

# Homework Assignment 2

## Exercise 1

Decide whether the following statements are tautologies or contradictions or neither.

1.  $(p \rightarrow q) \vee (q \rightarrow p)$ .
2.  $(p \wedge q) \vee (q \rightarrow \neg p)$ .
3.  $(p \vee \neg q) \rightarrow (q \wedge \neg p)$ .

## Exercise 2

. Let  $f$  and  $g$  be functions defined as:

$$f = \{(0, e), (1, a), (2, u), (3, i), (4, o)\}, \text{ and}$$
$$g = \{(a, 4), (e, 1), (i, 0), (o, 3), (u, 2)\}.$$

Show, computationally that  $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$

## Exercise 3

. Using Mathematical Induction, prove for all Natural Numbers  $n$ ,  $\sum_{i=0}^n i^3 = \frac{n^2(n+1)^2}{4}$ .

#### **Exercise 4**

Find the terms  $a_3$ ,  $a_4$ ,  $a_5$ , and  $a_6$  for the recursively defined sequence given by:  $a_0 = -1$ ,  $a_1 = 0$ ,  $a_2 = 1$  and  $a_n = 2(a_{n-1})(a_{n-3}) + (a_{n-2})^2$  for  $n \geq 3$ .

#### **Exercise 5**

For the function  $f: \mathbf{R} \rightarrow \mathbf{R}$  defined as  $f(x) = x^3 + 2$ , show:

- (a)  $f$  is One-To-One
- (b)  $f$  is Onto