Indeterminate Form and L'Hopital Rule

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Example: $\lim_{x\to\infty}(1+\frac{1}{x})^x=e.$

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Note: This rule also works if $c = \pm\infty$ or when $x \to c^+$ or $x \to c^-$.

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