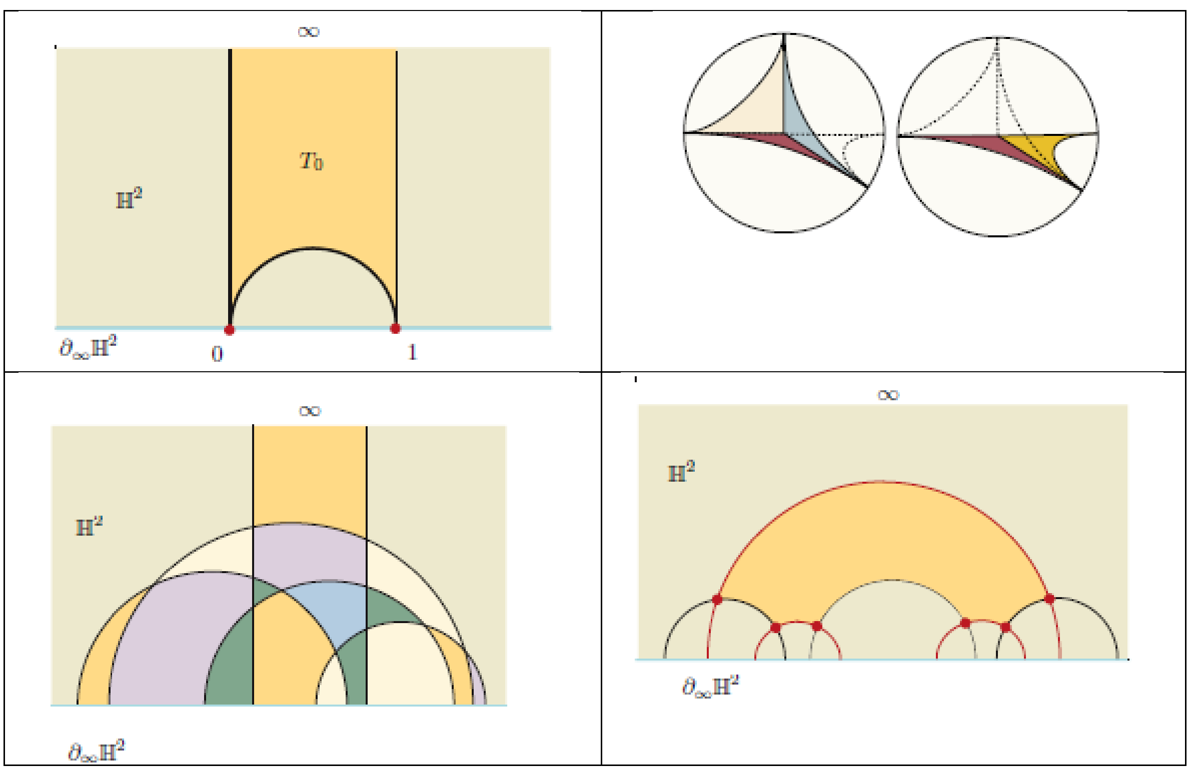
$$\frac{\sinh(a)}{\sin(\alpha)} = \frac{\sinh(b)}{\sin(\beta)} = \frac{\sinh(c)}{\sin(\gamma)}.$$

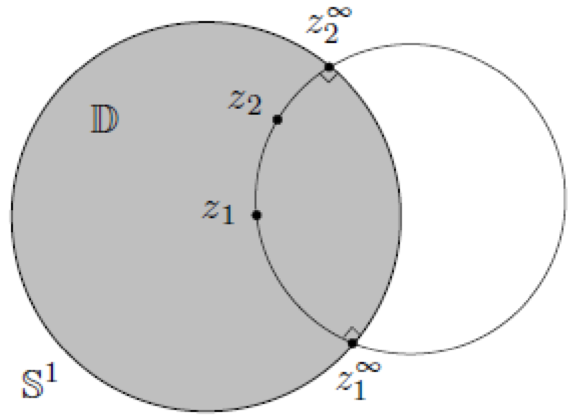
law of cosines II:

$$\cos(\gamma) = -\cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)\cosh(c).$$

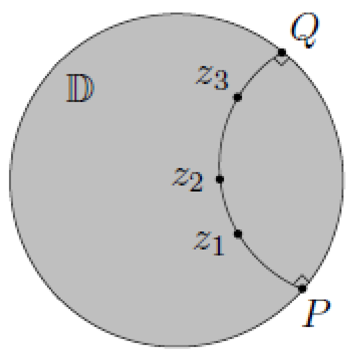
$$\cosh a = \cosh b \cosh c - \sinh b \sinh c \cos A$$

$$\text{aire}(T) = \pi - A - B - C.$$





$$d_{\mathbb{D}}(z_1, z_2) = -\log[z_1, z_2; z_1^\infty, z_2^\infty].$$



$$[z_1, z_3; P, Q] = \frac{(z_1 - P)(z_2 - Q)}{(z_2 - P)(z_1 - Q)} \frac{(z_2 - P)(z_3 - Q)}{(z_3 - P)(z_2 - Q)} = [z_1, z_2; P, Q][z_2, z_3; P, Q],$$

$$d_{\mathbb{D}}(z_1, z_3) = d_{\mathbb{D}}(z_1, z_2) + d_{\mathbb{D}}(z_2, z_3)$$

$$t \in (0, 1)$$

$$d_{\mathbb{D}}(0, t) = -\log[0, t; -1, 1] = -\log\left(\frac{(0 - (-1))(t - 1)}{(t - (-1))(0 - 1)}\right) = -\log\left(\frac{1 - t}{1 + t}\right).$$

$$\begin{aligned} d_{\mathbb{D}}(i_{\mathscr{C}}(z_1), i_{\mathscr{C}}(z_2)) &= -\log[i_{\mathscr{C}}(z_1), i_{\mathscr{C}}(z_2); i_{\mathscr{C}}(z_1)^\infty, i_{\mathscr{C}}(z_2)^\infty] \\ &= -\log[i_{\mathscr{C}}(z_1), i_{\mathscr{C}}(z_2); i_{\mathscr{C}}(z_1^\infty), i_{\mathscr{C}}(z_2^\infty)] \\ &= -\log \overline{[z_1, z_2; z_1^\infty, z_2^\infty]} = -\log[z_1, z_2; z_1^\infty, z_2^\infty] = d_{\mathbb{D}}(z_1, z_2). \end{aligned}$$

$$d_{\mathbb{D}}(z_1, z_2) = d_{\mathbb{D}}(0, t) = -\log\left(\frac{1-t}{1+t}\right)$$

$z_1, z_2 \in \mathbb{D}$. $\varphi(z_1) = 0$ $\varphi(z_2) = t \geq 0$.