واجبات مقرر 425 ريض – نشاط تمارين- ف1-1442 هـ

رموز مونج Mong:

$$p = z_x\,, \qquad q = z_y\,, \qquad z_{xx} = r, \qquad z_{xy} = s\,, \qquad z_{yy} = t$$

س1:

11. Make the change of independent variables $\xi = \log x$, $\eta = \log y$ and reduce the differential equation to one with constant coefficients. Obtain the general solution.

$$8xp - 5yq + 4z = x^2 \cos x$$

س2:

حل المعادلة:

$$u_x - 4u_y + 7u_z - u = x + y + z + 1$$

س3:

حل كلاً من المعادلات التالية:

(أ)

$$yp - xq = x^3y + xy^3$$

(ب)

$$(y^3x - 2x^4)p + (2y^4 - x^2y)q = 9z(x^3 - y^3)$$

(ج)

$$yp - xq + z + x^2 + y^2 - 1 = 0$$

(د)

$$(x^2 - y^2 - z^2)p + 2xyq = 2xz$$

(هـ)

$$(z + e^x)p + (z + e^y)q = z^2 e^{x+y}$$

س4:

(أ)

(b) The velocity potential Φ of a stationary velocity field of fluid flow is $\Phi(x,y,z) = xy + xyz^2$. The velocity field is $V = \nabla\Phi$. The trajectories of the field are the curves C such that the tangent to the curve at each point has the direction of V at the point, i.e., the field lines. Find the trajectories.

(ب)

(c) The potential Φ of an electrostatic field is $\Phi(x,y,z) = 1/x + 1/y + 1/z$. The electric field is $E = -\nabla\Phi$. Find the field lines of the E field.

س5:

Find the general solution of each of the following.

(أ)

$u_x - u_y + 7u_z + u = Ae^{ax+by+cz}$ A, a, b, c constants

(ب)

$$xzu_x + yzu_y - (x^2+y^2)u_z = zy^2 \sin y$$

(ج)

$$(z-y)u_x + yu_y - zu_z = y(x+z) - y^2$$

(د)

$$\sqrt{x}\,u_x + \sqrt{y}\,u_y + \sqrt{z}\,u_z + u = x+y$$

(هـ)

$$x_1\frac{\partial u}{\partial x_1} + x_1x_2\frac{\partial u}{\partial x_2} + x_1x_2x_3\frac{\partial u}{\partial x_3} + x_1x_2x_3x_4\frac{\partial u}{\partial x_4} + u = x_1^2x_2^2$$

(و)

$$(x^2+y^2)p + 2xyq = xz\,;\quad x=a,\ y^2+z^2=a^2$$

(ز)

$$z(x+z)p - y(y+z)q = z(x-y)\,;\quad x=1,\ y=t,\ z=\sqrt{t}$$

(ح)

$$(y^2-x^2+2xz)p + 2y(z-x)q = 0\,;\quad x=0,\ y^2+4z^2=4$$

س6:

Obtain the general solution of the following.

(أ)

$$xu_x + (z+w)u_y + (y+w)u_z = y+z$$

(ب)

$$x u_x + y u_y + z u_z = u + \frac{xy}{z}$$

(ج)

$$(s-x)u_x + (s-y)u_y + (s-z)u_z = s-u, \quad s = x+y+z+u$$

س7:

Determine the integral surface which passes through given curve.

(أ)

$$(x+z)p + (y+z)q = 0 \; ; \; x = 1-t, \; y = 1+t, \; z = t$$

(ب)

$$xzp + yzq + xy = 0 \; ; \; xy = a^2, \; z = h$$

س8:

22 If a unique solution exists, find the Power Series expansion to second degree term about the indicated point containing the given curve.

(أ)

$$yp - xq = 0 \; ; \; (1, 0, 1) \; ; \; x = b, \; y = 0, \; z = t^2$$

(ب)

$$yp - xq = (x^2 + y^2) \; ; \; (0, 1, 0) \; ; \; x = u, \; y = t, \; z = 0$$

س9:

Obtain a solution to each of the following Cauchy problems

(b) $u_x + x u_y + xy u_z = xyzu$; $u = x^2 + y^2$ on the surface $x = s$, $y = t$, $z = 0$

س10:

أوجد الحل العام:

(أ)

$$4z_{xy} + z_{yy} = \cos y + 1$$

(ب)

$$z_{xx} + z_x + x + y + 1 = 0$$

(ج)

$$r - t - 3p + 3q = xy + e^{x+2y}$$

س11:

أوجد حلاً خاصاً لكل من المعادلات الآتية. أيضا أوجد الحل العام.

(أ)

$$r + 5s + 6t = \log(y - 2x)$$

(ب)

$$r - s - 2t = (y - 1)e^x$$

(ج)

$$r - 4t = \frac{4x}{y^2} - \frac{y}{x^2}$$

س12:

6. Use simple integrations, reduction to an ordinary differential equation, etc., to obtain a solution involving two arbitrary functions.

(أ)

$$t - xq = x^2$$

(ب)

$$xr = c^2 p + x^2 y^2 \qquad c = \text{const}$$

س13:

أوجد الحل العام لكل من المعادلات التالية:

(أ)

$$x^2 r - 2xy s - 3y^2 t + xp - 3yq = 2\log xy + 4x$$

(ب)

$$x^2 z_{xx} + 2xy\, z_{xy} + y^2 z_{yy} - nx z_x - ny z_y + nz = x + y \qquad n = \text{const} \neq 0$$

(ج)

$$(ax D_x + by D_y)^2 z + \lambda^2 z = 0 \qquad a, b, \lambda \text{ constants}$$

س14:

أوجد الحل العام للمعادلة:

$$(y^2-x^2)(r-t)+4(xp+yq-z)=0$$

س15:

أوجد الحل العام للمعادلة:

$$x^2u_{xx}+2xyu_{xy}+y^2u_{yy}+2yzu_{yz}+2xzu_{xz}+z^2u_{zz}=\ln xyz$$

س16:

صنف المعادلة. حولها للشكل النظامي(القياسي) و أوجد الحل العام:

(أ)

$$4z_{xx}-4z_{xy}+5z_{yy}=0$$

(ب)

$$r-t+p+q+x+y+1=0$$

(ج)

$$xr-(x+y)s+yt=\frac{x+y}{x-y}(p-q)$$

(د)

$$\frac{\partial}{\partial x}\left(x^2\frac{\partial z}{\partial x}\right)=x^2\frac{\partial^2 z}{\partial y^2}$$

(هـ)

$$(\sec^2 y)r-t+(2\tan y)q=0$$

(و)

$$y^2 z_{xx} - 2y z_{xy} + z_{yy} = z_x + 6y$$

س17:

صنف كلاً من المعادلات التالية:

(أ)

$$\frac{\partial u}{\partial t} = k\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \quad k>0 \text{ a real constant}$$

(ب)

$$u_{xx} + u_{yy} - u_{tt} = 0$$

(ج)

$$u_{xx} + u_{yy} + u_{zz} + u_{tt} = 0$$

س18:

24. In each of the following Cauchy problems determine whether (i) there exists a unique solution, (ii) there exist infinitely many solutions, (iii) no solution exists. If i, find the solution.

(أ)

$$u_{xx} - u_{tt} = 0;\ u(x,0) = \sin x,\ u_t(x,0) = b,\ b \text{ a constant}$$

(ب)

$$z_{xx} - 2z_{xy} + z_{yy} = 4e^{3y};\ z(x,0) = x^2;\ z_y(x,0) = \frac{1}{5}$$

س19:

حل مسألة كوشي لكلٍ مما يلي:

(أ)

$$y\,u_{xx} + (x+y)\,u_{xy} + x\,u_{yy} = xy\,;\ u(0,y) = y^2,\ u_x(0,y) = 1,\ y > 0$$

(ب)

$$u_{xx} + (2\cos x)u_{xy} - (\sin^2 x)u_{yy} - (\sin x)u_y = 0\,;\ u = x^2,\ u_y = 1 \text{ on } C \text{ is the curve } y = \sin x$$

س20:

حل مسألة القيمة الابتدائية:

The initial-value problem

$$u_{tt} - c^2 u_{xx} = F(x,t) \qquad -\infty < x < \infty;\ t \geq 0$$

$$u(x,0) = f(x) \qquad u_t(x,0) = g(x) \qquad -\infty < x < \infty$$

$$F(x,t) = A \sin(kx - \omega t), f(x) = g(x) = 0, k = \omega c$$

س21:

19 The string with fastened ends at $x = 0$, $x = b$ executes free vibrations after release from the initial displacement $f(x)$ and with initial speed $g(x)$. Determine the series expression for the subsequent displacement $u(x,t)$.

(c) $f(x) = A \sin \omega x$, $g(x) = 1$, $0 \leq x \leq b$ (ω is a constant and is not an integral multiple of π/b)

س22:

21 (a) The string with fastened ends at $x = 0$, $x = b$ executes forced vibrations under the external force per unit mass

$$F(t) = F_0 \sin \omega t$$

where F_0, ω are given positive constants. The string is released from rest with zero displacement. Use Eq. (4-41) to obtain an expression for the displacement $u(x,t)$. Distinguish the cases (i) $\omega \neq \omega_k$, all k; (ii) $\omega = \omega_k$, some k. Case ii illustrates *resonance* for the vibrating string with fastened ends.

21 (a) For the case of nonresonance

$$u = \frac{4cF_0}{b} \sum_{n=1}^{\infty} \frac{\omega \sin \omega_{2n-1} t - \omega_{2n-1} \sin \omega t}{\omega_{2n-1}^2 (\omega^2 - \omega_{2n-1}^2)} \sin \frac{(2n-1)\pi x}{b}$$

If $\omega = \omega_m$ (resonance case), m even, solution is given by the above series. If $\omega = \omega_{2r-1}$, then in the series the term involving ω_{2r-1} is replaced by the term

$$\frac{\sin \omega_{2r-1} t - \omega_{2r-1} t \cos \omega_{2r-1} t}{\omega_{2r-1}^3} \sin \frac{(2n-1)\pi x}{b}$$

Q1] $\xi = \text{Log}(x)$, $\eta = \log(y)$, now $Z_x = Z_\xi \xi_x = Z_\xi \frac{1}{x}$

$Z_y = Z_\eta \eta_y = Z_\eta \frac{1}{y}$

$\Rightarrow 8xp - 5yq + 4z = 8Z_\xi - 5Z_\eta + 4z$ ①

now solving the homogeneous ① $= 0$

$Z_h = e^{-\frac{4\xi}{8}} f(-5\xi - 8\eta) = e^{-\frac{\text{Log}(x)}{2}} f(-5\,\text{Log}(x) - 8\log(y))$

now for specific solutions since $G(x,y) = x^2\cos(x)$ is function of x only $\Rightarrow 8xZ_x + 4z = x^2\cos(x) \Rightarrow$

$Z_x + \frac{Z}{2x} = \frac{1}{8} x\cos(x)$ solution for this Z_p

$= \left(\int \frac{1}{8} x\cos(x)\, e^{\int \frac{1}{2x} dx}\, dx\right) e^{-\int \frac{1}{2x} dx} \Rightarrow Z = Z_h + Z_p$ □

Q2] $U_x - 4U_y + 7U_z - u = x + y + z + 1$

a solution is of the form $u_h = e^{-Dx/A} f(ax + by + cz)$, $Aa + Bb + Cc = 0$

choose $a = 1$ $b = 2$, $C = 1 \Rightarrow 1(1) + (2)(-4) + 7(1) = 0$

$\Rightarrow$ solution for the homogenous equation is

$u_h = e^x f(x + 2y + z)$, for the non homogenous

try $u_p = ax + by + cz + d$ now plugging it in the equation yields $a - 4b + 7c - (ax + by + cz + d)$

$= x + y + z + 1$ equating the coefficients $\Rightarrow$

$-a = 1$, $-b = 1$, $-c = 1$, $a - 4b + 7c - d = 1 \Rightarrow$

$d = -1 + 4 - 7 - 1 = -5 \Rightarrow u_p = -x - y - z - 5 \Rightarrow$

$u = u_h + u_p$

Q3] (i) $yz_x - xz_y = x^3y + xy^3$

using Lagrange's method $\frac{dx}{y} = \frac{dy}{-x} = \frac{dz}{x^3y + xy^3}$

$\Rightarrow -x\,dx = y\,dy$ Integrating both sides we get

$\frac{x^2}{2} + \frac{y^2}{2} = c_1$, now $\frac{dy}{-x} = \frac{dz}{x(x^2y + y^3)} \Rightarrow$

$-dy = \frac{dz}{((2c_1 - y^2)y + y^3} \Rightarrow (-((2c_1 - y^2)y + y^3))\,dy = dz$

Integrating both sides $-2c_1\frac{y^2}{2} + \frac{y^4}{4} - \frac{y^4}{4} = z + c_2$

$c_1 = \frac{x^2}{2} + \frac{y^2}{2} \Rightarrow c_2 = -(\frac{x^2}{2} + \frac{y^2}{2})y^2 - z$

$\Rightarrow$ the General Solution $F(c_1, c_2) = F(\frac{x^2+y^2}{2}, -(\frac{x^2}{2} + \frac{y^2}{2})y^2 - z)$

for an arbitrary function F

Q3] (ii) $(y^3x - 2x^4)p + (2y^4 - x^3y)q = qz(x^3 - y^3)$

Q3] (i) $yp - xq + z + x^2 + y^2 - 1 = 0$

take $\frac{dy}{dx} = \frac{-x}{y} \Rightarrow \frac{x^2}{2} + \frac{y^2}{2} = c_1$, $\eta(x,y) = \frac{x^2}{2} + \frac{y^2}{2}$

$\xi = x$

$\zeta(\xi,\eta) = 1 \qquad A(\xi,\eta) = \sqrt{2\eta - \xi^2} \Rightarrow \int \frac{C}{A}\, d\xi$

$= \int \frac{1}{\sqrt{2\eta - \xi^2}}\, d\xi = \frac{1}{\sqrt{2\eta}} \sin^{-1}\left(\frac{\xi}{\sqrt{2\eta}}\right) \Rightarrow Z_h = f(\eta)\, e^{-\int \frac{C}{A} d\xi}$

$= f\left(\frac{x^2+y^2}{2}\right) e^{-\frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{x}{\sqrt{x^2+y^2}}\right)}$ to get Z_p try solution of the form $Z_p = ax^2 + by^2 + cxy + dx + ey + g$ o

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Q3] (ii) $(x^2 - y^2 - z^2) Z_x + 2xy Z_y = 2xz$

applying Lagrange's method, we have the Subsidiary equation

$\frac{dx}{x^2-y^2-z^2} = \frac{dy}{2xy} = \frac{dz}{2xz} \Rightarrow \frac{dy}{y} = \frac{dz}{z} \Rightarrow z\,dy - y\,dz$

$\Rightarrow \frac{z}{y} = c_1 \Rightarrow \frac{dx}{x^2 - y^2 - c_1^2 y^2} = \frac{dy}{2xy} \Rightarrow$

$\frac{2x\,dx}{x^2 - y^2 - c_1^2 y^2} = \frac{dy}{y}$ Integrating both sides gi

$\ln(x^2 - y^2 - c_1^2 y^2) - \ln(y) = c_2 \qquad c_1 = \frac{z}{y}$

$\Rightarrow \ln\left(\frac{x^2 - y^2 - z^2}{y}\right) = c_2 \Rightarrow \frac{x^2 - y^2 - z^2}{y} = c_3$

The general Solution $F(c_1, c_2) = F\left(\frac{z}{y}, \ln\left(\frac{x^2 - y^2 - z}{y}\right)\right.$

Q4] (b) $\Phi(x,y,z) = xy + xyz^2$, $\nabla\Phi = (y+yz^2)\vec{i} + (x+xz^2)\vec{j} + 2zxy\vec{k} \Rightarrow V = p\vec{i} + q\vec{j} + R\vec{k}$, the trajectories are the solutions to $pz_x + qz_y = R$, applying lagrange's Method,

$$\frac{dx}{y+yz^2} = \frac{dy}{x+xz^2} = \frac{dz}{2xyz} \Rightarrow \frac{dy}{(1+z^2)} = \frac{dz}{2yz}$$

$\Rightarrow (2yz)dy - (1+z^2)dz = 0 \Rightarrow 2y\,dy = \frac{1+z^2}{z}dz$

Integrating both sides $y^2 - \ln|z| - \frac{z^2}{2} = c_1$

now $y^2 = c_1 + \ln|z| - \frac{z^2}{2}$, $\frac{dx}{1+z^2} = \frac{dz}{2xz}$

$\Rightarrow x^2 - \ln|z| - \frac{z^2}{2} = c_2 \Rightarrow F(c_1, c_2)$ is the solution

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4] (c) $\Phi(x,y,z) = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$, $E = -\nabla\Phi$, the fields lines lie on the solutions of $pz + qz_y = R$, now

$E = -\left(\frac{-1}{x^2}\vec{i} - \frac{1}{y^2}\vec{j} - \frac{1}{z^2}\vec{k}\right) = \frac{1}{x^2}\vec{i} + \frac{1}{y^2}\vec{j} + \frac{1}{z^2}\vec{k}$

now $\frac{1}{x^2}z_x + \frac{1}{y^2}z_y = \frac{1}{z^2}$, $\frac{dx}{\frac{1}{x^2}} = \frac{dy}{\frac{1}{y^2}} = \frac{dz}{\frac{1}{z^2}}$

$\Rightarrow x^2dx = y^2dy = z^2dz \Rightarrow$ by Integrating both equations $x^2dx = y^2dy$, $y^2dy = z^2dz$

$\Rightarrow \frac{x^3}{3} - \frac{y^3}{3} = c_1$, $\frac{y^3}{3} - \frac{z^3}{3} = c_2 \Rightarrow$

a solution of the form $F(c_1, c_2)$

Q5] (i) $u_x - u_y + 7u_z + u = Ae^{ax+by+cz}$, a, b, c, A consts

first solution to the homogeneous equation, $a'=1, b'=-1, c'=7, d=1$

$u_h = e^{-\frac{d}{a'}x} f(Ax + By + Cz)$, where $Aa' + Bb' + Cc' = 0$

choose $A=2, B=9, C=1 \Rightarrow u_h = e^{-x} f(2x + 9y + z)$

Now u_p is of the form $MAe^{ax+by+cz}$, where M is const

substituting in the equation yields $(ma - bm + 7cm) Ae^{ax+by+cz} = Ae^{ax+by+cz} \Rightarrow ma - bm + 7cm = 1 \Rightarrow m = \frac{1}{a-b+7c}$

$\Rightarrow u = u_h + u_p$

Q5] (ii) $xz u_x + yz u_y - (x^2 + y^2) u_z = z y^2 \sin(y)$

to solve this we have the subsidiary equations

$\frac{dx}{xz} = \frac{dy}{yz} = \frac{dz}{-(x^2+y^2)} = \frac{du}{zy^2\sin(y)}$, first we have

$\frac{dx}{x} = \frac{dy}{y} \Rightarrow y\,dx - x\,dy = 0 \Rightarrow \left(\frac{y}{x}\right) = c_1$

now we have $\frac{dx}{xz} = \frac{dz}{-(x^2 + c_1^2 x^2)} \Rightarrow -\frac{(x^2 + c_1^2 x^2)}{x} dx = 2dz$

$\Rightarrow -\frac{x^2}{2} - c_1^2 \frac{x^2}{2} - \frac{z^2}{2} = c_2$, $c_1 = \frac{y}{x}$

$\Rightarrow -\frac{x^2}{2} - \left(\frac{y}{x}\right)^2 \frac{x^2}{2} - \frac{z^2}{2} = c_2$

now $\frac{dy}{yz} = \frac{du}{zy^2\sin(y)} \Rightarrow y\sin(y)\,dy = du$

$\int y\sin(y)\,dy = u \Rightarrow -y\cos(y) + \sin(y) - u = c_3$

$\Rightarrow$ the general solution of the form $F(c_1, c_2, c_3)$

Q5] (1) $\sqrt{x}\,u_x + \sqrt{y}\,u_y + \sqrt{z}\,u_z + u = x + y$ ①

take $\xi = \sqrt{x}$, $\eta = \sqrt{y}$, $\nu = \sqrt{z}$

$\Rightarrow u_x = u_\xi \xi = u_\xi \frac{1}{2\sqrt{x}}$, $u_y = u_\eta \eta_y = u_\eta \frac{1}{2\sqrt{y}}$, $u_z = u_\nu \nu_z = u_\nu \frac{1}{2\sqrt{z}}$

$\Rightarrow$ ① Becomes $\frac{1}{2} u_\xi + \frac{1}{2} u_\eta + \frac{1}{2} u_\nu + u = \xi^2 + \eta^2$

$A = \frac{1}{2}$, $B = \frac{1}{2}$, $C = \frac{1}{2}$, $d = 1$, to solve the homogenous equation choose $a = 2$, $b = 2$, $c = -4$ $\Rightarrow Aa + Bb + Cc = 0$

$\Rightarrow u_h = e^{-D\xi/A} f(2\xi + 2\eta - 4\nu) = e^{-2\sqrt{x}} f(2\sqrt{x} + 2\sqrt{y} - 4\sqrt{z})$

to solve ① Let $u_p = A\xi^2 + B\eta^2 + C\xi\eta + D\xi + E\eta + F$, plugging it in the equation we get

$\frac{1}{2}(2A\xi + C\eta + D) + \frac{1}{2}(2B\eta + C\xi + E\eta) + A\xi^2 + B\eta^2 + C\xi\eta + D\xi + E\eta + F = \xi^2 + \eta^2$

$A = B = 1$, $A + \frac{1}{2}C + D = 0$, $B + \frac{1}{2}C + E = 0$, but $E = 0$ $F = 0$

$\Rightarrow D = E = -1$

$\Rightarrow u_p = \xi^2 + \eta^2 - \xi - \eta = x + y - \sqrt{x} - \sqrt{y}$

$\Rightarrow u = u_h + u_p$

Q(7) (i) $(x+z)p + (y+z)q = 0$, $x = 1-t$, $y = 1+t$, $z = t$

answer:

the Subsiday equations $\frac{dx}{x+z} = \frac{dy}{y+z} = \frac{dz}{0} \Rightarrow z = c_1$

now $\frac{dx-dy}{x-y} = \frac{dy}{y+c_1} \Rightarrow \frac{x-y}{y+c_1} = c_2$, $c_1 = z$

to find relation between c_1, c_2 take $x = 1-t$, $y = 1+t$

$\Rightarrow \frac{-2t}{1+t+c_1} = c_2$ for $t = 1 \Rightarrow \frac{-2}{2+c_1} = c_2 \Rightarrow$

$c_2(c_1+2) + 2 = 0 \Rightarrow \left(\frac{x-y}{y+z}\right)(z+2) + 2 = 0$ is the Integral surface

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$(xz)p + (yz)q = -xy$, $xy = a^2$, $z = h$

answer:

the Subsiday equations $\frac{dx}{xz} = \frac{dy}{yz} = \frac{dz}{-xy}$

now we have $\frac{dx}{x} = \frac{dy}{y} \Rightarrow \frac{y}{x} = c_1 \Rightarrow y = xc_1$

$\frac{dx}{xz} = \frac{dz}{-x^2 c_1} \Rightarrow -xc_1\,dx = z\,dz \Rightarrow$

$-\frac{x^2 c_1}{2} - \frac{z^2}{2} = c_2$

but $c_1 = \frac{y}{x}$

$\Rightarrow \frac{-xy}{2} - \frac{z^2}{2} = c_2$

$\Rightarrow -\frac{a^2}{2} - \frac{h^2}{2} = c_2$

Q8)(i) $yp - xq = 0$, $(1, 0, 1)$, $x = t$, $y = 0$, $z = t^2$

$P = y$, $Q = -x$, $R = 0$ are analytic functions in $\mathbb{R}^3$

& $f(t) = t$, $g(t) = 0$, $h(t) = t^2$ the point $(1, 0, 1)$

when $t_0 = 1$, $P_0(x_0, y_0, z_0) = y_0 = 0$

$Q_0(x_0, y_0, z_0) = -x_0 = -1$, $R(x_0, y_0, z_0) = 0$

$f'(t) = 1$, $g'(t) = 0$, $h'(t) = 2t$

$f'(t_0) = f'(1) = 1$ $g'(t_0) = 0$

now $P_0 g'_{t_0} - Q_0 f' = 0 - (-1)(1) = 1 \neq 0$

$$\frac{dx}{y} = \frac{dy}{-x} = \frac{dz}{0} \Rightarrow \frac{x^2}{2} + \frac{y^2}{2} = \frac{c}{2}, \quad z = c_1$$

$$z = f\left(\frac{x^2}{2} + \frac{y^2}{2}\right), \quad t^2 = f\left(\frac{t^2}{2}\right)$$

$$\Rightarrow f(t) = 2t = \varphi(x, y) = 2\left(\frac{x^2}{2} + \frac{y^2}{2}\right)$$

now $P_0 = \frac{\partial z}{\partial x}\Big|_{(1,1)} = 2$, $q_0 \frac{\partial z}{\partial y}\Big|_{(1,1)} = 2$

$\varphi_{xx}(1,1) = 2$, $\varphi_{xy}(1,1) = 0$, $\varphi_{yy}(1,1) = 2$

$$\Rightarrow z = \varphi(x, y) = 1 + 2(x-1) + 2(y-1)$$
$$+ \frac{1}{2}(2)(x-1)^2 + \frac{1}{2}(2)(y-1)^2 + \dots \neq 0$$

Q 9) (b) $u_x + xu_y + yu^2 = xyzu$, $u = x^2 + y^2$

$x = s,\ y = t,\ z = 0$

the Subsiday equation $\frac{dx}{1} = \frac{dy}{x} = \frac{dz}{xy} = \frac{du}{xyzu}$

now $\frac{dx}{1} = \frac{dy}{x} \Rightarrow y = \frac{x^2}{2} = c_1$

$\frac{du}{xyzu} = \frac{dz}{xy} \Rightarrow z\,dz = \frac{du}{u} \Rightarrow \frac{z^2}{2} - \ln(u) = c_2$

$\frac{dy}{x} = \frac{dz}{xy} \Rightarrow y\,dy = dz \Rightarrow \frac{y^2}{2} - z = c_3$

note that $u = x^2 + y^2$, $x = s$ $y = t$, $z = 0$

$-\ln(s^2 + t^2) = c_2$, $t - \frac{s^2}{2} = c_1$, $\frac{t^2}{2} = c_3$

$\Rightarrow s^2 + t^2 = e^{-c_2}$, $t = \frac{s^2}{2} - \frac{t^2}{2} = t$

note that $t = c_1 + \frac{s^2}{2} \Rightarrow \frac{t^2}{2}\left(c_1 + \frac{s^2}{2}\right)^2 = c_3$

$\frac{1}{2}\left(c_1 + \frac{s^2}{2}\right)^2 = c_3 = e^{-c_2} - s^2$

take $s^2 = K$

$\Rightarrow \frac{1}{2}\left(c_1 + \frac{K}{2}\right)^2 - c_3 = \frac{1}{2}\left(c_1 + \frac{K}{2}\right)^2 - e^{-c} - K^2$

now we solve to get fuchional Relation $G(c_1, c_2, c_3) = 0$ to get the simple take $K = 0 \Rightarrow$

$\frac{1}{2}c_1^2 - c_3 = \frac{1}{2}c_1^2 - e^{-c_2} \Rightarrow$

$-(-\ln(x^2+y^2))$

$\frac{1}{2}\left(y - \frac{x^2}{2}\right)^2 - \frac{y^2}{2} - z = \frac{1}{2}\left(y - \frac{x^2}{2}\right)^2 - e$

Kingdom of Saudi Arabia
KING SAUD UNIVERSITY
College of Science
Department of Mathematics

المملكة العربية السعودية
جامعة الملك سعود
كلية العلوم
قسم الرياضيات

واجب رقم 1 (1442) للمقرر 425 ريض

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س) ٢٠ م

$$\frac{dx}{1} = \frac{dy}{-4} = \frac{dz}{7} = \frac{du}{x+y+z+u+1}$$

$-4\,dx = dy \Rightarrow -4x = y + c_1^* \Rightarrow -y-4x = c_1^* \Rightarrow \boxed{y+4x = c_1}$

$7\,dy = -4\,dz \Rightarrow 7y = -4z + c_2 \Rightarrow \boxed{7y+4z = c_2}$

$7\,dx = dz \Rightarrow \boxed{7x - z = c_3}$

$F(y+4x,\ 7y+4z,\ 7x-z) = 0$

(P) (س ۳

$$\frac{dx}{y} = \frac{dy}{-x} = \frac{du}{x^3y+xy^3}$$

$\frac{dx}{y} = \frac{dy}{-x} \Rightarrow -x\,dx = y\,dy$

$\Rightarrow -x^2 - y^2 = 2C_1^*$

$\Rightarrow \boxed{x^2+y^2 = C_1}$

$\frac{dx}{y} = \frac{du}{xy(x^2+y^2)} = \frac{du}{xy\,C_1}$

$x\,dx = \frac{1}{C_1}\,du$

$\frac{x^2}{2} = \frac{1}{C_1}u + C_2^*$

$x^2 = \frac{2u}{(x^2+y^2)} + C_2^*$

$\Rightarrow \boxed{x^2 - \frac{2u}{x^2+y^2} = C_2}$

$$\therefore F\left(x^2+y^2,\ x^2 - \frac{2u}{x^2+y^2}\right) = 0$$

(ب ۲،۳،۲

$$\frac{dx}{y^3x-2x^4} = \frac{dy}{2y^4-x^3y} = \frac{dz}{9z(x^3-y^3)}$$

$$\frac{\frac{dx}{x}+\frac{dy}{y}+\frac{dz}{3z}}{y^3-2x^3+2y^3-x^3+3x^3-3y^3} = \frac{\frac{dx}{x}+\frac{dy}{y}+\frac{dz}{3z}}{0}$$

$\Rightarrow \frac{dx}{x}+\frac{dy}{y}+\frac{dz}{3z} = 0 \Rightarrow \ln|x| + \ln|y| + \frac{1}{3}\ln|z| = C_1^*$

$\Rightarrow xyz^{1/3} = C_{1_0}^*$

$= \boxed{x^3y^3z = C_1}$

$$\frac{x^2dx + y^2dy}{2y^6 - 2x^6} = \frac{dz}{9z(x^3-y^3)}$$

$$\frac{x^2dx + y^2dy}{(y^3-x^3)(y^3+x^3)} = \frac{2}{9}\frac{dz}{z(x^3-y^3)}$$

$$\frac{x^2dx + y^2dy}{x^3+y^3} = -\frac{2}{9}\frac{dz}{z}$$

$$\frac{1}{3}\ln|x^3+y^3| = -\frac{2}{9}\ln|z| + C_2^*$$

$$\ln|x^3+y^3| + \frac{2}{3}\ln|z| = 3C_2^*$$

$$\ln|(x^3+y^3)z^{2/3}| = 3C_2^*$$

$$(x^3+y^3)z^{2/3} = C_2 \Rightarrow F(x^3y^3z, (x^3+y^3)z^{2/3}) = 0$$

p.p (2.

$$\frac{dx}{y} = \frac{dy}{-x} = \frac{dz}{-z-x^2-y^2+1}$$

$$\frac{dx}{y} = \frac{dy}{-x} \Rightarrow \boxed{x^2+y^2 = C_1} \qquad y = \sqrt{C_1 - x^2}$$

$$\frac{dx}{\sqrt{C_1 - x^2}} = \frac{dz}{-z-C_1+1}$$

$$\sin^{-1}\left(\frac{x}{\sqrt{C_1}}\right) + \ln|-z-C_1+1| = C_2$$

$$\Rightarrow \boxed{\sin^{-1}\left(\frac{x}{\sqrt{x^2+y^2}}\right) + \ln|-z-x^2-y^2+1| = C_2}$$

$$F\left(x^2+y^2, \sin^{-1}\left(\frac{x}{\sqrt{x^2+y^2}}\right) + \ln|-z-x^2-y^2+1|\right) = 0$$

$$\frac{dx}{x^2-y^2-z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}$$ ر.م (2)

$$\frac{dy}{2xy} = \frac{dz}{2xz} \Rightarrow \frac{y}{z} = c_1^* \Rightarrow \frac{z}{y} = \frac{1}{c_1^*} \Rightarrow \boxed{\frac{z}{y} = c_1} \Rightarrow z = c_1 y$$

$$\frac{dx}{x^2-y^2-y^2c_1} = \frac{dy}{2xy}$$

$$(2xy)\,dx - (x^2-y^2-y^2c_1)\,dy = 0$$

$$2xy\,dx - x^2\,dy = -y^2\,dy - y^2c_1\,dy$$

$$y^2\,d\left(\frac{x^2}{y}\right) = -y^2\,dy - y^2c_1\,dy$$

$$d\left(\frac{x^2}{y}\right) = -dy - c_1\,dy \qquad (y \neq 0)$$

$$\frac{x^2}{y} = -y - c_1 y + c_2^*$$

$$\frac{x^2}{y} = -y - \frac{z}{y}\cdot y + c_2^*$$

$$\boxed{\frac{x^2}{y} + y + z = c_2}$$

$$\Rightarrow F\left(\frac{z}{y}, \frac{x^2}{y} + y + z\right) = 0$$

(b)

$$\frac{dx}{z+e^x} = \frac{dy}{z+e^y} = \frac{dz}{z^2-e^{x+y}}$$

$$\frac{e^y dx - z dy + dz}{ze^y + e^{x+y} - z^2 - ze^y + z^2 - e^{x+y}} = \frac{e^y dx - z dy + dz}{0}$$

$$\Rightarrow e^y dx - z dy + dz = 0$$

$$\Rightarrow dx - z e^{-y} dy + e^{-y} dz = 0$$

$$\Rightarrow dx + d(z e^{-y}) = 0$$

$$\Rightarrow \boxed{x + z e^{-y} = c_1}$$

$$\frac{-z dx + e^x dy + dz}{-z^2 - ze^x + ze^{y+x} + z^2 - e^{x+y}} \Rightarrow -z dx + e^x dy + dz = 0$$

$$\Rightarrow -ze^{-x} dx + e^x dz + dy = 0$$

$$\Rightarrow d(z e^{-x}) + dy = 0$$

$$\Rightarrow \boxed{z e^{-x} + y = c_2}$$

$$F(x + z e^{-y},\ y + e^{-x} z) = 0$$

(س٤)

(أ)

$$V = \nabla\varphi = \frac{\partial\varphi}{\partial x}\,i + \frac{\partial\varphi}{\partial y}\,j + \frac{\partial\varphi}{\partial z}\,k$$

$$= (y + yz^2)\,i + (x + xz^2)\,j + (2xyz)\,k$$

$$\frac{dx}{y+yz^2} = \frac{dy}{x+xz^2} = \frac{dz}{2xyz}$$

$$\frac{dx}{y(1+z^2)} = \frac{dy}{x(1+z^2)} \Rightarrow \boxed{x^2 - y^2 = C_1}$$

$$\frac{dy}{x(1+z^2)} = \frac{dz}{2xyz}$$

$$2y\,dy = \left(\frac{1}{z} + z\right)dz$$

$$2y^2 = 2\ln|z| + z^2 + C_2$$

$$\boxed{2y^2 - 2\ln(z) - z^2 = C_2}$$

(ب)

$$\varphi = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$

$$E = -\nabla\varphi = \frac{1}{x^2}\,i + \frac{1}{y^2}\,j + \frac{1}{z^2}\,k$$

$$x^2\,dx = y^2\,dy = z^2\,dz$$

$$x^3 = y^3 = z^3 = C_1$$

$$x = y = z = c$$

∴ خطوط الحقل هي: $x = C_1 = y = z$

(س

$$\frac{dx}{xz} = \frac{dy}{yz} = \frac{dz}{-(x^2+y^2)} = \frac{du}{zy^2\sin y}$$ ٢٠ (ب)

$$\frac{dx}{xz} = \frac{dy}{yz} \Rightarrow \ln|x| = \ln|y| + c_1^*$$

$$\Rightarrow \ln|x| - \ln|y| = c_1^*$$

$$\Rightarrow \ln\left|\frac{x}{y}\right| = c_1^*$$

$$\Rightarrow \frac{x}{y} = c_1' \Rightarrow \boxed{\frac{y}{x} = c_1} \Rightarrow y = c_1 x$$

$$\frac{dy}{yz} = \frac{du}{zy^2\sin y}$$

$$y\sin y\, dy = du$$

$$-y\cos y - \int -\cos y\, dy = u + c_3^*$$

$$-y\cos y + \sin y = u + c_3^*$$

$$\boxed{\sin y - y\cos y - u = c_3}$$

$$\frac{dx}{xz} = -\frac{dz}{x^2+y^2}$$

$$(x + c_1^2 x)\, dx = -z\, dz$$

$$x^2 + c_1^2 x^2 = -z^2 + 2c_2^*$$

$$\boxed{x^2 + y^2 + z^2 = c_2}$$

$$F(c_1, c_2, c_3) = 0$$

$$\frac{dx_1}{x_1} = \frac{dx_2}{x_1 x_2} = \frac{dx_3}{x_1 x_2 x_3} = \frac{dx_4}{x_1 x_2 x_3 x_4} = \frac{du}{x_1^2 x_2^2 - u} \quad \text{p.p (*)}$$

$$\frac{dx_1}{x_1} = \frac{dx_2}{x_1 x_2} \Rightarrow x_1 = \ln|x_2| + c_1 \Rightarrow \boxed{x_1 - \ln|x_2| = c_1} \Rightarrow \begin{matrix} \ln|x_2| = x_1 - c_1 \\ x_2 = e^{x_1 - c_1} \end{matrix}$$

$$\frac{dx_2}{x_1 x_2} = \frac{dx_3}{x_1 x_2 x_3} \Rightarrow x_2 = \ln|x_3| + c_2 \Rightarrow \boxed{x_2 - \ln|x_3| = c_2}$$

$$\frac{dx_3}{x_1 x_2 x_3} = \frac{dx_4}{x_1 x_2 x_3 x_4} \Rightarrow \boxed{x_3 - \ln|x_4| = c_3}$$

$$\frac{dx_1}{x_1} = \frac{du}{x_1^2 x_2^2 - u} \Rightarrow x_1^2 e^{2(x_1 - c_1)} dx_1 - u\,dx_1 - x_1\,du = 0$$

$$\Rightarrow x_1^2 e^{2x_1 - 2c_1} dx_1 - d(x_1 u) = 0$$

$$\Rightarrow \int \underbrace{x_1^2}_{u} \underbrace{e^{2x_1 - 2c_1}}_{dv} dx_1 - x_1 u = c_4^*$$

$$\Rightarrow \frac{x_1^2 e^{2x_1 - 2c_1}}{2} - \int \underbrace{x_1}_{u} \underbrace{e^{2x_1 - 2c_1}}_{dv} dx_1 - x_1 u = c_4^*$$

$$\Rightarrow \frac{x_1^2 e^{2x_1 - 2c_1}}{2} - \left[\frac{x_1}{2} e^{2x_1 - 2c_1} - \int \frac{e^{2x_1 - 2c_1}}{2} dx_1\right] - x_1 u = c_4^*$$

$$\Rightarrow \frac{x_1^2 e^{2x_1 - 2c_1}}{2} - x_1 \frac{e^{2x_1 - 2c_1}}{2} + \frac{1}{4} e^{2x_1 - 2c_1} - x_1 u = c_4^*$$

$$\Rightarrow 2x_1^2 e^{2x_1 - 2c_1} - 2x_1 e^{2x_1 - 2c_1} + e^{2x_1 - 2c_1} - 4x_1 u = 4c_4^*$$

$$\Rightarrow 2x_1^2 e^{2x_1 - 2x_1 + 2\ln|x_2|} - 2x_1 e^{2x_1 - 2x_1 + 2\ln|x_2|} + e^{2x_1 - 2x_1 + 2\ln x_2} - 4x_1 u = c_4^*$$

$$\Rightarrow \boxed{2x_1^2 x_2^2 - 2x_1 x_2^2 + x_2^2 - 4x_1 u = c_4}$$

$$F(c_1, c_2, c_3, c_4) = 0$$

$$\frac{dx}{x^2+y^2} = \frac{dy}{2xy} = \frac{dz}{xz} \quad ; x=a,\ y^2+z^2=a^2$$

$$\boxed{y^2+z^2=x^2}$$

$$\frac{dx}{x^2+y^2} = \frac{dy}{2xy} \Rightarrow 2xy\,dx - x^2dy = y^2dy$$

$$\Rightarrow y^2 d\left(\frac{x^2}{y}\right) = y^2 dy \quad ; y\neq 0$$

$$\frac{x^2}{y} = y + c_1^*$$

$$\boxed{\frac{x^2}{y} - y = c_1}$$

$$\frac{dy}{2xy} = \frac{dz}{xz} \Rightarrow \frac{1}{2}\ln|y| = \ln|z| + c_2^*$$

$$\Rightarrow \ln\left|\frac{y^{1/2}}{z}\right| = c_2^*$$

$$\Rightarrow \frac{y^{1/2}}{z} = c_2^* \Rightarrow \boxed{\frac{y}{z^2} = c_2}$$

$$F\left(\frac{x^2}{y} - y,\ \frac{y}{z^2}\right) = 0$$

$$\frac{dx}{x} = \frac{dy}{z+u} = \frac{dz}{y+u} = \frac{du}{y+z}$$

$$\frac{dy-dz}{z-y} = \frac{dx}{x} \Rightarrow -\ln|z-y| = \ln|x| + c_1^*$$

$$\Rightarrow \boxed{\frac{z-y}{x} = c_1} \Rightarrow z-y = x c_1$$

$$\frac{dy-du}{u-y} = \frac{dx}{x} \Rightarrow -\ln|u-y| = \ln|x| + c_2^*$$

$$\Rightarrow \boxed{\frac{u-y}{x} = c_2} \Rightarrow u-y = x c_2$$

$$\boxed{\frac{z-y}{u-y} = \frac{x c_1}{x c_2} = c_3}$$

$$\therefore F\left(\frac{z-y}{x}, \frac{u-y}{x}, \frac{z-y}{u-y}\right) = 0$$

ب) م.م

$$\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z} = \frac{du}{u + \frac{xy}{z}}$$

$$\frac{dx}{x} = \frac{dy}{y}$$

$$\ln|x| = \ln|y| + c_1^*$$

$$\Rightarrow \boxed{\frac{x}{y} = C_1}$$

$$\frac{dy}{y} = \frac{dz}{z}$$

$$\ln|y| = \ln|z| + c_2^*$$

$$\Rightarrow \boxed{\frac{y}{z} = C_2}$$

$$\frac{dx}{x} = \frac{du}{u + \frac{xy}{z}} = \frac{du}{u + xc_2}$$

$$(u + xc_2)\,dx - x\,du = 0$$

$$M = u + xc_2 \quad , \quad N = -x$$

$$M_u = 1 \quad , \quad N_x = -1$$

$$\frac{M_u - N_x}{N} = \frac{1+1}{-x} = \frac{2}{-x}$$

$$\Rightarrow \mu = e^{-2\ln x} = \frac{1}{x^2}$$

$$\left(\frac{u}{x^2} + \frac{c_2}{x}\right) dx - \frac{1}{x}\,du = 0$$

$$F(x,u) = \int \frac{u}{x^2} + \frac{c_2}{x}\,dx = -\frac{u}{x} + c_2 \ln|x| + f(u)$$

$$F_u(x,u) = -\frac{1}{x} + f'(u) = -\frac{1}{x} \Rightarrow f'(u) = 0$$

$$f(u) = C_3$$

$$\therefore -\frac{u}{x} + \frac{y}{z}\ln x = C_3$$

$$F\left(\frac{x}{y}, \frac{y}{z}, -\frac{u}{x} + \frac{y}{z}\ln x\right) = 0$$

$$\frac{dx}{s-x} = \frac{dy}{s-y} = \frac{dz}{s-z} = \frac{du}{s-u} \qquad (2.)$$

$$\frac{dx-dy}{-x+y} = \frac{dy-dz}{-y+z} \Rightarrow -\ln|-x+y| = -\ln|-y+z| + c_1^*$$

$$\Rightarrow -\ln(-x+y) + \ln(-y+z) = c_1^*$$

$$\Rightarrow \ln\left|\frac{-y+z}{-x+y}\right| = c_1^* \Rightarrow \boxed{\frac{y-z}{x-y} = c_1}$$

$$\frac{dx-dy}{-x+y} = \frac{dz-du}{-z+u} \Rightarrow -\ln|-x+y| = -\ln|-z+u| + c_2^*$$

$$\Rightarrow \ln\left|\frac{-z+u}{-x+y}\right| = c_2^*$$

$$\Rightarrow \boxed{\frac{z-u}{x-y} = c_2}$$

$$\frac{y-z}{\cancel{x-y}} \cdot \frac{\cancel{x-y}}{z-u} = \frac{c_1}{c_2} = c_3 \Rightarrow \boxed{\frac{y-z}{z-u} = c_3}$$

$$F(c_1, c_2, c_3) = 0$$

(س 7)

$\Gamma: x = 1 - t, \; y = 1 + t, \; z = t$ (1)

$$\frac{dx}{x+z} = \frac{dy}{y+z}, \quad dz = 0$$
$$z = c_2$$

$$\frac{dx}{x+c_2} = \frac{dy}{y+c_2} \Rightarrow \ln(x+c_2) = \ln(y+c_2) + c_2^*$$

$$\Rightarrow \ln\left(\frac{x+c_2}{y+c_2}\right) = c_1^*$$

$$\Rightarrow \frac{x+c_2}{y+c_2} = c_1 \Rightarrow \boxed{\frac{x+z}{y+z} = c_1}$$

$$z = c_2 = t, \quad \frac{1-t+t}{1+t+t} = c_1 \Rightarrow \frac{1}{1+2t} = c_1$$

$$\Rightarrow \frac{1}{1+2c_2} = c_1$$

$$\Rightarrow \frac{1}{1+2z} = \frac{x+z}{y+z}$$

$$\Rightarrow y + z = (x+z)(1+2z)$$
$$= x + 2xz + z + 2z^2$$

$$\Rightarrow \boxed{y = x + 2xz + 2z^2}$$

$xy = a^2, \; z = h$ (1)

$$\frac{dx}{xz} = \frac{dy}{yz} = \frac{dz}{-xy} \qquad (1.5$$

$$\frac{dx}{zx} = \frac{dy}{zy} \Rightarrow \boxed{\frac{x}{y} = c_1} \Rightarrow x = c_1 y \quad \Big| \quad \frac{dy}{yz} = \frac{dz}{-c_1 y^2}$$

$$\frac{x}{y} = c_1$$

$$y^2 \frac{x}{y} = y^2 c_1$$

$$yx = y^2 c_1$$

$$a^2 = y^2 c_1$$

$$\Rightarrow -c_1 y \, dy = z \, dz$$

$$\Rightarrow -c_1 y^2 = z^2 + 2c_2^*$$

$$\Rightarrow -\frac{x}{y} y - z^2 = 2c_2^*$$

$$\Rightarrow xy + z^2 = -2c_2^*$$

$$\Rightarrow \boxed{xy + z^2 = c_2}$$

$$a^2 + h^2 = c_2$$

$$y^2 c_1 + h^2 = c_2$$

$$\Rightarrow y^2 \frac{x}{y} + h^2 = xy + z^2$$

$$\Rightarrow yx + h^2 = xy + z^2$$

$$\Rightarrow \boxed{z^2 - h^2 = 0}$$

س 8 (i)

$\Gamma : x = t \ , \ y = 0 \ , \ z = t^2$

$t = 1$

$\frac{dx}{y} = \frac{dy}{-x}$ ، م.م

$dz = 0$

$\boxed{z = c_2}$

$-x\,dx = y\,dy \Rightarrow$

$-x^2 = y^2 + 2c_1^*$

$y^2 + x^2 = -2c_1^*$

$\boxed{x^2 + y^2 = c_1}$

$\begin{vmatrix} 1 & 0 \\ y & -x \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & -t \end{vmatrix} = -t \neq 0$

حل وحيد

$z = f(x^2 + y^2)$

$z = c_2 = t^2 \ , \quad t^2 + 0 = c_1 \Rightarrow c_1 = t^2$

$z = c_2 = t^2 = c_1$

$c_1 = c_2$

$z = x^2 + y^2 = f(x, y)$

$z = f(a,b) + \frac{1}{1!}\left[f_x(x-a) + f_y(x-a)\right] + \frac{1}{2!}\left[f_{xx}(x-a)^2 + 2f_{xy}(x-a)(x-b) + f_{yy}(x-b)^2\right]$

$f_x = 2x, \ f_y = 2y, \ f_{xx} = 2, \ f_{yy} = 2, \ f_{xy} = 0$

$z = 1 + [2(x-1) + 0] + \frac{1}{2!}\,2(x-1)^2 + 0 + y^2$

$= 1 + 2x - 2 + \frac{1}{2!}\,x(x^2 - 2x + 1) + xy^2$

$= x + 2x - 2 + x^2 - 2x + 1 + y^2$

$= x^2 + y^2$

ب)

$\Gamma: \quad x = 0 \,,\; y = t \,,\; z = 0$

$\boxed{t = 1}$

$$\begin{vmatrix} 0 & 1 \\ y & -x \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ t & 0 \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1 \neq 0$$

حل وحيد

$$\frac{dx}{y} = \frac{dy}{-x} = \frac{dz}{x^2+y^2}$$

$$-x\,dx = y\,dy \Rightarrow \boxed{x^2 + y^2 = c_1}$$

$$\frac{dx}{y} = \frac{dx}{\sqrt{c_1 - x^2}} = \frac{dz}{c_1}$$

$$\sin^{-1}\left(\frac{x}{\sqrt{c_1}}\right) = \frac{z}{c_1} + c_2^*$$

$$\boxed{\sin^{-1}\left(\frac{x}{\sqrt{x^2+y^2}}\right) - \frac{z}{x^2+y^2} = c_2}$$

$$\sin^{-1}\left(\frac{x}{\sqrt{x^2+y^2}}\right) - \frac{z}{x^2+y^2} = F(x^2+y^2)$$

$$t^2 = c_1$$

$$c_2 = 0$$

لا يوجد علاقة بين

$c_2 \,,\, c_1$

$\therefore$ لا يمكن تمثيلها

(i) $K\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{\partial u}{\partial z} = 0$, answer: let

$b_{11} = \text{coef } u_{xx}$, $b_{12} = \text{coef } u_{xy}$, $b_{13} = \text{coef } u_{xz}$, $b_{21} = \text{coef } u_{yx}$, $b_{22} = \text{coef } u_{yy}$

$b_{23} = \text{coef } u_{yz}$, $b_{31} = \text{coef } u_{zx}$, $b_{32} = \text{coef } u_{zy}$, $b_{33} = \text{coef } u_{zz}$

let A be the matrix of coefficients:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} K & 0 & 0 \\ 0 & K & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ now the eigenvalues

of this matrix are $\lambda = K$, $\lambda = 0$, since it has zero eigenvalue the equation above is parabolic ∎

(ii) $u_{xx} + u_{yy} - u_{zz} = 0$, again let A be the matrix of coefficients as described in (i) $\Rightarrow$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$, now the eigenvalues of

A are $\lambda_1 = 1$, $\lambda_2 = 1$, $\lambda_3 = -1$ $\Rightarrow$ since all eigenvals are non-zero two of the same ~~type~~ sign and one of different sign $\Rightarrow$

the equation is hyperbolic ∎

(iii) $u_{xx} + u_{yy} - u_{zz} - u_{tt} = 0$, answer: let A be the matrix of coefficients of 2nd derivatives, since all 2nd partials Except $\frac{\partial^2 u}{\partial x^2}$, $\frac{\partial^2 u}{\partial y^2}$, $\frac{\partial^2 u}{\partial z^2}$, $\frac{\partial^2 u}{\partial t^2}$ are zero we have

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Since all eigenvalues are non-zero also,

the number of positive is 2 & negative is 2

the equation is ultrahyperbolic

ID: 433100322 M425 خالد الشال

Q 18] (i) $u_{xx} - u_{tt} = 0$, $u(x,0) = \sin(x)$, $u_t(x,0) = b$ constant

answer: since all coefficients are analytic $C(x,t) = -1 \neq 0$ also the functions $\sin(x)$ & b are analytic on $\mathbb{R}$, the equation has unique solution, solving the equation by factoring the operator yields $(D_x - D_t)(D_x + D_t)(u) = 0 \Rightarrow$ Solution is $u(x,t) = f(x+t) + g(x-t)$, if we apply the conditions $\Rightarrow u(x,0) = f(x) + g(x) = \sin(x)$ ①, also $u_t(x,0) = f'(x) - g'(x) = b \Rightarrow$ by Integration with respect to x that $f(x) - g(x) = bx + c$ ② $\Rightarrow$ Solving ① and ② $\Rightarrow$

$f(x) = \dfrac{\sin(x) + bx + c}{2} \Rightarrow g(x) = \dfrac{\sin(x) - bx - c}{2}$

$\Rightarrow$ the general solution is $u(x,t) = \frac{1}{2}(\sin(x+t) + b(x+t) + c) + \dfrac{\sin(x-t) - b(x-t) + c}{2}$ to get solution unique the B.C

we must have $c = 0 \Rightarrow$ the unique solution to the cauchy problem is $u(x,t) = \frac{1}{2}\left[\sin(x+t) + b(x+t) + \sin(x-t) - b(x-t)\right]$

(ii) $z_{xx} - 2z_{xy} + z_{yy} = 4e^{3y}$, $z(x,0) = x^2$, $z_y(x,0) = \frac{1}{3}$

answer: all the coefficients are analytic $C(x,y) = 1 \neq 0$ and $x^2 \wedge \frac{1}{3}$ are analytic on $\mathbb{R} \Rightarrow$ the equation has unique solution $\Rightarrow$ by solving the homogeneous equation $z_{xx} - 2z_{xy} + z_{yy} = 0 \Rightarrow (D_x - D_y)^2(z) = 0 \Rightarrow$ the solution is $x f(x+y) + g(x+y)$, now particular solution to the non-homogeneous equation try Ae^{3y} $\Rightarrow$ by plugging it in the equation $\Rightarrow 9Ae^{3y} = 4e^{3y} \Rightarrow A = \frac{4}{9} \Rightarrow$ solution $z(x,y) = x f(x+y) + g(x+y) + \frac{4}{9}e^{3y}$, now applying the conditions $z(x,0) = x f(x) + g(x) + \frac{4}{9} = x^2$ ①

also differentiating with respect to $y \Rightarrow z_y(x,0) = x f'(x) + g'(x) + \frac{4}{3} = \frac{1}{3}$ ②, if we differentiate ① with respect to x and substract ② $\Rightarrow f(x) - x^2 - \frac{1}{3} = 0 \Rightarrow f(x) = x^2 + 1 \Rightarrow g(x) = -x^3 - x + x^2 - \frac{4}{9}$ from ① $\Rightarrow$ the ~~general~~ unique solution is $z(x,y) = x((x+y)^2 + 1) - (x+y)^3 - (x+y) + (x+y)^2 - \frac{4}{9} + \frac{4}{9}e^{3y}$

q 20] $u_{tt} - c^2 u_{xx} = F(x,t) \quad -\infty < x < \infty \;,\; t > 0$

$u(x,0) = f(x)$, $u_t(x,0) = g(x)$, $f(x) = g(x) = 0$, $F(x,t) = A\sin(kx - \omega t)$

answer:

We'll Apply the D'alembert method

we have: $u(x,t) = \frac{1}{2}[f(x+ct) + f(x-ct)] + \frac{1}{2c}\int_{x-ct}^{x+ct} g(\alpha)\,d\alpha$

$$+ \frac{1}{2c}\int_0^t \int_{x-c(t-\tau)}^{x+c(t-\tau)} F(s,\tau)\,ds\,d\tau$$

however we have $f(x) = g(x) = 0 \Rightarrow$ the solution is

$$u(x,t) = \frac{1}{2c}\int_0^t \int_{x-c(t-\tau)}^{x+c(t-\tau)} A\sin(ks - \omega\tau)\,ds\,d\tau \quad , \text{ so Integrating}$$

with respect to (s) first, $\Rightarrow$

$$u(x,t) = \frac{1}{2c}\int_0^t -\frac{A}{k}\cos(ks - \omega\tau)\Big]_{s = x-c(t-\tau)}^{s = x+c(t-\tau)}\,d\tau \Rightarrow$$

$$u(x,t) = \frac{1}{2c}\left(-\frac{A}{k}\right)\int_0^t [\cos(k(x+c(t-\tau)) - \omega\tau) - \cos(k(x-c(t-\tau)) - \omega\tau)]\,d\tau$$

$$= \frac{-A}{2ck}\int_0^t [\cos(kx + kct - (kc+\omega)\tau) - \cos(kx - kct + (kc-\omega)\tau)]\,d\tau$$

$$= \frac{-A}{2ck}\left[\frac{\sin(kx + kct - (kc+\omega)\tau)}{-(kc+\omega)} - \frac{\sin(kx - kct + (kc-\omega)\tau)}{kc-\omega}\right]_0^t$$

is the solution □

Q15] $x^2 u_{xx} + 2xy\,u_{xy} + y^2 u_{yy} + 2yz\,u_{yz} + 2xz\,u_{xz} + z^2 u_{zz} = \ln x y z$

answer: Let $\xi = \ln(x)$, $\eta = \ln(y)$, $\tau = \ln(z)$

now

$$u_x = u_\xi \xi_x + u_\eta \eta_x + u_\tau \tau_x = u_\xi \xi_x \Rightarrow u_{xx} = (u_\xi \xi_x)_x$$

$$= u_{\xi\xi} \xi_x^2 + u_{\xi\eta} \eta_x \xi_x + u_{\xi\tau} \tau_x \xi_x + u_\xi \xi_{xx} = u_{\xi\xi} \xi_x^2 + u_\xi \xi_{xx}$$

By the same argument $u_{yy} = u_{\eta\eta} \eta_y^2 + u_\eta \eta_{yy}$, $u_{zz} = u_{\tau\tau} \tau_z^2 + u_\tau \tau_{zz}$

$$u_{xy} = (u_\xi \xi_x)_y = u_{\xi\xi} \xi_y \xi_x + u_{\xi\eta} \eta_y \xi_x + u_{\xi\tau} \tau_y \xi_x + u_\xi \xi_{xy} = u_{\xi\eta} \eta_y \xi_x$$

because other terms will vanish

By the same argument $u_{yz} = u_{\eta\tau} \eta_y \tau_z$, $u_{xz} = u_{\xi\tau} \xi_x \tau_z$

the equation becomes

$$u_{\xi\xi} - u_\xi + 2u_{\xi\eta} + u_{\eta\eta} - u_\eta + 2u_{\eta\tau} + 2u_{\xi\tau} + u_{\tau\tau} - u_\tau = \xi + \tau + \eta$$

becomes

$$= u_{\xi\xi} + u_{\eta\eta} + u_{\tau\tau} + 2(u_{\xi\eta} + u_{\eta\tau} + u_{\xi\tau}) - (u_\xi + u_\eta + u_\tau) = \xi + \tau + \eta$$

this the operator: for the homogeneous

$$[D_\xi^2 + D_\eta^2 + D_\tau^2 + 2(D_\xi D_\eta + D_\eta D_\tau + D_\xi D_\tau) - (D_\xi + D_\eta + D_\tau)](u) = 0$$

it is factorable

$(D_\xi + D_\eta + D_\tau - 1)(D_\xi + D_\eta + D_\tau)(u) = 0$, solving this

now $u_\eta = e^{\xi} f(\xi - \eta, \xi - \tau) + g(\xi - \eta, \xi - \tau)$

For Particular solution $u_p = -\frac{\xi^2}{2} - \frac{\eta^2}{2} - \frac{\tau^2}{2} - \xi - \eta - \tau$

$\Rightarrow$ the general solution is $u = e^{\xi} f(\xi - \eta, \xi - \tau) + g(\xi - \eta, \xi - \tau) + u_p$

change to the original variables

$$u(x, y, z) = e^{\ln(x)} f(\ln(x) - \ln(y), \ln(x) - \ln(z)) + g(\ln(x) - \ln(y), \ln(x) - \ln(z))$$

$$- \left(\frac{\ln^2(x)}{2} + \frac{\ln^2(y)}{2} + \frac{\ln^2(z)}{2} + \ln(x) + \ln(y) + \ln(z)\right)$$

□

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س١)

(أ)

$$4z_{xy} + z_{yy} = \cos y + 1$$

$$\frac{\partial}{\partial y}\left(4z_x + z_y\right) = \cos y + 1$$

$$4z_x + z_y = \sin y + y + g(x)$$

م.م

$$\frac{dx}{4} = dy = \frac{dz}{\sin y + y + g(x)}$$

$$\frac{dx}{4} = dy$$

$$x = 4y + c_1$$

$$y = \frac{x - c_1}{4}$$

$$dy = \frac{dz}{\sin y + y + g(x)} = \frac{dz}{\sin y + y + g(4y + c_1)}$$

$$\Rightarrow \int \left[\sin y + y + g(4y + c_1)\right] dy = \int dz$$

$$\Rightarrow -\cos y + \frac{y^2}{2} + \frac{1}{4} G(4y + c_1) = z + c_2$$

$$\Rightarrow -\cos y + \frac{y^2}{2} + \frac{1}{4} G(x) = z + c_2$$

$$\Rightarrow c_2 = -\cos y + \frac{y^2}{2} + \frac{1}{4} G(x) - z = F(c_1)$$

$$\Rightarrow \boxed{z = -\cos y + \frac{y^2}{2} + \frac{1}{4} G(x) - F[x - 4y)}$$

ب)

$$z_{xx} + z_x + x + y + 1 = 0$$

$$z_{xx} + z_x = -x - y - 1$$

بتكامل بالنسبة لـ x

$$z_x + z = -\frac{x^2}{2} - yx - x + g(y)$$

$$e^x(z_x + z) = e^x\left(-\frac{x^2}{2} - yx - x + g(y)\right)$$

$$(e^x z)_x = e^x\left(-\frac{x^2}{2} - yx - x + g(y)\right)$$

$$e^x z = -\frac{1}{2}\int x^2 e^x dx + \int yxe^x dx - \int xe^x dx + \int e^x g(y)\, dx$$

$$e^x z = -\frac{1}{2}\left[x^2 e^x - \int 2xe^x dx\right] - y\left[xe^x - e^x\right] - \left[xe^x - e^x\right] + e^x g(y) + F(y)$$

$$e^x z = -\frac{1}{2}\left[x^2 e^x - 2\left(xe^x - e^x\right)\right] - yxe^x + ye^x - xe^x + e^x + e^x g(y) + F(y)$$

$$z = -\frac{1}{2}x^2 + x - 1 - yx + y - x + 1 + g(y) + e^{-x}F(y)$$

$$\boxed{z = -\frac{1}{2}x^2 + y(1-x) + g(y) + e^{-x}F(y)}$$

$$z_{xx} - z_{yy} - 3z_x + 3z_y = xy + e^{x+2y}$$

$$z_h : (D_x^2 - D_y^2 - 3D_x + 3D_y)\, z = 0$$

$$(D_x - D_y)(D_x + D_y - 3)\, z = 0$$

$$z_h = f(-x-y) + e^{3x} g(-x+y)$$

$$z_h = F(x+y) + e^{3x} g(-x+y)$$

$$z_{p_1} = Axe^{x+2y}$$

$$2Ae^{x+2y} + Axe^{x+2y} - 4Axe^{x+2y} - 3Ae^{x+2y} + 6Ae^{x+2y} = e^{x+2y}$$

$$2A = 1 \Rightarrow A = \tfrac{1}{2} \Rightarrow z_{p_1} = \tfrac{1}{2} xe^{x+2y}$$

$$z_{p_2} = \frac{1}{(D_x - D_y)(D_x + D_y - 3)} xy$$

$$z_g = F(x+y) + e^{3x} g(-x+y) + \tfrac{1}{2} xe^{x+2y} + \frac{1}{(D_x - D_y)(D_x + D_y - 3)} xy$$

(س 11)

$$z_{xx} + 5z_{xy} + 6z_{yy} = \ln(y-2x) \qquad (1)$$

$$(D_x^2 + 5D_{xy} + 6D_y^2)\, z = \ln(y-2x)$$

$$(D_x + 3D_y)(D_x + 2D_y)\, z = \ln(y-2x)$$

let $(D_x + 2D_y)\, z = v$

$$(D_x + 3D_y)\, v = \ln(y-2x)$$

$$\frac{dx}{1} = \frac{dy}{3} = \frac{dv}{\ln(y-2x)}$$

$$3dx = dy \Rightarrow 3x = y + c'$$

$$\boxed{y - 3x = c_1}$$

$$y - 2x = c_1 + x$$

$$\frac{dx}{1} = \frac{dv}{\ln(y-2x)}$$

$$\int \ln(c_1 + x)\, dx = \int dv$$

$$\Rightarrow x\ln(x+c_1) - \int \frac{x}{x+c_1}\, dx = v + c_2'$$

$$\Rightarrow x\ln(x+c_1) - \int \frac{x + c_1 - c_1}{x+c_1}\, dx = v + c_2'$$

$$\Rightarrow x\ln(x+c_1) - [x - c_1\ln x + c_1] = v + c_2'$$

$$\Rightarrow x\ln(x+c_1) - x + c_1\ln(x+c_1) = v + c_2'$$

$$\Rightarrow (x+c_1)\ln(x+c_1) - x = v + c_2'$$

$$v = (x+c_1)\ln(x+c_1) - x + c_2 \; ; \; c_2 = -c_2'$$

$$v = (x+c_1)\ln(x+c_1) - x + f(c_1)$$

$$= (x+c_1)\ln(x+c_1) - x + f(y-3x)$$

$$v = (y-2x)\ln(y-2x) - x + f(y-3x)$$

(ii)

$$(D_x + 2D_y)\, z = v$$

$$\frac{dx}{1} = \frac{dy}{2} = \frac{dz}{v}$$

$$2x - y = c'$$

$$\boxed{y - 2x = c_1}$$

$$\frac{dx}{1} = \frac{dz}{(y-2x)\ln(y-2x) - x + f(y-3x)}$$

$$= \frac{dz}{c_1\ln c_1 - x + f(c_1 - x)}$$

$$\Rightarrow z = (c_1\ln c_1)x - \frac{x^2}{2} + g(c_1 - x) + c_2$$

$$g(c_1 - x) = \int f(c_1 - x)\, dx$$

$$z = x(y-2x)\ln(y-2x) - \frac{x^2}{2} + g(y-3x) + h(y-2x)$$

$$z_{xx} - z_{xy} - 2z_{yy} = (y-1)e^x$$

z_h : $m^2 - m - 2 = 0$

$(m-2)(m+1) = 0$

$m = 2, -1$

$$z_h = f(2x+y) + g(-x+y)$$

let $z_p = (Ay+B)e^x \cdot y = (Ay^2+By)e^x$

$(z_p)_x = (Ay^2+By)e^x$ ، $(z_p)_{xx} = (Ay^2+By)e^x$

$(z_p)_{xy} = (2Ay+B)e^x$

$(z_p)_y = (2Ay+B)e^x$ ، $(z_p)_{yy} = 2Ae^x$

$$(Ay^2+By)e^x - (2Ay+B)e^x - 4Ae^x = (y-1)e^x$$

$$Ay^2 + By - 2Ay - B - 4A = y - 1$$

y^2 جملات : $A = 0$ ، y جملات : $B - 2A = 1 \Rightarrow B = 1$

بقیه : $-B - 4A = -1 \Rightarrow B = 1$

$$z_p = ye^x$$

$$\Rightarrow \boxed{z_g = f(2x+y) + g(-x+y) + ye^x}$$

$$z_{xx} - 4z_{yy} = \frac{4x}{y^2} - \frac{y}{x^2}$$

(2.

$$z_{xx} - 4z_{yy} = 0$$

$$m^2 - 4 = 0 \Rightarrow m = \pm 2$$

$$z_h = f(-2x+y) + g(2x+y)$$

$$z_p = y\ln x + x\ln y$$

$$z_g = F(-2x+y) + G(2x+y) + y\ln x + x\ln y$$

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$$z_{yy} - x z_y = x^2$$

$$\mu = e^{-\int x\, dy} = e^{-xy}$$

$$z_y \cdot e^{-xy} = \int x^2 e^{-xy}\, dy$$

$$\Rightarrow z_y\, e^{-xy} = \frac{x^2 e^{-xy}}{-x} + g(x)$$

$$\Rightarrow z_y = -x + e^{xy} g(x)$$

$$z = -xy + \frac{e^{xy}}{x} g(x) + h(x)$$

$$\boxed{z = -xy + \frac{1}{x} e^{xy} g(x) + h(x)}$$

(ب

$$z_{xx} = c^2 z_x + x^2 y^2 \qquad c = \text{const}$$

$$z_{xx} - c^2 z_x = x^2 y^2$$

$$z_x = w \;:\; w_x - c^2 w = x^2 y^2$$

$$\mu = e^{-\int c^2 dx} = e^{-c^2 x}$$

$$w\, e^{-c^2 x} = \int (x^2 y^2)\, e^{-c^2 x}\, dx$$

$$= y^2 \left[\frac{x^2 e^{-c^2 x}}{-c^2} - \int \frac{e^{-c^2 x}}{-c^2} 2x\, dx \right]$$

$$= y^2 \left[-\frac{x^2}{c^2} e^{-c^2 x} + \frac{2}{c^2} \int x e^{-c^2 x}\, dx \right]$$

$$we^{-c^2x} = y^2\left[\frac{-x^2}{c^2}e^{-c^2x} + \frac{2}{c^2}\left(\frac{xe^{-c^2x}}{-c^2} - \int\frac{e^{-c^2x}}{-c^2}\,dx\right)\right]$$

$$we^{-c^2x} = y^2\left[\frac{-x^2}{c^2}e^{-c^2x} - \frac{2xe^{-c^2x}}{c^4} - \frac{2}{c^6}e^{-c^2x}\right] + g(y)$$

$$w = \frac{-y^2x^2}{c^2} - \frac{2xy^2}{c^4} - \frac{2y^2}{c^6} + e^{c^2x}g(y), \quad c \neq 0$$

$w = z_x$

$$z = \frac{-y^2x^3}{3c^2} - \frac{x^2y^2}{c^4} - \frac{2xy^2}{c^6} + \frac{e^{c^2x}}{c^2}g(y) + f(y) \qquad c \neq 0$$

(1) — $x^2 Z_{xx} - 2xy Z_{xy} - 3y^2 Z_{yy} + x Z_x - 3y Z_y = 2\ln xy + 4x$ (س١٣
(أ)

let $x = e^{\xi}$, $y = e^{\eta}$

$$(D_\xi(D_\xi - 1) - 2D_{\xi\eta} - 3D_\eta(D_\eta - 1) + D_\xi - 3D_\eta) Z = 2\ln(e^{\xi+\eta}) + 4e^{\xi}$$

$$(D_\xi(D_\xi - 1)) - 2D_{\xi\eta} - 3D_\eta(D_\eta - 1) + D_\xi - 3D_\eta) Z = 2\xi + 2\eta + 4e^{\xi}$$

$$(D_\xi^2 - 2D_{\xi\eta} - 3D_\eta^2) Z = 2\xi + 2\eta + 4e^{\xi}$$

$$(D_\xi - 3D_\eta)(D_\xi + D_\eta) Z = 2\xi + 2\eta + 4e^{\xi}$$

$$Z_h = f(-3\xi - \eta) + g(\xi - \eta) = F(3\xi + \eta) + g(\xi - \eta)$$

$$Z_p = (A\xi^2 + B)\xi^2 + (C\eta + D)\eta^2 + Me^{\xi}$$

نشتق ونعوض في (1)

نحصل على

$A = \frac{1}{3}$, $C = \frac{1}{3}$

$M = -\frac{1}{2}$

$$Z_p = \frac{1}{3}\xi^2 - \frac{1}{3}\eta^2 - \frac{1}{2}e^{\xi}$$

$$Z_g = F(3\xi + \eta) + g(\xi - \eta) + \frac{1}{3}\xi^2 - \frac{1}{3}\eta^2 - \frac{1}{2}e^{\xi}$$

$$= F(3\ln x + \ln y) + g(\ln x - \ln y) + \frac{1}{3}(\ln x)^2 - \frac{1}{3}(\ln y)^2 - \frac{1}{2}x$$

$$\boxed{Z_g = F^*(x^3 y) + G^*\left(\frac{x}{y}\right) + \frac{1}{3}(\ln x)^2 - \frac{1}{3}(\ln y)^2 - \frac{1}{2}x}$$

$$(ax D_x + by D_y)^2 z + \lambda^2 z = 0$$

$$\left((ax D_x + by D_y)^2 + \lambda^2\right) z = 0$$

let $x = e^{\xi}$, $y = e^{\eta}$

$$xD_x = D_\xi \;,\; yD_y = D_\eta$$

$$\Rightarrow \left[(aD_\xi + bD_\eta)^2 + \lambda^2\right] z = 0$$

$$(aD_\xi + bD_\eta + \lambda i)(aD_\xi + bD_\eta - \lambda i) z = 0$$

$$z = e^{-\frac{\lambda i}{a}} f(b\xi - a\eta) + e^{\frac{\lambda i \xi}{a}} g(b\xi - a\eta)$$

$$= e^{-\frac{\lambda i}{a}\ln x} f(b\ln x - a\ln y) + e^{\frac{\lambda i \ln x}{a}} g(b\ln x - a\ln y)$$

$$= e^{-\frac{\lambda i \ln x}{a}} f(\ln x^b + \ln y^{-a}) + e^{\frac{\lambda i \ln x}{a}} g(\ln x^b + \ln y^{-a})$$

$$\Rightarrow \boxed{z = e^{-\frac{\lambda i \ln x}{a}} f(\ln(x^b y^{-a})) + e^{\frac{\lambda i \ln x}{a}} g(\ln x^b y^{-a})}$$

س 17

(أ)

$$\frac{\partial u}{\partial t} = k\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$A = k \;, B = 0, \; C = k$$

$$\varphi = B^2 - 4AC = -4(k)(k) = -4k^2 < 0$$

∴ قطع ناقص

(ب)

$$u_{xx} + u_{yy} - u_{tt} = 0$$

$$A = 1 \;, B = 0 \;, \; C = 1$$

$$\varphi = B^2 - 4AC = 0 - 4(1)(1) = -4 < 0$$

∴ قطع ناقص

(ج)

$$u_{xx} + u_{yy} - u_{zz} + u_{tt} = 0$$

$$A = 1 \;, B = 0 \;, \; C = 1$$

$$\varphi = B^2 - 4AC = 0 - 4(1)(1) = -4 < 0$$

∴ قطع ناقص

$$\begin{cases} u_{xx} - u_{tt} = 0 \\ u(x,0) = \sin x \\ u_t(x,0) = b \end{cases} \quad , a,b \text{ constant} \qquad (1)$$

$$(D_x^2 - D_t^2)\,u = 0$$

$$\Rightarrow (D_x - D_t)(D_x + D_t)\,u = 0$$

$$u = f(x+t) + g(x-t)$$

$$u(x,0) = f(x) + g(x) = \sin x \quad - (1)$$

نشتق (1)

$$u_t = f'(x+t) - g'(x-t) \quad , \; u_t(x,0) = f'(x) + g'(x) = \cos x \quad - (2)$$

$$u_t(x,0) = f'(x) - g'(x) = b \quad - (2)$$

$$2f' = \cos x + b$$

$$f(x) = \frac{\sin x}{2} + \frac{b}{2} + C_1$$

$$g(x) = \sin x - f(x) = \frac{\sin x}{2} - \frac{b}{2} - C_1$$

$$\Rightarrow u = \frac{\sin(x+t)}{2} + \frac{b}{2} + C_1 + \frac{\sin(x-t)}{2} - \frac{b}{2} - C_1$$

$$\Rightarrow \boxed{u = \frac{\sin(x+t) + \sin(x-t)}{2}}$$

(٥

$$\begin{cases} z_{xx} - 2z_{xy} + z_{yy} = 4e^{3y} & -(*) \\ z(x,0) = x^2 \\ z_y(x,0) = \frac{1}{3} \end{cases}$$

z_h : $z_{xx} - 2z_{xy} + z_{yy} = 0$

$m^2 - 2m + 1 = 0 \Rightarrow m = 1,1$

$\Rightarrow z_h = f(x+y) + x g(x+y)$

$z_p = Ae^{3y}$ $(z_p)_{xx} = 0$, $(z_p)_{xy} = 0$ $(z_p)_{yy} = 9Ae^{3y}$

نعوض في (*)

$9Ae^{3y} = 4e^{3y} \Rightarrow \boxed{A = \frac{4}{9}} \Rightarrow z_p = \frac{4}{9}e^{3y}$

$z = z_g = f(x+y) + x g(x+y) + \frac{4}{9}e^{3y}$

$z(x,0) = f(x) + x g(x) + \frac{4}{9} = x^2$ — ①

$z_y(x,y) = f' + x g' + \frac{12}{9}e^{3y}$

$z_y(x,0) = f'(x) + x g'(x) + \frac{12}{9} = \frac{1}{3}$

$f' + x g' = -1$ — ②

$f' + g + x g' = 2x$ — ③ نشتق ① لحدها

③ − ② : $g = 2x + 1$

$f(x) + x(2x+1) + \frac{4}{9} = x^2$

$\Rightarrow f(x) = -x^2 - x - \frac{4}{9}$

من ①

$z = -(x+y)^2 - (x+y) - \frac{4}{9}$

$+ x(2(x+y)+1) + \frac{4}{9}e^{3y}$

(س٢٠

$$\begin{cases} u_{tt} - c^2 u_{xx} = F(x,t) \\ u(x,0) = f(x) \\ u_t(x,0) = g(x) \end{cases}$$

$F(x,t) = A\sin(kx - wt)$, $f(x) = g(x) = 0$, $w = kc$

$$u(x,t) = \frac{1}{2}[f(x+ct) + f(x-ct)] + \frac{1}{2c}\int_{x-ct}^{x+ct} g(s)\,ds + \frac{1}{2c}\int_0^t \int_{x-c(t-s)}^{x+c(t-s)} F(y,s)\,dy\,ds$$

$$= \frac{1}{2c}\int_0^t A\sin(ky - w)\,dy\,ds = \frac{-A}{2\underbrace{w}_{kc}}\int_0^t \cos(ky - ws)\Big|_{y=x-c(t-s)}^{y=x+c(t-s)}\,ds$$

$$= \frac{-A}{2w}\int_0^t [\cos(kx + kc(t-s) - ws) - \cos(kx - kc(t-s) - ws)]\,ds$$

$$= \frac{-A}{2w}\int_0^t [\cos(kx + wt - 2ws) - \cos(kx - wt)]\,ds$$

$$= \frac{-A}{2w}\left[\frac{\sin(kx + wt - 2ws)}{2w} - t\cos(kx - wt)\right]_0^t$$

$$= \frac{-A}{2w}\left[\frac{\sin(kx - wt)}{-2w} - t\cos(kx - wt)\right] + \frac{A}{2w}\left[\frac{\sin(kx + wt)}{-2w} - 0\right]$$

$$u(x,t) = \frac{A}{4w^2}\left[\sin(kx - wt) - \sin(kx + wt)\right] + \frac{At}{2w}\cos(kx - wt)$$

بعد التبسيط

$$u(x,t) = A\left[\frac{t\cos(kx - wt)}{2w} - \frac{\cos(kx)\sin wt}{w^2}\right]$$

Q10] (1) $4Z_{xy} + Z_{yy} = \cos(y) + 1$

answer: $D_y(4D_x + D_y)Z = 0 \Rightarrow D_yZ = 0$ or $(4D_x + D_y)Z = 0$

$\Rightarrow f(x)$ or $g(x-4y) \Rightarrow Z_h = f(x) + g(x-4y)$, f, g arbitrary

particular solution is $Z_p = -\cos(y) + \frac{y^2}{2} \Rightarrow Z(x,y) = Z_h + Z_p$, is the general sol

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ب) $Z_{xx} + Z_x + x + y + 1 = 0$, answer: solving the homogeneous

$Z_{xx} + Z_x = 0 \Rightarrow D_x(D_x + 1)Z = 0 \Rightarrow D_xZ = 0$ or $D_x + 1Z = 0$

$\Rightarrow f(y)$ or $e^{-x}g(y) \Rightarrow Z_h = f(y) + e^{-x}g(y)$, for particular solution for $Z_{xx} + Z_x = -x - y - 1$ try solution of the form $ax^2 + bxy + cy^2 + ex + fy + g$ plugging it gives

$2a + 2ax + by + e = -x - y - 1 \Rightarrow 2a = -1$, $b = -1$, $2a + e = -1 \Rightarrow e = 0$

$\Rightarrow Z(x,y) = f(y) + e^{-x}g(y) - \frac{1}{2}x^2 - xy + cy^2 - 0 + fy + g$, c, f, g const

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ج) $Z_{xx} - Z_{yy} - 3Z_x + 3Z_y = xy + e^{x+2y}$, answer: solving the homogenous $(D_x^2 - D_y^2 - 3D_x + 3D_y)(z) = 0$ factoring the operator $(D_x - D_y)(D_x + D_y - 3)(z) = 0 \Rightarrow$

$(D_x - D_y)(z) = 0 \Rightarrow$ the solution is $f(x+y)$ or $(D_x + D_y - 3)(z) =$

$\Rightarrow$ solution is $e^{3x}g(x-y) \Rightarrow Z_h = f(x+y) + e^{3x}g(x-y)$

ID: 433100322 خالد فيصل الدخال

Q2#3 (i) $r+6s+6t = \text{Log}(y-2x)$; answer if we factor the operator $D_x^2+5D_{xy}+6D_y^2$ this gives $(D_x+3D_y)(D_x+2D_y)$

now solve $(D_x+3D_y)(z) = \text{Log}(y-2x)$, note that $D_x(x\text{Log}(x)-x)=\text{Log}x$
$D_y(y\text{Log}(y)-y)=\text{Log}(x)$

So particular solution for this is $z_p=(y-2x)\text{Log}(y-2x)-(y-2x)$

$(D_x+3D_y)(z_p) = -2\text{Log}(y-2x)+3\text{Log}(y-2x) = \text{Log}(y-2x)$ ✓

now try solution to the following $L_2(z)=z_p$ $\Rightarrow$
$(D_x+2D_y)(z) = (y-2x)\text{Log}(y-2x)-(y-2x)$, now z_p for $(y-2x)\text{Log}(y-2x)$, is of the form $(ax^2+bxy+cy^2+dx+ey+f)\log(y-2x)$ and z_p for $(y-2x)$ is also quadratic polynomial plugging and equating coefficients yield $z_p = -(4x^2-3xy+\frac{y^2}{2})\text{Log}(y-2x) + 7x^2-6xy+\frac{5}{4}y^2$

$\Rightarrow$ the general solution is $f(y-3x)+g(2x-y)+z_p$ □

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ii) $r-s-2t = (y-1)e^x$, answer: the operator $D_x^2-D_{xy}-2D_y^2$ is factorable $\sim(D_x-2D_y)(D_x+D_y)$, to obtain particular solution solve $(D_x-2D_y)(z)=(y-1)e^x$ now z_p for this is ye^x+e^x, we have $(D_x-2D_y)(ye^x+e^x) = ye^x+e^x-2e^x = ye^x-e^x=(y-1)e^x$, now obtain solution for $(D_x+D_y)(z)=ye^x+e^x$ is ye^x — these were found by trial and error $(D_x+D_y)(ye^x)=ye^x+e^x$

so particular solution for the original equation is $z_p=ye^x$
$L_1L_2(ye^x)=L_1(ye^x+e^x)=(y-1)e^x$, now solution for the homo eq is $f(2x+y)+g(x-y)$ $\Rightarrow$ the general solution is
$z(x,y)=f(2x+y)+g(x-y)+ye^x$ □

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iii) $r-4t = \frac{4x}{y^2}-\frac{y}{x^2}$; answer: the operator is $D_x^2-4D_y^2=(D_x-2D_y)(D_x+2D_y)$
to find particular solution solve $(D_x-2D_y)(z)=\frac{4x}{y^2}-\frac{y}{x^2}$ again by trial and error we see that solution to this $z_p=$ $\text{Log}(y)+\frac{y}{x}+\frac{2x}{y}+2\text{Log}(x)$ if we plug this we see that
$(D_x-2D_y)(\text{Log}(y)+\frac{y}{x}+\frac{2x}{y}+2\text{Log}(x)) = (-\frac{y}{x^2}+\frac{2}{y}+\frac{2}{x}-\frac{2}{y}+\frac{4x}{y^2}-\frac{2}{x})$
$=\frac{4x}{y^2}-\frac{y}{x^2}$, now solve $(D_x+2D_y)(z)=\text{Log}(y)+\frac{y}{x}+\frac{2x}{y}+2\text{Log}(x)$
we see $z_p=x\text{Log}(y)+y\text{Log}(x)$ $\Rightarrow$ the general solution is
$f(2x+y)+g(2x-y)+x\text{Log}(y)+y\text{Log}(x)$ □

Q12] (i) $t - xq = x^2$, answer: $Z_{yy} - xZ_y = x^2$, put $u = Z_y$

$\Rightarrow u_y - xu = x^2$, we have Integrating factor for this $e^{\int -x\,dy} = e^{-xy}$

$\Rightarrow e^{-xy}u_y - xe^{-xy}u = x^2e^{-xy} \sim (e^{-xy}u)_y = x^2e^{-xy}$ Integrating both sides with respect to $y \Rightarrow e^{-xy}u = \frac{x^2e^{-xy}}{-x} + f(x) \Rightarrow$ by multiplying both sides by e^{xy}, $u = -x + f(x)e^{xy}$, $u = Z_y$ Integrating both sides with respect to $y \Rightarrow Z = -xy + \frac{f(x)e^{xy}}{x} + g(x)$ □

(ii) $xr = c^2p + x^2y^2$, answer: $xZ_{xx} - c^2Z_x = x^2y^2$, dividing by "x"

$\Rightarrow Z_{xx} - \frac{c^2}{x}Z_x = xy^2$, the Integrating factor is

$e^{\int -\frac{c^2}{x}dx} = e^{-c^2\ln(x)} = \frac{1}{x^{c^2}}$, if we multiply the equation by it $\frac{Z_{xx}}{x^{c^2}} - \frac{c^2Z}{x^{c^2+1}}x = xy^2 \Rightarrow \left(\frac{Z_x}{x^{c^2}}\right)_x = xy^2$ Integrating with respect to "x" $\Rightarrow \frac{Z_x}{x^{c^2}} = \frac{x^2}{2}y^2 + g(y)$

$\Rightarrow Z_x = \frac{x^{c^2+2}}{2}y^2 + g(y)x^{c^2}$, if we Integrate with x

$\Rightarrow Z$

$\hookrightarrow Z = \frac{x^{c^2+2+1}}{2(c^2+3)}y^2 + g(y)\frac{x^{c^2+1}}{c^2+1} + f(y)$ □

Q13](i) $x^2 z_{xx} - 2xy\, z_{xy} - 3y^2 z_{yy} + x z_x - 3y z_y = 2\log(xy) + 4x$

answer: use the transformation $\xi = \log(x)$, $\eta = \log(y)$

$\Rightarrow z_{xx} = z_{\xi\xi}\xi_x^2 + z_\xi \xi_{xx}$, $z_{xy} = z_{\xi\eta}\xi_x\eta_y$, $z_{yy} = z_{\eta\eta}\eta_y^2 + z_\eta \eta_{yy}$

$\xi_x = \frac{1}{x}$, $\xi_{xx} = \frac{-1}{x^2}$, $\eta_y = \frac{1}{y}$, $\eta_{yy} = \frac{-1}{y^2}$

$\Rightarrow x^2 z_{xx} = z_{\xi\xi} - z_\xi$, $-2xy\, z_{xy} = -2z_{\xi\eta}$, $-3y^2 z_{yy} = -3z_{\eta\eta} + 3z_\eta$

$x z_x = z_\xi$, $-3y z_y = -3z_\eta \Rightarrow$ by substitution in the equation

$z_{\xi\xi} - 2z_{\xi\eta} - 3z_{\eta\eta} = 2\xi + 2\eta + 4e^\xi$, for particular solution

for exponential solution try $ae^\xi \Rightarrow$ by plugging it that

$ae^\xi = 4e^\xi \Rightarrow z_{p_1} = 4e^\xi$, z_{p_2} for $z_{\xi\xi} - 2z_{\xi\eta} - 3z_{\eta\eta} = 2\xi + 2\eta$

try cubic $a\xi + b\eta^3$ plugging it in $\Rightarrow$

$6a\xi - 18b\eta = 2\xi + 2\eta \Rightarrow 6a = 2 \Rightarrow a = \frac{1}{3}$, $-18b = 2 \Rightarrow b = \frac{-1}{9}$

$\Rightarrow z_{p_2} = \frac{\xi^3}{3} - \frac{\eta^3}{9}$, now z_h, $(D_\xi - 3D_\eta)(D_\xi + D_\eta)(z) = 0$

$\Rightarrow z_h = f(3\xi + \eta) + g(\xi - \eta) \Rightarrow z = f(3\xi + \eta) + g(\xi - \eta) + \frac{\xi^3}{3} - \frac{\eta^3}{9} + 4e^\xi$

$\Rightarrow z = f(3\log(x) + \log(y)) + g(\log(x) - \log(y)) + \frac{\log(x)^3}{3} - \frac{\log(y)^3}{9} + 4x$ □

(ii) $x^2 z_{xx} + 2xy\, z_{xy} + y^2 z_{yy} - nx z_x - ny z_y + nz = x + y \quad n \neq 0$

answer: Let $\xi = \log(x)$, $\eta = \log(y)$ this transformation yields $z_{xx} = z_{\xi\xi}\xi_x^2 + z_\xi \xi_{xx}$, $z_{yy} = z_{\eta\eta}\eta_y^2 + z_\eta \eta_{yy}$, $z_x = z_\xi \xi_x$, $z_y = z_\eta \eta_y$

noting that $\xi_x = \frac{1}{x}$, $\eta_y = \frac{1}{y}$ the equation becomes

$z_{\xi\xi} + 2z_{\xi\eta} + z_{\eta\eta} - (n+1)z_\xi - (n+1)z_\eta + nz = e^\xi + e^\eta$

solving the hom. eq $(D_\xi^2 + 2D_\xi D_\eta - (n+1)D_\xi + D_\eta^2 - (n+1)D_\eta + n)(z) =$

if we factor the operator $\Rightarrow$

$(D_\xi + D_\eta - 1)(D_\xi + D_\eta - n)(z) = 0$ this equation

$z_h = e^\xi f(\xi - \eta) + e^{n\xi} g(\xi - \eta)$

for particular solution try exponential solution

first plug ae^ξ in the equation $\Rightarrow ae^\xi - (n+1)ae^\xi + nae^\xi = e^\xi$

$\Rightarrow 0 = e^\xi$, so we try solution of the form $a\xi e^\xi$ if we plug it

$2ae^\xi + a\xi e^\xi - (n+1)(ae^\xi + a\xi e^\xi) + na\xi e^\xi \Rightarrow (-(n+1)+2)ae^\xi$

$= e^\xi \Rightarrow a = \frac{1}{-n+1}$ also by symmetry for e^η, we choose

$b\eta e^\eta$, $b = \frac{1}{-n+1} \Rightarrow z = e^\xi f(\xi - n) + e^{n\xi} g(\xi - n) + \frac{1}{-n+1}(\xi e^\xi + \eta e^\eta)$

we can substitute $\xi = \log(x)$, $\eta = \log(y)$

Q6](i) $4z_{xx} - 4z_{xy} + 5z_{yy} = 0$, answer: $b^2 - ac = 4 - 20 = -16 < 0$

$\Rightarrow$ the equation is elliptic, now $\frac{-b \pm \sqrt{b^2 - ac}}{a} = \frac{1}{2} \pm i$,

by solving $\frac{dy}{dx} = -(\frac{1}{2} + i) = -m \Rightarrow y + \frac{1}{2}x + ix = c$

is solution $\Rightarrow$ now choose $\xi = y + \frac{1}{2}x$ the Real part and

$\eta = x$ the Imaginay part $\Rightarrow z_{xx} = z_{\xi\xi}\xi_x^2 + 2z_{\xi\eta}\eta_x\xi_x + z_{\eta\eta}\eta_x^2 + z_\xi \xi_{xx} + z_\eta \eta_{xx}$

and $z_{yy} = z_{\xi\xi}\xi_y^2 + 2z_{\xi\eta}\xi_y\eta_y + z_{\eta\eta}\eta_y^2 + z_\xi \xi_{yy} + z_\eta \eta_{yy}$, $z_{xy} =$

$z_{\xi\eta} = z_{\xi\xi}\xi_x\xi_y + z_{\xi\eta}(\xi_x\eta_y + \xi_y\eta_x) + z_{\eta\eta}\eta_x\eta_y + z_\xi \xi_{xy} + z_\eta \eta_{xy}$

Noting that many termes will vanish, now $4z_{xx} = 4z_{\xi\xi} + 4z_{\xi\eta} + 4z_{\eta\eta}$

$-4z_{xy} = -2z_{\xi\xi} - 4z_{\xi\eta}$, $5z_{yy} = 5z_{\xi\xi}$, adding Hence

$\Rightarrow 4z_{\xi\xi} + 4z_{\xi\eta} + 4z_{\eta\eta} - 2z_{\xi\xi} - 4z_{\xi\eta} + 5z_{\xi\xi} = 4(z_{\xi\xi} + z_{\eta\eta}) = 0$

$\Rightarrow z_{\xi\xi} + z_{\eta\eta} = 0$, solution is $z = f(\xi + i\eta) + g(\xi - i\eta)$.

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ii) $z_{xx} - z_{yy} + z_x + z_y + x + y + 1 = 0$, answer: $b^2 - ac = 1 > 0$

$\Rightarrow$ the equation is hyperbolic, $\frac{-b \pm \sqrt{b^2 - ac}}{a} = \pm 1$, by solving

$\frac{dy}{dx} = -1$, $\frac{dy}{dx} = 1 \Rightarrow y + x = \xi$, $y - x = \eta$

$z_{xx} = z_{\xi\xi}\xi_x^2 + 2z_{\xi\eta}\eta_x\xi_x + z_{\eta\eta}\eta_x^2 + z_\xi \xi_{xx} + z_\eta \eta_{xx}$

$z_{yy} = z_{\xi\xi}\xi_y^2 + 2z_{\xi\eta}\eta_y\xi_y + z_{\eta\eta}\eta_y^2 + z_\xi \xi_{yy} + z_\eta \eta_{yy}$

$\xi_x = +1$, $\xi_y = +1$, $\eta_x = -1$, $\eta_y = 1$

$\Rightarrow z_{xx} - z_{yy} = -4z_{\xi\eta}$, $z_x = z_\xi \xi_x + z_\eta \eta_x = z_\xi - z_\eta$, $z_y = z_\xi \xi_y + z_\eta \eta_y = z_\xi + z_\eta$

$x = \frac{\xi - \eta}{2}$, $y = \frac{\xi + \eta}{2}$

$\Rightarrow -4z_{\xi\eta} + 2z_\xi + \xi + 1 = 0$. dividing by -4

$\Rightarrow z_{\xi\eta} + (-\frac{z_\xi}{2}) + (-\frac{\xi}{4}) - \frac{1}{4} = 0$, solving the homogeneous

$z_{\xi\eta} - \frac{z_\xi}{2} = 0$ Integrating with respect to ξ

$\Rightarrow z_\eta - \frac{z}{2} = g(\eta)$ $\Rightarrow z_\eta = \frac{z}{2} + g(\eta)$ Integrating with η

$\Rightarrow z = (\int \frac{z}{2} d\eta) + \int f(\eta) + h(\xi)$, to obtain particular solution

take $z_p = -\frac{\xi^2}{4} - \frac{\xi}{2}$.

2) $xz_{xx} - (x+y)z_{xy} + yz_{yy} - \left(\frac{x+y}{x-y}\right)(z_x - z_y) = 0$, answer

$b^2 = \left(-\frac{x+y}{2}\right)^2 = \frac{1}{4}(x^2 + 2xy + y^2) - xy = \frac{x^2}{4} - \frac{xy}{2} + y^2 = \left(\frac{x-y}{2}\right)^2 > 0$

for $x \neq y$ is hyperbolic, $\frac{\frac{1}{2}(x+y) \pm \frac{(x-y)}{2}}{x} = \frac{1}{2}\frac{(2x)}{x} = m_1$, $\frac{\frac{1}{2}(2y)}{x}$

$m_1 = 1$, $m_2 = \frac{y}{x}$, $\frac{dy}{dx} = -1 \Rightarrow y + x = c_1$, $\frac{dy}{dx} = -\frac{y}{x} = \ln(y) + \ln(x) = c$

$xy = c_1$

$\xi = y + x$, $\eta = xy$

$z_{xx} = z_{\xi\xi}\xi_x^2 + 2z_{\xi\eta}\xi_x\eta_x + z_{\eta\eta}\eta_x^2 + z_\xi \xi_{xx} + z_\eta \eta_{xx}$

$z_{xy} = z_{\xi\xi}\xi_x\xi_y + z_{\xi\eta}(\xi_x\eta_y + \xi_y\eta_x) + z_{\eta\eta}\eta_x\eta_y + z_\xi \xi_{xy} + z_\eta \eta_{xy}$

$z_{yy} = z_{\xi\xi}\xi_y^2 + 2z_{\xi\eta}\xi_y\eta_y + z_{\eta\eta}\eta_y^2 + z_\xi \xi_{yy} + z_\eta \eta_{yy}$

$\Rightarrow z_{xx} = z_{\xi\xi} + 2z_{\xi\eta}y + z_{\eta\eta}y^2$, $z_{xy} = z_{\xi\xi} + z_{\xi\eta}(x+y) + z_{\eta\eta}xy + z_\eta$, $z_{yy} = z_{\xi\xi} + 2z_{\xi\eta}x + z_{\eta\eta}x^2 + 0$

$xz_{xx} = x z_{\xi\xi} + 2xy z_{\xi\eta} + xy^2 z_{\eta\eta}$, $-(x+y)z_{xy} = -xz_{xy} - yz_{xy} =$

$-x z_{\xi\xi} - x z_{\xi\eta}(x+y) - x^2 y z_{\eta\eta} - x z_\eta - y z_{\xi\xi} - y z_{\xi\eta}(x+y) - y^2 x z_{\eta\eta} - y z_\eta$

$+ y z_{yy} = y z_{\xi\xi} + 2xy z_{\xi\eta} + yx^2 z_{\eta\eta}$, if we add all these term

$\Rightarrow 2xy z_{\xi\eta} - x^2 z_{\xi\eta} - xy z_{\xi\eta} - xy z_{\xi\eta} - y^2 z_{\xi\eta} - y z_\eta$

$= (2xy - x^2 - xy - xy - y^2 + 2xy) z_{\xi\eta} - y z_\eta$, now the origin equation becomes

$\Rightarrow = (-x^2 + 2xy - y^2) z_{\xi\eta} - y z_\eta - \left(\frac{x+y}{x-y}\right)(z_\xi \xi_x + z_\eta \eta_x - z_\xi \xi_y - z_\eta \eta_y)$

$= -(x-y)^2 z_{\xi\eta} - y z_\eta - \frac{(x+y)}{x-y}(z_\xi + z_\eta y - z_\xi - z_\eta x)$

$= z_{\xi\eta} + \frac{y}{(x-y)} z_\eta + \frac{x z_\eta}{(x-y)^2} \frac{(x+y)}{(x-y)^3}(z_\eta)(y-x)$

$= z_{\xi\eta} + z_\eta\left(\frac{y+x}{(x-y)^2} - \frac{(x+y)}{(x-y)^2}\right) = 0 \Rightarrow$

$z_{\xi\eta} = 0 \Rightarrow z = f(\xi) + g(\eta) = f(x+y) + g(xy)$ is solution

1) $\frac{\partial}{\partial x}\left(x^2\frac{\partial z}{\partial x}\right) = x^2\frac{\partial^2 z}{\partial x^2}$, answer: This becomes $2x\frac{\partial z}{\partial x} + x^2\frac{\partial^2 z}{\partial x^2} = x^2\frac{\partial^2 z}{\partial y^2}$

$\Rightarrow x^2 z_{xx} - x^2 z_{yy} + 2x z_x = 0$, $b^2 - ac = x^4 > 0$ for $x \neq 0$

$\Rightarrow$ it is hyperbolic for $x \neq 0$, parabolic $x = 0$

$\frac{-b \pm \sqrt{b^2-ac}}{a} = \pm\frac{x^2}{x^2} = \pm 1$, $m_1 = 1$, $m_2 = -1$, $\frac{dy}{dx} = -m_1$, $\frac{dy}{dx} = -m_2$

solving this gives $x + y = \xi$, $y - x = \eta$

$z_{xx} = z_{\xi\xi}\xi_x^2 + 2z_{\xi\eta}(\eta_x \xi_x) + z_{\eta\eta}\eta_x^2 + z_\eta \eta_{xx} + z_\xi \xi_{xx}$

$z_{yy} = z_{\xi\xi}\xi_y^2 + 2z_{\xi\eta}(\eta_y \xi_y) + z_{\eta\eta}\eta_y^2 + z_\eta \eta_{yy} + z_\xi \xi_{yy}$

$z_x = z_\xi \xi_x + z_\eta \eta_x$, because $\eta_{xx} = \xi_{xx} = \eta_{yy} = \xi_{yy} = 0$

$x^2 z_{xx} = x^2 z_{\xi\xi} + 2x^2 z_{\xi\eta}(-1) + x^2 z_{\eta\eta}$ many terms will vanish

$-x^2 z_{yy} = (-1)x^2 z_{\xi\xi} + (-1)2x^2 z_{\xi\eta} + (-1)x^2 z_{\eta\eta}$

$2x z_x = 2x z_\xi - 2x z_\eta$ if we add those terms

$\Rightarrow -4x^2 z_{\xi\eta} + 2x z_\xi - 2x z_\eta$

$\Rightarrow z_{\xi\eta} + \frac{1}{2x}(z_\eta - z_\xi) = 0$, $x = \frac{\xi - \eta}{2} \Rightarrow \frac{1}{2x} = \frac{1}{\xi - \eta}$

$\Rightarrow z_{\xi\eta} + \frac{1}{\xi - \eta}(z_\eta - z_\xi)$ is the standard form

3) $\sec^4(y)r - t + (2\tan(y))q = 0$, answer: $b^2 - ac = \sec^4(y) > 0$

where $\sec(y)$ is defined $\Rightarrow$ the equation is hyperbolic $\Rightarrow \frac{-b \pm \sqrt{b^2-ac}}{a} = \frac{\pm \sec^2(y)}{\sec^4(y)} = \frac{1}{\sec^2(y)}$, solve $\frac{dy}{dx} = \frac{-1}{\sec^2(y)}$, $\frac{dy}{dx} = \frac{1}{\sec^2(y)} \Rightarrow$

$\xi = \tan(y) + x$, $\eta = \tan(y) - x$, now we have

$z_{xx} = z_{\xi\xi}\xi_x^2 + 2z_{\xi\eta}(\xi_x\eta_x) + z_{\eta\eta}\eta_x^2 + z_\xi \xi_{xx} + z_\eta \eta_{xx}$

$z_{yy} = z_{\xi\xi}\xi_y^2 + 2z_{\xi\eta}(\xi_y\eta_y) + z_{\eta\eta}\eta_y^2 + z_\xi \xi_{yy} + z_\eta \eta_{yy}$

$z_y = z_\xi \xi_y + z_\eta \eta_y \Rightarrow$

$\Rightarrow z_{xx} = z_{\xi\xi} + 2z_{\xi\eta}(-1) + z_{\eta\eta}$

$z_{yy} = z_{\xi\xi}\sec^4(y) + 2z_{\xi\eta}(\sec^2(y)) + z_{\eta\eta}\sec^4(y) + 2\sec^2(y)\tan(y)(z_\xi + z_\eta)$

now $\Rightarrow \sec^4(y)(z_{xx}) - z_{yy} = -4\sec^4(y) z_{\xi\eta} - 2\sec^2(y)\tan(y)(z_\xi + z_\eta)$ ①

we have $2\tan(y) z_y = (z_\xi + z_\eta)\sec^2(y)\,2\tan(y)$ ②

$\Rightarrow$ if we add ② to ① we get $-4\sec^4(y) z_{\xi\eta} = 0$

$\Rightarrow z_{\xi\eta} = 0 \Rightarrow z = f(\xi) + g(\eta) = f(\tan(y)+x) + g(\tan(y) - x)$ is standard form

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4) $y^2 z_{xx} - 2y z_{xy} + z_{yy} = z_x + 6y$, answer: $b^2 - ac = (-y)^2 - y^2 = 0$

$\Rightarrow$ the equation is parabolic , $\frac{-b}{a} = \frac{y}{y^2} = \frac{-1}{y}$, solving

$\frac{dy}{dx} = \frac{-1}{y} \Rightarrow \frac{1}{2}y^2 + x = \xi$, choose $\eta = y$, now

$z_{xx} = z_{\xi\xi}\xi_x^2 + 2z_{\xi\eta}(\xi_x\eta_x) + z_{\eta\eta}\eta_x^2 + z_\eta \eta_{xx} + z_\xi \xi_{xx}$

$z_{xy} = z_{\xi\xi}\xi_x\xi_y + z_{\xi\eta}(\xi_x\eta_y + \xi_y\eta_x) + z_{\eta\eta}\eta_x\eta_y + z_\eta \eta_{xy} + z_\xi \xi_{xy}$

$z_{yy} = z_{\xi\xi}\xi_y^2 + 2z_{\xi\eta}(\eta_y\xi_y) + z_{\eta\eta}\eta_y^2 + z_\xi \xi_{yy} + z_\eta \eta_{yy}$

$z_x = z_\xi \xi_x + z_\eta \eta_x = z_\xi$

$z_{xx} = z_{\xi\xi}(0) + 2z_{\xi\eta}(0) + z_{\eta\eta}(0) + 0 + 0$

$z_{xy} = z_{\xi\xi}y + z_{\xi\eta}(1) + 0 + 0$

$z_{yy} = z_{\xi\xi}y^2 + 2z_{\xi\eta}(y) + z_{\eta\eta} + z_\xi + 0$

$\Rightarrow y^2 z_{xx} - 2y z_{xy} + z_{yy} - z_x - 6y =$

$y^2 z_{\xi\xi} + \cancel{y} - 2y^2 z_{\xi\xi} - 2\cancel{y} z_{\xi\eta} + z_{\xi\xi}y^2 + 2\cancel{y} z_{\xi\eta} + z_{\eta\eta} - \cancel{z_\xi} - \cancel{z_\xi} - 6y = 0$

$\Rightarrow z_{\eta\eta} - 6y = 0$, $y = \eta \Rightarrow z_{\eta\eta} - 6\eta = 0$ the standard form

$\Rightarrow z_\eta = 3\eta^2 + g(\xi)$ again Integrating with respect to η yields

$z = \eta^3 + g(\xi)\eta + f(\xi)$ where f & g are arbitrary

$\Rightarrow z = y^3 + g(\frac{1}{2}y^2 + x)y + f(\frac{1}{2}y^2 + x)$ is the general solution