Homework 1

1. (a) Two cards are selected in sequence from a standard deck. Find the probability that the second card is a queen, given that the first card is a king. (Assume that the king is not replaced.)

   (b) Two cards are selected, with replacement, from a standard deck. Find the probability of selecting a king and then selecting a queen.

   (c) What is the probability that a poker hand is a full house? A poker hand consists of five random selected cards from an ordinary deck of 52 cards. It is a full house if three cards are of the one denomination and two cards are of another denomination: for example, three queens and two 4's.

   (a) **Solution:**
   Let A represent the first card drawn is a king, B the second card drawn is a queen. so the probability to get the event of interest is

   $$P(B|A) = \frac{4}{51}.$$

   (b) **Solution:**
   Because of sampling with replacement, $P(B|A) = P(B)$. Therefore, the probability to get the event of interest is

   $$P(A \cap B) = P(A)P(B|A) = P(A)P(B) = \frac{4}{52} \times \frac{4}{52}.$$

   (c) **Solution:**
   Let C be the event that a pork hand is full house. Then, the probability to get a hand poker of full house is

   $$P(C) = \frac{\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}}{\binom{52}{5}}.$$

2. An experiment consists of tossing a pair of 6-sided dice.

(a) list the elements of the sample space S;

(b) list the elements corresponding to the event A that the sum is greater than 9;

(c) list the elements corresponding to the event B that a 5 occurs on either dice;

(d) list the elements corresponding to the event A';

(e) list the elements corresponding to the event $A' \cap B$;

(f) list the elements corresponding to the event $A \cup B$;

Solution:
Let x and y represent the the numbers showing up in dice1 and dice2, respectively.

(a) $S = \{(x, y)|x = 1, 2, 3, 4, 5, 6, \ y = 1, 2, 3, 4, 5, 6\}$.

(b) $A = \{(x, y)|x + y > 9\}$

(c) $B = \{(x, y)|x = 5 \text{ or } y = 5\}$

(d) $A' \cap B = \{(1, 5), (2, 5), (3, 5), (4, 5), (5, 1), (5, 2), (5, 3), (5, 4)\}$.

(e) $A \cup B = \{(x, y)|x + y < 9 \text{ or } x = 5 \text{ or } y = 5\}$

3. Suppose that we have two urns, cleverly named Urn I and Urn II. Suppose that Urn I has 2 red marbles and 2 blue marbles, and Urn II has 1 red and 3 blue marbles. We flip a fair coin to select an urn. If head occurs, Urn I is selected, otherwise, Urn II. Having selected an urn we select a marble without looking in the urn. It so happens that the marble we chose is red. Our question is: what is the probability that we chose Urn I?

Solution: Let

$$A = \{\text{The Urn I is selected.}\}$$
$$B = \{\text{The Urn II is selected.}\}$$
$$R = \{\text{The red marble is selected.}\}$$
$$P(A) = 1/2, \quad P(B) = 1/2, \quad P(R|A) = 1/2 \quad P(R|B) = 1/4;$$

$$\begin{aligned} P(A|R) &= \frac{P(A)P(R|A)}{P(A)P(R|A) + P(B)P(R|B)} \\ &= \frac{1/2 \times 1/2}{1/2 \times 1/2 + 1/2 \times 1/4} \\ &= \frac{2}{3}. \end{aligned}$$

4. Suppose that we roll a pair of fair 6-sided dice, so each of the 36 possible outcomes is equally likely. Let A denote the event that the first dice lands on 4, let B be the event that the sum of the dice is 7.

(a) Are A and B disjoint (mutually exclusive)?

(b) Are A and B independent?

(c) **True or False**. Determine whether the statement is true or false.

i. __*F* If two events, say E_1 and E_2, are independent, then E_1 and E_2 are disjoint.

ii. <u>   *F*</u> If two events, say E_1 and E_2, are disjoint, then E_1 and E_2 are independent.

Solution:
Let x and y represent the numbers showing up in first and second dice, respectively.

		y					
	(x, y)	1	2	3	4	5	6
	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
x	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
	4	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	5	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Let's define one more event that the sum of the dice is 11.

$$
\begin{aligned}
P(A) =& \frac{1}{6};\\
P(B) =& \frac{1}{6};\\
P(C) =& \frac{1}{18}.
\end{aligned}
$$

Since $A \cap B = \{(4, 3)\}$, $P(A \cap B) = \frac{1}{36}$. We can see $P(AB) = P(A)P(B)$, so A and B are independent but obviously they are not disjoint.

Apparently, $A \cap C = \phi$, are disjoint, but they are not independent

$$P(A \cap C) \neq P(A)P(C).$$