M 106

Chapter 1 & 2

Chapter 1 :

Ex 1.1

(1) $\int \frac{1}{\sqrt{x}}\, dx = \int x^{-\frac{1}{2}}\, dx$

$= 2x^{\frac{1}{2}} + c$

(2) $\int \frac{\tan x}{\cos x}\, dx = \int \tan x \cdot \frac{1}{\cos x}\, dx$

$= \int \tan x \cdot \sec x\, dx$

$= \sec x + c$

Ex 1.2

(2) $\int (x^{\frac{3}{4}} + x^2 + 1)\, dx$

$= \frac{4}{7} x^{\frac{7}{4}} + \frac{x^3}{3} + x + c$

(7) $\int \frac{x^2 - 1}{x^4}\, dx$

$= \int \left(\frac{x^2}{x^4} - \frac{1}{x^4}\right) dx = \int \left(\frac{1}{x^2} - \frac{1}{x^4}\right) dx$

$= \int (x^{-2} - x^{-4})\, dx$

$= -x^{-1} + \frac{x^{-3}}{3} + c$

(13) $\int f'(x)\, dx = \int (4x^3 + 2x + 1)\, dx$

$f(x) = x^4 + x^2 + x + c$

$f(0) = 0^4 + 0^2 + 0 + c = 1$

$\Rightarrow c = 1$

$\Rightarrow f(x) = x^4 + x^2 + x + 1$

Ex 1.3

① $\int x\sqrt{1+x^2}\,dx$

$= \int x(1+x^2)^{\frac{1}{2}}\,dx$

$= \frac{1}{2}\int 2x(1+x^2)^{\frac{1}{2}}\,dx$

$= \frac{1}{2}\frac{(1+x^2)^{\frac{3}{2}}}{3/2} + c$

$= \frac{1}{3}(1+x^2)^{\frac{3}{2}} + c$

② $\int x\sqrt{x-1}\,dx$

$u = x-1 \Rightarrow du = dx$

$\Downarrow$

$u+1 = x$ by sub.

$\int (u+1)\,u^{\frac{1}{2}}\,du$

$= \int (u^{\frac{3}{2}} + u^{\frac{1}{2}})\,du$

$= \frac{2}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}} + c$

$= \frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{2}{3}(x-1)^{\frac{3}{2}} + c$

③ $\int x^2\sqrt{x-1}\,dx$

$u = x-1 \Rightarrow du = dx$

$\Downarrow$

$u+1 = x$

$\int (u+1)^2\,u^{\frac{1}{2}}\,du$

$\int (u^2+2u+1)\,u^{\frac{1}{2}}\,du$

$= \int (u^{\frac{5}{2}} + 2u^{\frac{3}{2}} + u^{\frac{1}{2}})\,du = \ldots$

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(16) $\int \sin^2 3x \cos 3x \, dx$

Let $u = \sin 3x$

$\Rightarrow du = 3\cos 3x \, dx$

by sub.

$\frac{1}{3}\int u^2 \, du = \frac{1}{3} \frac{u^3}{3} + c$

$= \frac{1}{9} \sin^3 3x +$

Review (chapter 1)

(41) $\int \frac{\sin^2 \sqrt{x}}{\sqrt{x} \cos^2 \sqrt{x}} \, dx$

$= \int \frac{\tan^2 \sqrt{x}}{\sqrt{x}} \, dx$

Note: $1 + \tan^2 x = \sec^2 x$

$\int \frac{\sec^2 \sqrt{x} - 1}{\sqrt{x}} \, dx$

$= \int \frac{\sec^2 \sqrt{x}}{\sqrt{x}} \, dx - \int \frac{1}{\sqrt{x}} \, dx$

$= 2 \int \frac{\sec^2 \sqrt{x}}{2\sqrt{x}} \, dx - \int x^{-\frac{1}{2}} \, dx$

$= 2 \tan \sqrt{x} - 2 x^{\frac{1}{2}} + c$

(62) $\int \frac{x^3}{\sqrt{x^4 - 1}} \, dx = \int x^3 (x^4 - 1)^{-\frac{1}{2}} \, dx$

$u = x^4 - 1 \Rightarrow du = 4x^3 \, dx$

$\sqrt{}$ 1

(65) $\int \frac{\sin x}{\sqrt{2 + \cos x}} \, dx = \int \sin x (2 + \cos x)^{-\frac{1}{2}} \, dx$

$u = 2 + \cos x \Rightarrow du = -\sin x \, dx$

$\sqrt{}$ 1

(66) $\int \frac{\sin(\tan x)}{\cos^2 x}\,dx$

$= \int \sin(\tan x)\sec^2 x\,dx$

$u = \tan x \Rightarrow du = \sec^2 x\,dx$

by sub.

$\int \sin(u)\,du = -\int -\sin u\,du$

$= -\cos u + c$

$= -\cos(\tan x) + c$

Chapter 2.

Ex 2.1

(1) $\sum_{i=1}^{3}(i+1)$

$= 2+3+4 = 9$

(2) $\sum_{k=1}^{n}(k-1)$

$= \sum_{k=1}^{n} k - \sum_{k=1}^{n} 1$

$= \frac{n(n+1)}{2} - n$

$= \frac{n(n+1)-2n}{2}$

Ex 2.2:

(1)

$[x_{k-1}, x_k]$	Δx_k
$[-1, 0]$	1
$[0, 1.3]$	1.3
$[1.3, 4]$	2.7
$[4, 4.1]$	0.1
$[4.1, 5]$	0.9

$\|P\| = 2.7$

(10) $\int_c^a f(x)\,dx + \int_b^c f(x)\,dx$

$= -\int_a^c f(x)\,dx - \int_c^b f(x)\,dx$

$= -\left(\int_a^c f(x)\,dx + \int_c^b f(x)\,dx\right)$

$= -\int_a^b f(x)\,dx = -2$

Ex 2.4 :

(1) $\int_0^3 (2 - x + x^2)\,dx$

$= \left[2x - \frac{x^2}{2} + \frac{x^3}{3}\right]_0^3$

$= \left(6 - \frac{9}{2} + \frac{27}{3}\right) - (0)$

$= 9 - \frac{9}{2} = \frac{9}{2}$

(7) $\int_{\pi/4}^{\pi/3} \sec x\,(\tan x + \sec x)\,dx$

$= \int_{\pi/4}^{\pi/3} (\sec x \tan x + \sec^2 x)\,dx$

$= \left[\sec x + \tan x\right]_{\pi/4}^{\pi/3}$ - - - -

(9) $\int_{-1}^{1} |3x+1|\,dx$

$3x+1 \geq 0$

$x \geq \frac{-1}{3}$

$|3x+1| = \begin{cases} 3x+1 & : x \geq -\frac{1}{3} \\ -3x-1 & : x < -\frac{1}{3} \end{cases}$

$\int_{-1}^{1} |3x+1|\,dx = \int_{-1}^{-1/3} (-3x-1)\,dx + \int_{-1/3}^{1} (3x+1)\,dx$

$= \ldots + \ldots$

9. $\Delta x = \frac{3-0}{5} = \frac{3}{5}$

$x_0 = 0$

$x_1 = 0 + (1)\frac{3}{5} = \frac{3}{5}$

$x_2 = 0 + 2\left(\frac{3}{5}\right) = \frac{6}{5}$

$x_3 = 0 + 3\left(\frac{3}{5}\right) = \frac{9}{5}$

$x_4 = 0 + 4\left(\frac{3}{5}\right) = \frac{12}{5}$

$x_5 = 0 + 5\left(\frac{3}{5}\right) = 3$

$P = \left\{ 0, \frac{3}{5}, \frac{6}{5}, \frac{9}{5}, \frac{12}{5}, 3 \right\}$

13.

$[x_{k-1}, x_k]$	w_k	$f(w_k)$	Δx_k	$f(w_k)\Delta x_k$
$[0, 1]$	0	1	1	1
$[1, 3]$	1	2	2	4
$[3, 4]$	3	10	1	10
			$R_P = \sum f(w_k)\Delta x_k$	15

14, 15 كالطالب
بنفس الطريقة راجع مثال 2.7

Ex 2.3:

7. $\int_a^b \left(6f(x) - \frac{g(x)}{3}\right) dx$

$= 6\int_a^b f(x)\,dx - \frac{1}{3}\int_a^b g(x)\,dx$

$= 6(2) - \frac{1}{3}(3)$

$= 12 - 1 = 11$

(12) $f(x) = 1 - x^3 \quad , \quad [-2, 0]$

$f(x)$ is continuous

$$\int_{-2}^{0} f(x)\,dx = (0-(-2))(1-z^3)$$

$$\int_{-2}^{0} (1-x^3)\,dx = 2\,(1-z^3)$$

$$6 = 2\,(1-z^3)$$

$$1 - z^3 = 3 \Rightarrow z^3 = -2$$

$$\Rightarrow z = \sqrt[3]{-2} \in (-2, 0)$$

(17) $f_{av} = \dfrac{1}{2-0} \displaystyle\int_{0}^{2} (x^3 + x^2 - 1)\,dx$

$=$ — — كمل

(24) $\dfrac{d}{dx} \displaystyle\int_{x+1}^{3(x-1)} \dfrac{1}{t-1}\,dt$

$$= \frac{1}{3(x-1)-1}(3) - \frac{1}{(x+1)-1}(1)$$

$$= \frac{3}{3x-4} - \frac{1}{x}$$

(29) $F(x) = \displaystyle\int_{2}^{x} \sqrt{3t^2+1}\,dt$,

$F(2) = \displaystyle\int_{2}^{2} \sqrt{3t^2+1}\,dt = 0$

$F'(x) = \sqrt{3x^2+1} \Rightarrow F'(2) = \sqrt{12+1} = \sqrt{\ }$

$F''(x) = \dfrac{6x}{2\sqrt{3x^2+1}} = \dfrac{3x}{\sqrt{3x^2+1}} \Rightarrow F''(2) = \ldots$ كمل

Review (chapter 2)

(17) $\Delta x = \frac{3-1}{n} = \frac{2}{n}$

where

P is regular

$x_k = x_0 + k\Delta x \quad , x_0 = 1$

$w_k \in [x_{k-1}, x_k] \qquad \Delta x = \frac{2}{n}$

Let $w_k = x_k = x_0 + k\Delta x$

$\Rightarrow w_k = 1 + \frac{2k}{n}$

$f(w_k) = \left(1 + \frac{2k}{n}\right) + 3 = \frac{2k}{n} + 4$

$$R_P = \sum_{k=1}^{n} \left(\frac{2k}{n} + 4\right)\frac{2}{n}$$

$$= \frac{2}{n}\left[\frac{2}{n}\sum_{k=1}^{n} k + \sum_{k=1}^{n} 4\right]$$

$$= \frac{2}{n}\left[\frac{2}{n}\,\frac{n(n+1)}{2} + 4n\right]$$

$$= \frac{4(n+1)}{2n} + 8$$

$$\lim_{n\to\infty}\left(\frac{\cancel{4}^{2}(n+1)}{\cancel{2}n} + 8\right) = 2 + 8 = 10$$

$$= \frac{33}{2}$$

$$\lim_{n\to\infty}\left(\frac{2(n+1)}{n} + 8\right)$$

$$= 2 + 8 = 10$$

(19) نفس الطريقة ↑

لكن

$$\sum k^2 = \frac{n(n+1)(2n+1)}{6}$$

(84) $\sum_{k=1}^{n^2}(k-1) = \sum_{k=1}^{n^2} k - \sum_{k=1}^{n^2} 1$

$\frac{n^2(n^2+1)}{2} - n^2$

$= \frac{n^4+n^2-2n^2}{2} = \frac{n^4-n^2}{2}$

$= \frac{n^2(n^2-1)}{2}$

(87) $\sum_{k=1}^{4}(k+a) = 14$

$\sum_{k=1}^{4} k + \sum_{k=1}^{4} a = 14$

$\frac{4(4+1)}{2} + 4a = 14$

$10 + 4a = 14 \Rightarrow 4a = 4$

$a = 1$

(92) $|x-1| = \begin{cases} x-1 : & x \geq 1 \\ -x+1 : & x < 1 \end{cases}$

in $[0,1]$ we have

$f(x) = -x+1$

$\int_0^1 (-x+1)\,dx = (1-0)(-z+1)$

$\frac{1}{2} = -z+1 \Rightarrow z = \frac{1}{2} \in (0,1)$